“Complementi di Fisica”
Lecture 24

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Trieste, 18-11-2015

In this lecture

- **Contents**
  - **Drift of electrons and holes in practice (numbers…):**
    - **conductivity σ as a function of impurity concentration**
    - **“band bending”: representation of macroscopic electric fields**
  - **Diffusion in practice (numbers…):**
    - **Diffusion coefficient *D***
    - **Induced (“Built-in”) electric field**
    - **Einstein relation between σ and *D***
- **Reference textbooks**
  - **D.A. Neamen, *Semiconductor Physics and Devices*, McGraw-Hill, 3rd ed., 2003, p.154-188 (“5 Carrier Transport Phenomena”)**
  - **R.Pierret, Advanced Semiconductor Fundamental, Prentice Hall, 2nd ed., p.175-215 (“6 Carrier Transport”)**

From the Boltzmann Equation…

- The continuity equations for the electrical current density in semiconductors can be obtained from the Boltzmann equation:

$$\frac{\partial f}{\partial t} = -\vec{v}_g \cdot \vec{\nabla}_{\vec{r}} f - \frac{\vec{F}}{\hbar} \cdot \vec{\nabla}_{\vec{k}} f - \frac{f - f_0}{\tau} \qquad \vec{F} = -e\vec{\mathrm{E}}$$

Force on electrons

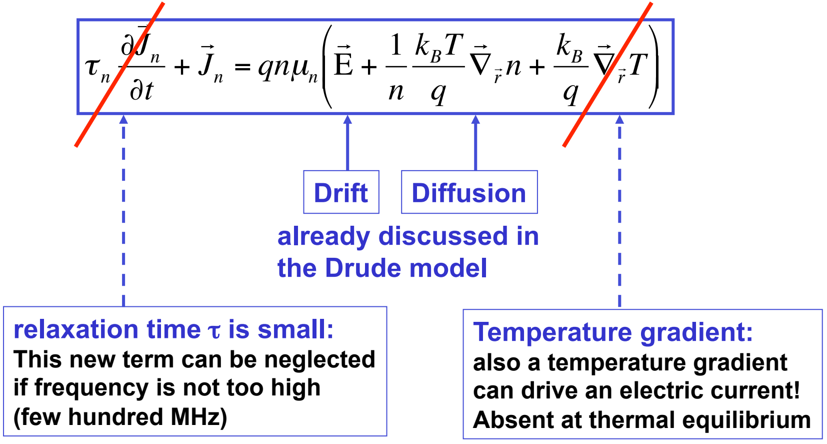
- Multiplying by the group velocity and integrating over the momentum space $dk_x\ dk_y\ dk_z$:

$$\int \vec{v}_g \frac{\partial f}{\partial t} d^3\vec{k} = -\int \vec{v}_g \left(\vec{v}_g \cdot \vec{\nabla}_{\vec{r}} f\right) d^3\vec{k} - \int \vec{v}_g \left(\frac{\vec{F}}{\hbar} \cdot \vec{\nabla}_{\vec{k}} f\right) d^3\vec{k} - \int \vec{v}_g \frac{f - f_0}{\tau} d^3\vec{k}$$

- One obtains the expression for current density (detailed derivation: see FELD p.187-194, MOUT p.100-104)

Drift-diffusion continuity equation

For electrons (similar for holes):



$$\boxed{\cancel{\tau_n \frac{\partial \vec{J}_n}{\partial t}} + \vec{J}_n = qn\mu_n \left(\vec{\mathrm{E}} + \frac{1}{n}\frac{k_B T}{q} \vec{\nabla}_{\vec{r}} n + \cancel{\frac{k_B}{q} \vec{\nabla}_{\vec{r}} T} \right)}$$

Drift **Diffusion**

already discussed in the Drude model

relaxation time τ is small:
This new term can be neglected if frequency is not too high (few hundred MHz)

Temperature gradient:
also a temperature gradient can drive an electric current! Absent at thermal equilibrium

Drift and diffusion: Einstein relation

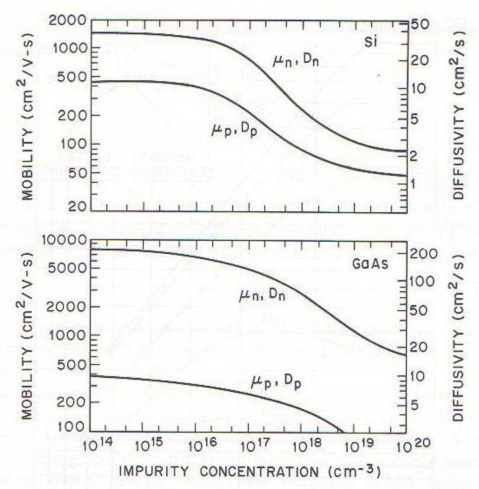
- **Drift (mobility) and diffusion (diffusivity) coefficients are correlated!**

thermal velocity **drift velocity** **external field**

$$D \equiv v_{th} l \qquad v_n = -\mu_n E \qquad v_p = \mu_p E$$

$$\boxed{D_n = \left(\frac{kT}{q}\right)\mu_n \qquad D_p = \left(\frac{kT}{q}\right)\mu_p}$$

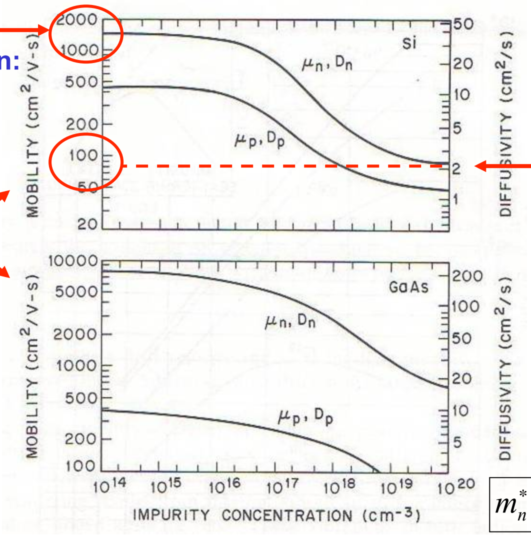
- **Why ?**
  - **See previous lecture**
  - **Also: no current at equilibrium, see next**



Mobility and diffusivity vs. concentration

Low doping concentration: **Lattice scattering limit**

mobility μ (cm^2 V^{-1} s^{-1})



T ~ 300 K

High doping concentration: **Impurity scattering limit**

diffusivity D (cm^2 s^{-1})

Electrons and holes:

$$m_n^* < m_p^* \Rightarrow \mu_n > \mu_p$$

"Band bending": "built-in" electrical field

Non-uniform doping: "built-in" field

p-type doping, thermal equilibrium (no "external" el.field applied!):

$$J_p = |q|\mu_n p\,\mathcal{E} - |q|D_n \frac{dp}{dx} = 0$$

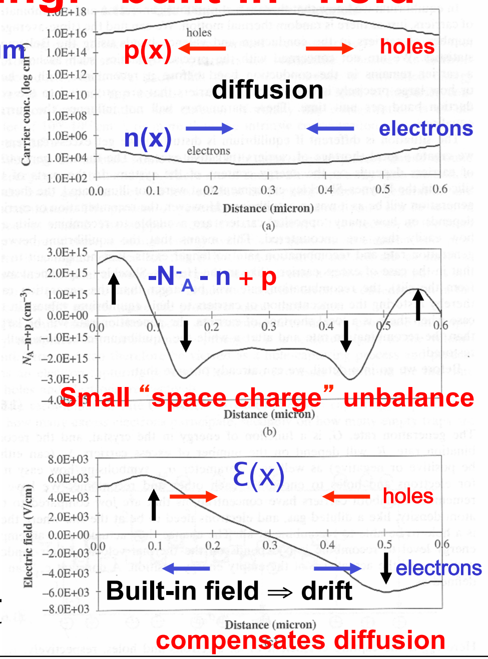
$$= |q|\mu_n p\,\mathcal{E} - |q|\frac{kT}{|q|}\mu_n \frac{dp}{dx} =$$

$$= |q|\mu_n p\left(\mathcal{E} - V_{th}\frac{1}{p}\frac{dp}{dx}\right) = 0$$

$\mathcal{E}$ is the "built-in" electric field:

$$\mathcal{E} = V_{th}\frac{1}{p}\frac{dp}{dx} \approx V_{th}\frac{1}{N_A}\frac{dN_A}{dx}$$

$$V_{th} \equiv \frac{kT}{q}$$ "thermal voltage equivalent"



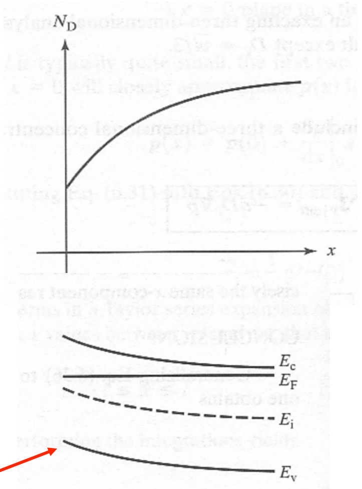
Non-uniform doping in equilibrium

- Under thermal and diffusive equilibrium conditions the Fermi level (= ***total*** chemical potential) inside a material (or group of materials in intimate contact) is invariant as a function of position

$$\frac{dE_F}{dx} = \frac{dE_F}{dy} = \frac{dE_F}{dz} = 0$$

- A non-zero ("built-in") electric field is established in nonuniformly doped semiconductors under equilibrium conditions

$$\mathcal{E}_x = (1/q)(dE_C/dx) =$$
$$= (1/q)(dE_i/dx) =$$
$$= (1/q)(dE_V/dx)$$



"band bending", as for an external field; here the field is "built-in"

Einstein relation - 1

Relation between electron concentration, donor concentration, Fermi level E_F and intrinsic Fermi level E_i in the "quasi-neutrality" approximation (local neutrality cannot be exact, otherwise there would be no built-in field!):

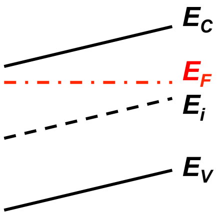
$$n(x) = n_i e^{(E_F - E_i(x))/kT} \approx N_D(x)$$

At equilibrium, E_F is constant ⇒ "band bending":

$$E_F - E_i = kT \ln\left(\frac{N_D(x)}{n_i}\right) \quad \Rightarrow \quad -\frac{dE_i}{dx} = \frac{kT}{N_D(x)} \frac{dN_D(x)}{dx}$$

Induced ("built-in") electric field:

$$\varepsilon_x = -\frac{dV}{dx} = -\frac{1}{-|q|}\frac{dE_i}{dx} = -\frac{kT}{|q|}\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}$$



Einstein relation - 2

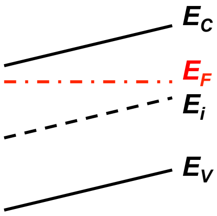
At equilibrium the total electric current (diffusion + drift) must be zero; using for the electric field ε_x the expression just derived one obtains:

$$J_{n,x} = 0 = |q| n \mu_n \varepsilon_x + |q| D_n \frac{dn}{dx} \approx$$

$$\approx |q| N_D(x) \mu_n \varepsilon_x + |q| D_n \frac{dN_D(x)}{dx} =$$

$$= |q| \cancel{N_D(x)} \mu_n \left(-\frac{kT}{|q|} \frac{1}{\cancel{N_D(x)}} \frac{dN_D(x)}{dx} \right) + |q| D_n \frac{dN_D(x)}{dx}$$

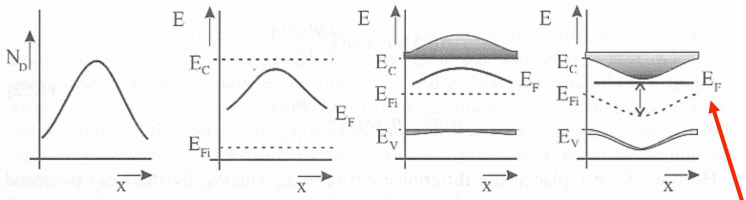
⇒ Einstein relation for electrons (similar for holes):



$$\frac{D_n}{\mu_n} = \frac{kT}{|q|} \qquad \left(\frac{D_p}{\mu_p} = \frac{kT}{|q|} \right)$$

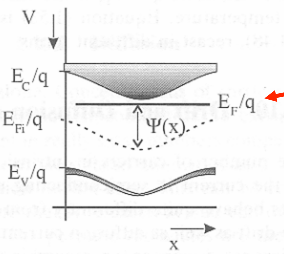
Non-uniform doping: an example

Donor concentration: **"band bending":**



⇒ Built-in electric field:

$\varepsilon_x = (1/q)(dE_i/dx) =$
$= (1/q)(d(E_F-E_i)/dx) =$
$= - d\Psi/dx$

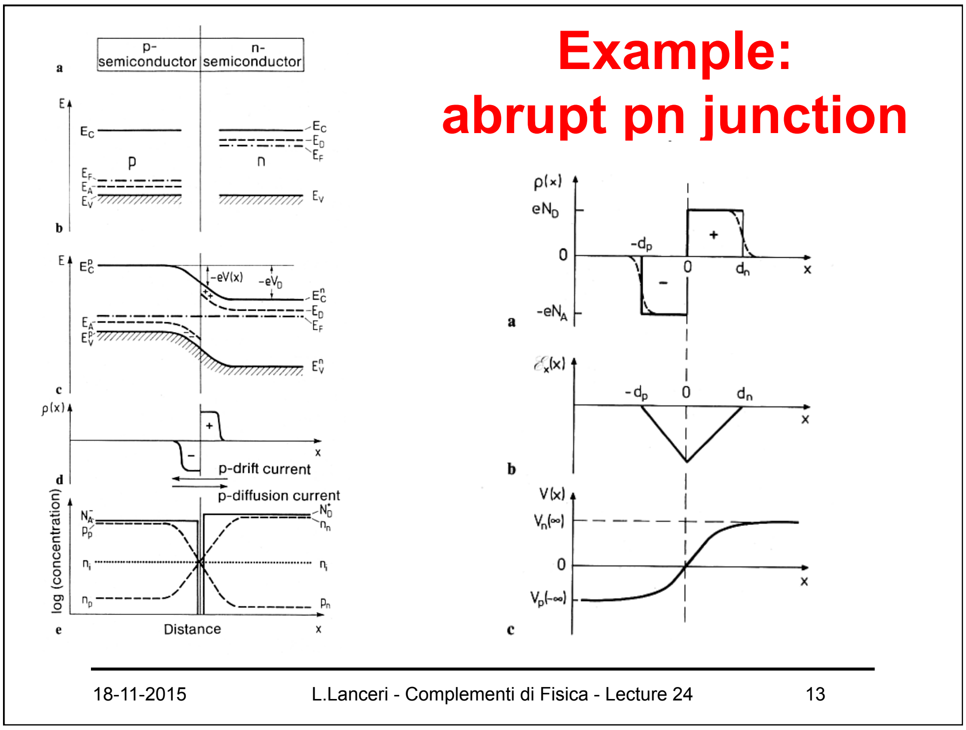


Equilibrium:

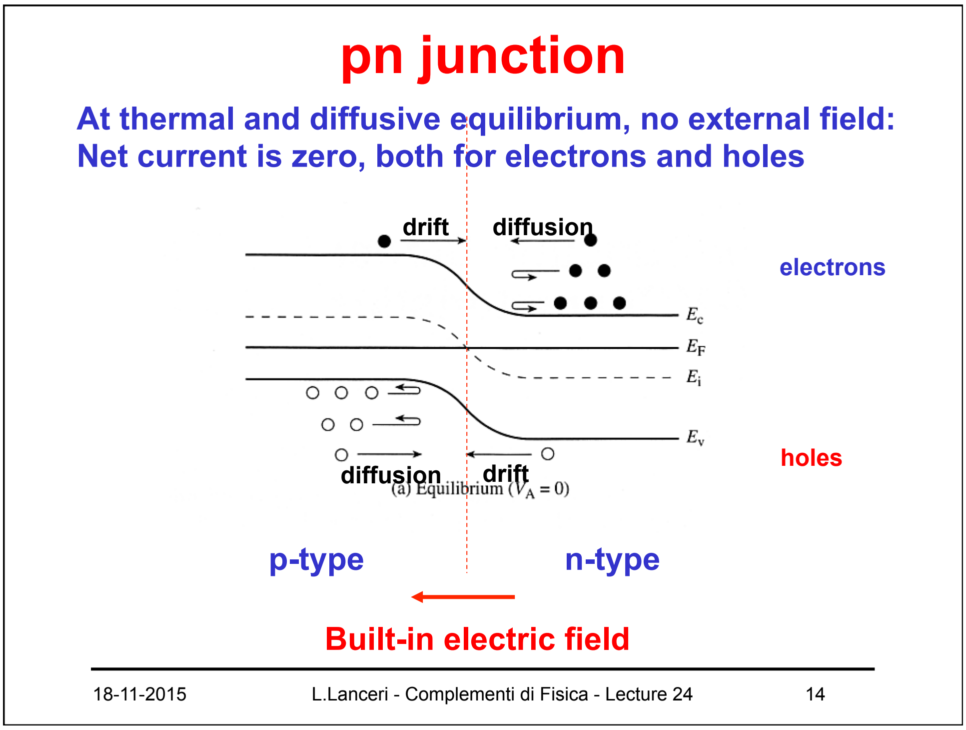
***E_F* = constant**

Electric potential

$V(x) = \Psi(x) = (1/q)(E_F - E_i(x))$



Example: abrupt pn junction



pn junction

At thermal and diffusive equilibrium, no external field:
Net current is zero, both for electrons and holes

Built-in electric field

Einstein relation – numerical examples

- **In the "non-degenerate" limit:**

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \qquad \frac{D_p}{\mu_p} = \frac{kT}{q}$$

- **Typical sizes of mobility and diffusivity:**

$$T = 300K \quad \Rightarrow \quad \frac{kT}{q} \approx 0.026\ \mathrm{V}$$

$$\mu_\mathrm{n} = 1000\ \mathrm{cm}^2/\mathrm{Vs} \quad \Rightarrow \quad D_n \approx 26\ \mathrm{cm}^2/\mathrm{s}$$

Current density equations

- **When an *external* electric field $\mathcal{E}$ is present *in addition* to the concentration gradient:** *no equilibrium!*
  - ***Both* drift and diffusion currents will flow**
  - **The total current density is *different from zero* in this case!**
- **For electrons and holes: $J = J_n + J_p$**

$$J_n = J_{n,drift} + J_{n,diff} = |q|\mu_n n\,\mathcal{E} + |q| D_n \frac{dn}{dx}$$

drift diffusion

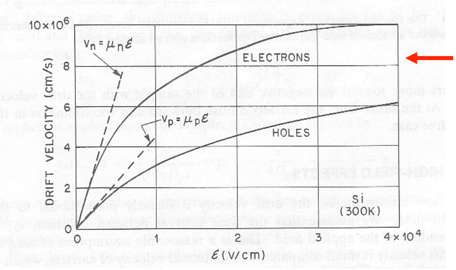
$$J_p = J_{p,drift} + J_{p,diff} = |q|\mu_p p\,\mathcal{E} - |q| D_p \frac{dp}{dx}$$

High field effects

Drift velocity saturation

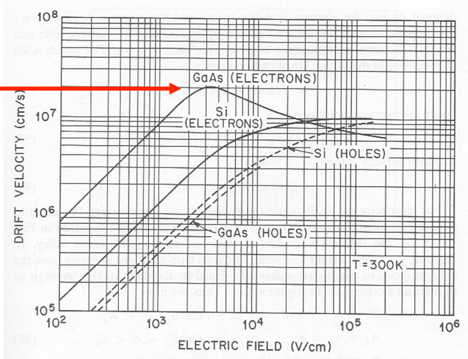
Avalanche processes

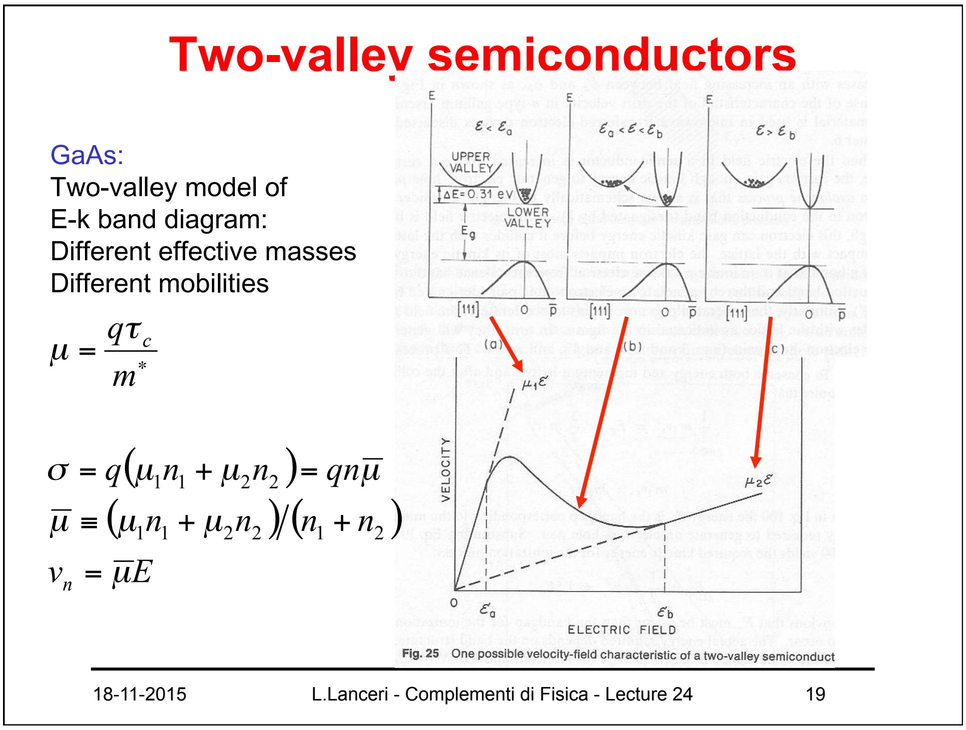
Drift velocity saturation



Silicon:
electrons and holes drift velocity
- increases linearly at small fields
- saturates (~ thermal velocity) at high fields

GaAs:
electrons and holes drift velocity
- Peculiar behavior
- see next slide for an explanation





Two-valley semiconductors

GaAs:
Two-valley model of
E-k band diagram:
Different effective masses
Different mobilities

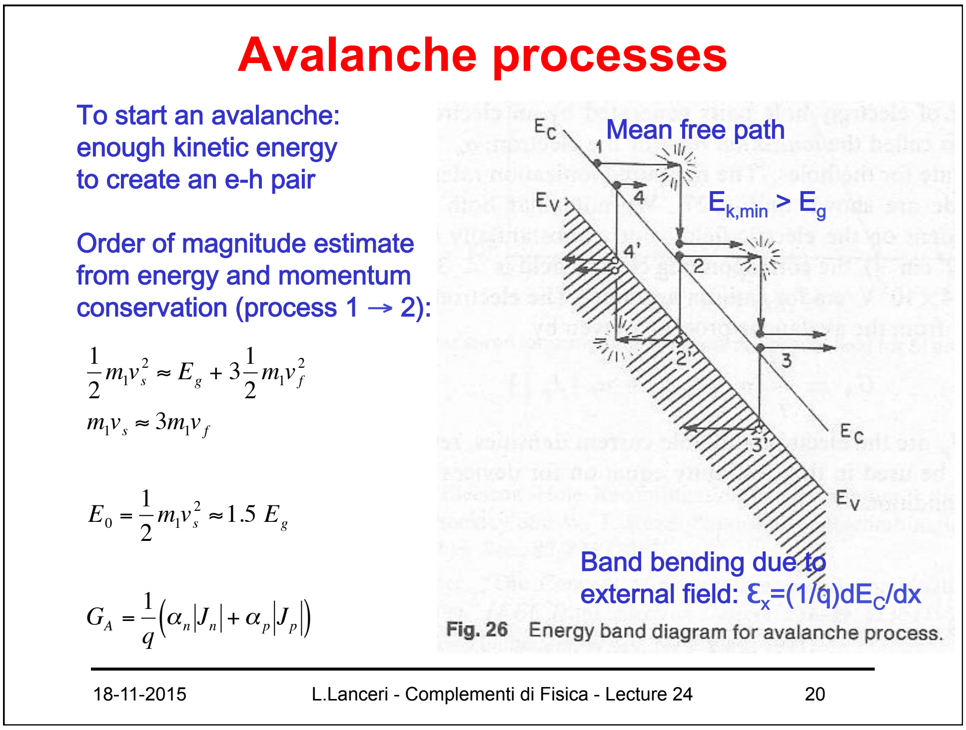
$$\mu = \frac{q\tau_c}{m^*}$$

$$\sigma = q(\mu_1 n_1 + \mu_2 n_2) = qn\overline{\mu}$$

$$\overline{\mu} \equiv (\mu_1 n_1 + \mu_2 n_2)/(n_1 + n_2)$$

$$v_n = \overline{\mu}E$$

Fig. 25 One possible velocity-field characteristic of a two-valley semiconduct



Avalanche processes

To start an avalanche:
enough kinetic energy
to create an e-h pair

Order of magnitude estimate
from energy and momentum
conservation (process 1 → 2):

$$\frac{1}{2}m_1 v_s^2 \approx E_g + 3\frac{1}{2}m_1 v_f^2$$

$$m_1 v_s \approx 3 m_1 v_f$$

$$E_0 = \frac{1}{2}m_1 v_s^2 \approx 1.5\ E_g$$

$$G_A = \frac{1}{q}\left(\alpha_n |J_n| + \alpha_p |J_p|\right)$$

Mean free path

$E_{k,min} > E_g$

Band bending due to external field: $\mathcal{E}_x = (1/q)dE_C/dx$

Fig. 26 Energy band diagram for avalanche process.

Ionization rates and generation rate

α_n, α_p ionization rates (cm^{-1})

e – h generation rate (cm^{-3}s^{-1})

$$G_A = \frac{1}{q}\left(\alpha_n |J_n| + \alpha_p |J_p|\right)$$

In Silicon:
$E_0 = 3.2\ E_g$ (el.)
$E_0 = 4.4\ E_g$ (h.)

$\mathcal{E} \sim 3\times10^5$ V/cm
For $\alpha \sim 10^4$ cm^{-1}

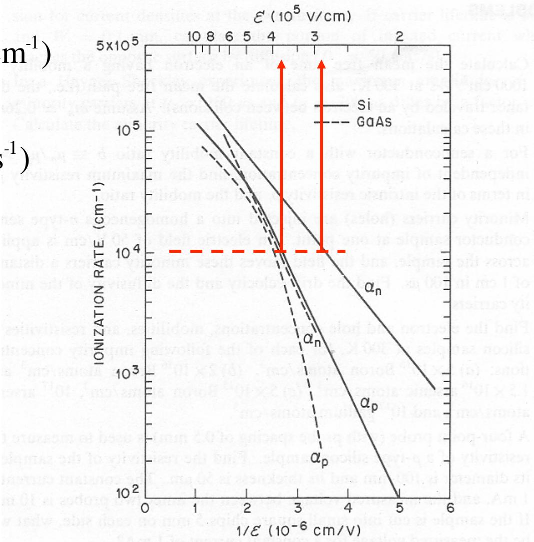


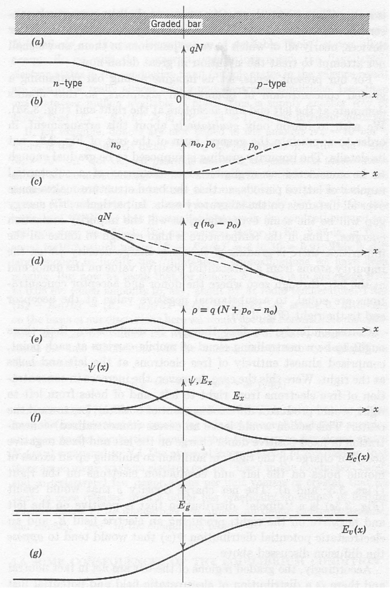
Fig. 27 Measured ionization rates versus reciprocal field for Si and GaAs.[13]

Lecture 24 - exercises

- Exercise 1: **Find the electron and hole concentrations, mobilities and resistivities of silicon samples at 300K, for each of the following impurity concentrations: (a) $5\text{x}10^{15}$ boron atoms/cm^3; (b) $2\text{x}10^{16}$ boron atoms/cm^3 together with $1.5\text{x}10^{16}$ arsenic atoms/cm^3; and (c) $5\text{x}10^{15}$ boron atoms/cm^3, together with 10^{17} arsenic atoms/cm^3, and 10^{17} gallium atoms/cm^3.**
- Exercise 2: **For a semiconductor with a constant mobility ratio $b \equiv \mu_n\mu_p > 1$ independent of impurity concentration, find the maximum resistivity ρ_m in terms of the intrinsic resistivity ρ_i and of the mobility ratio.**
- Exercise 3: **A semiconductor is doped with N_D (N_D>>n_i) and has a resistance R_1. The same semiconductor is then doped with an unknown amount of acceptors $_{NA}$ (N_A>>N_D), yielding a resistance of $0.5R_1$. Find N_A in terms of N_D if the ratio of diffusivities for electrons and holes is $D_n/D_p=50$.**

Back-up slides

Graded np junction



$N = N_D - N_A$

N: net *fixed* charge

n_0, p_0: majority carrier concentrations

net *mobile* charge

net *total* charge

potential Ψ

El.field $\mathcal{E}_x$

electron energy-band diagram