

The Tully-Fisher and Faber-Jackson Relations

There are three important properties for galaxies: radius R , luminosity L , and mass M , which are connected to “observables” with:

$$\theta = R/d \quad (1)$$

$$f = L/(4\pi d^2) \quad (2)$$

$$v^2 = GM/R \quad (3)$$

where θ is the angular diameter, d is the distance galaxy-observer, and the third relation is the virial theorem. The surface brightness I is independent of distance and is given by:

$$I = f/\theta^2 = Lv^4/(4\pi G^2 M^2) \quad (4)$$

and then

$$L = v^4/I \times 1/[4\pi G^2 (M/L)^2] \quad (5)$$

where M/L is the mass-to-light ratio. If **we can assume** that, for a give class of galaxies, the central surface brightness I and the M/L are constant, then

$$L \propto v^4 \quad (6)$$

which is the basis of **the most useful distance indicators in cosmology**.

For galaxy disks, where the rotation velocity is measured from HI 21cm line, the observed relation “the Tully-Fisher relation”, is:

$$L \propto V_{rot}^{3-4}, \quad (7)$$

where the precise exponent depend on the band of observations, e.g L_B or L_R ...

For elliptical parameters, the appropriate velocity is the central velocity dispersion which can be measured from the size of the absorption lines in the stellar spectrum. The “the Faber-Jackson relation” is observed:

$$L \propto \sigma_0^4. \quad (8)$$

However, this relation is very scattered and there is the need to introduce another parameter.

The Fundamental Plane

The observational quantities are: effective radius r_e , luminosity L , mean surface brightness $\langle I \rangle_e$ (called also $\langle \mu_e \rangle$), velocity dispersion σ_V .

From the observational point of view, $\langle I_e \rangle$ correlates with L and σ_V correlates with r_e , but a much better correlation is obtained combining σ_V , $\langle I_e \rangle$, and r_e , i.e.

The origin of the fundamental plane is the following. Assume that ellipticals form a “homologous” family, e.g. dispersion σ_V ; note that L , $\langle I \rangle_e$, $\langle \mu_e \rangle$ are not all independent since they are connected by the typical I profile of elliptical galaxies (de Vaucouleurs law):

$$L = c_1 I_e r_e^2, \quad (9)$$

Moreover, from the virial theorem

$$M = c_2 \frac{\sigma_0^2 r_e}{G}. \quad (10)$$

Combining them, we obtain

$$r_e = c_2/c_1 \times (M/L)^{-1} \sigma_0^2 I_e^{-1}. \quad (11)$$

If one has $M/L \propto L^{\sim 0.2}$, one recovers the observed fundamental plane. Alternatively, one could have constant M/L with a structure which varies relative to one of more of the fundamental variables. This study of this topic is ongoing.