“Complementi di Fisica”
Lectures 25-26

Livio Lanceri
Università di Trieste

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in these lectures

- **Introduction**
  - **Non or quasi-equilibrium: "excess" carriers injection**
  - **Processes for "generation" and "recombination" of carriers**
  - **Continuity equations for carriers**
- **Continuity equations: three important special cases**
  - **Steady-state injection from one side**
    - **"diffusion length" L_p**
  - **Minority carriers recombination at the surface**
    - **diffusion length and "surface recombination velocity" S_{lr}**
  - **The Haynes-Shockley experiment**
    - **Evidence for simultaneous diffusion, drift and recombination**
- **Are we describing the behaviour of *minority* carriers alone? What about *majority* carriers?**
  - **Why are "minorities" important? Some examples…**
  - **Built-in electric field (Gauss!) and *"ambipolar" transport equations***

In these lectures

- **Reference textbooks**
  - **D.A. Neamen, *Semiconductor Physics and Devices*, McGraw-Hill, 3rd ed., 2003, p.189-230 ("6 Nonequilibrium excess carriers in semiconductors")**
  - **R.Pierret, Advanced Semiconductor Fundamental, Prentice Hall, 2nd ed., p.134-174 ("5 Recombination-Generation Processes")**
  - **J.Nelson, The Physics of Solar Cells, Imperial College Press, p. 79-117 (Ch.4, "Generation and Recombination")**

Injection of “excess” carriers

Non-equilibrium!

(in some cases: quasi-equilibrium)

Carrier injection - introduction

- "carrier injection" = process of introducing "excess" carriers in a semiconductor, so that: $\boxed{np > n_i^2}$
  - Optical excitation:
    - shine a light on a semiconductor crystal;
    - if the energy of the photons is $h\nu > E_g$, then
      - Photons absorbed
      - "excess" electron-hole *pairs* are created: $\boxed{\Delta n = \Delta p}$
  - Other methods:
    - Forward-bias a pn junction
    - …
  - In an extrinsic semiconductor, the *relative* effect of $\Delta n = \Delta p$ is *very different for "majority" and "minority" carriers*, since $n \neq p$
  - Let us work out an example (n-type Si, $n_0 > p_0$ at equilibrium)

Carrier injection

Example: n-type Si at 300K

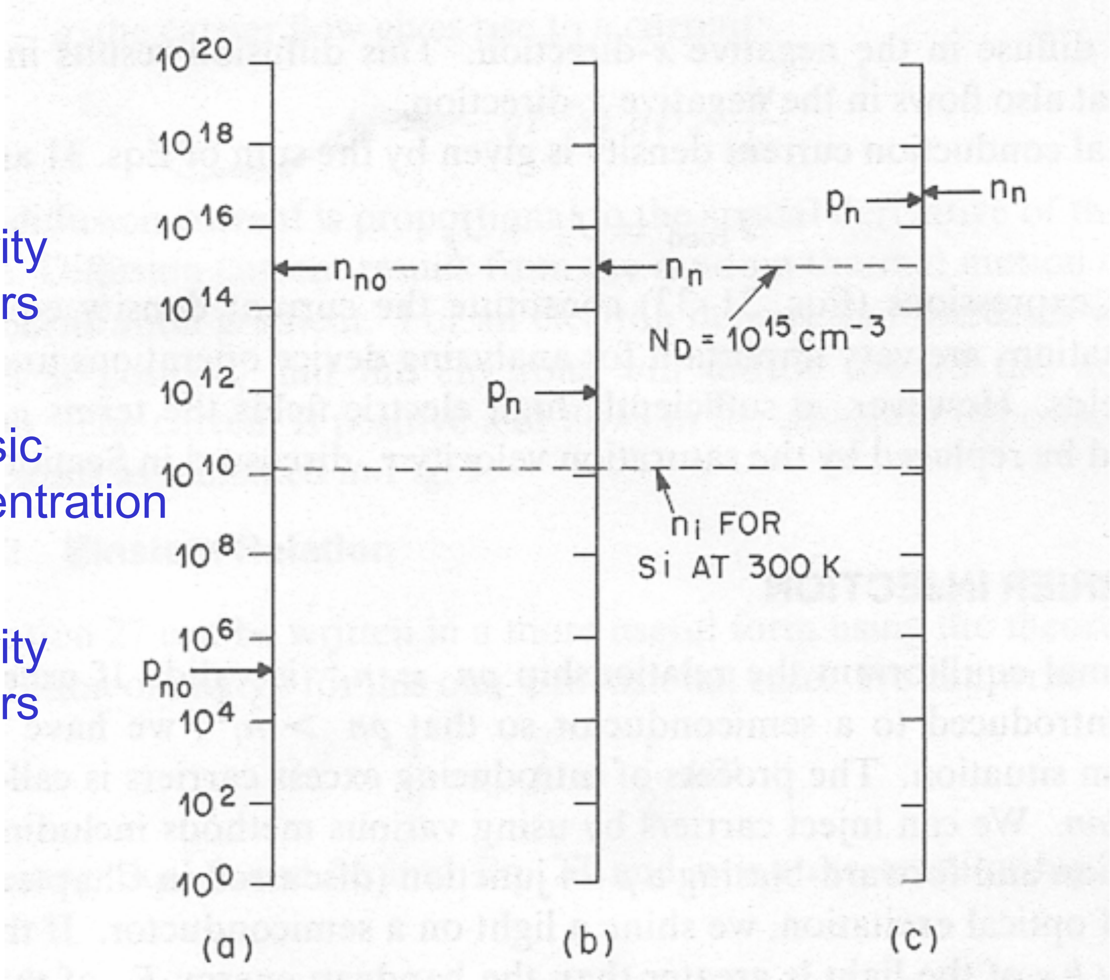
thermal equilibrium

$$n_0 p_0 = n_i^2$$

$$n_i = 1.45\times10^{10}\ \mathrm{cm}^{-3}$$

$$n_0 \approx N_D = 10^{15}\ \mathrm{cm}^{-3}$$

$$p_0 \approx n_i^2 / N_D = 2.1\times10^{5}\ \mathrm{cm}^{-3}$$



thermal equilibrium

low injection

high injection

majority carriers

intrinsic concentration

minority carriers

Carrier injection

Example: n-type Si at 300K

"excess" carriers
low-injection

$$\Delta n = \Delta p = 10^{12}\ \text{cm}^{-3} << N_D$$

$$\Rightarrow np > n_i^2$$

$$n = n_0 + \Delta n \approx n_0$$

$$= 10^{15} + 10^{12} \approx 10^{15}\,\text{cm}^{-3}$$

$$p = p_0 + \Delta p \approx \Delta p$$

$$= 10^{5} + 10^{12} \approx 10^{12}\,\text{cm}^{-3}$$

thermal equilibrium

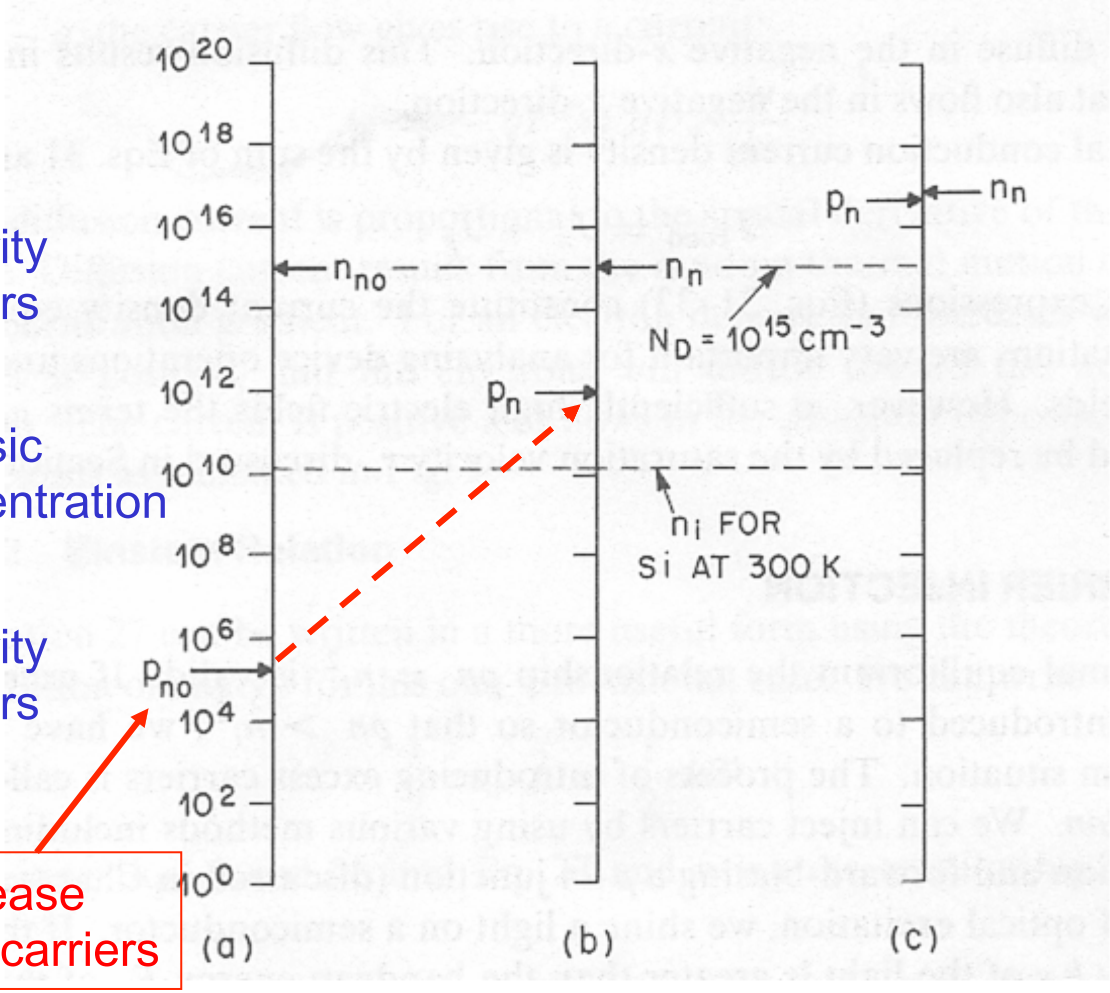
low injection

high injection

majority carriers

intrinsic concentration

minority carriers



Large increase in minority carriers

Carrier injection

Example: n-type Si at 300K

“excess” carriers
high-injection

$$\Delta n = \Delta p > N_D$$

$$\Rightarrow np > n_i^2$$

$$n = n_0 + \Delta n \approx \Delta n$$

$$p = p_0 + \Delta p \approx \Delta p$$

Large increase for both carriers
Similar concentrations

thermal equilibrium

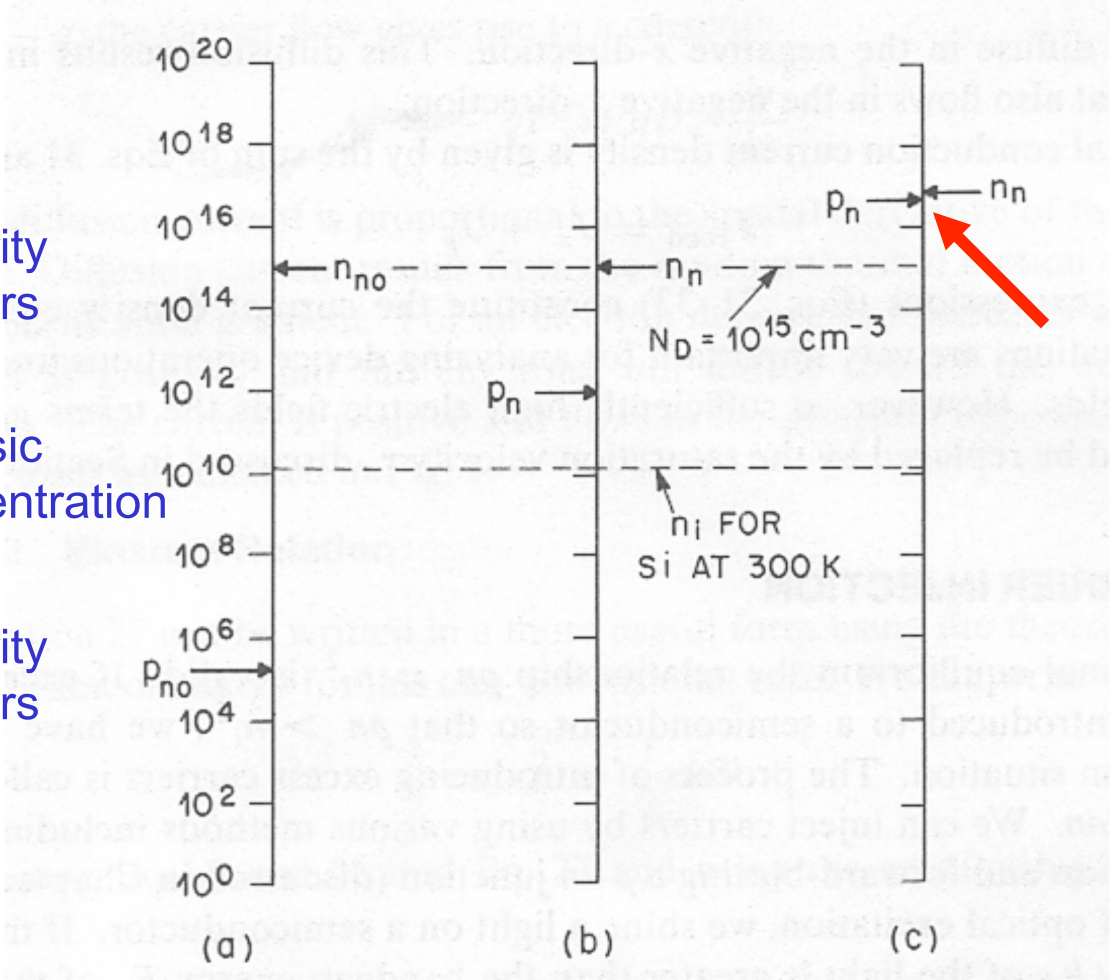
low injection

high injection

majority carriers

intrinsic concentration

minority carriers



Carrier injection - summary

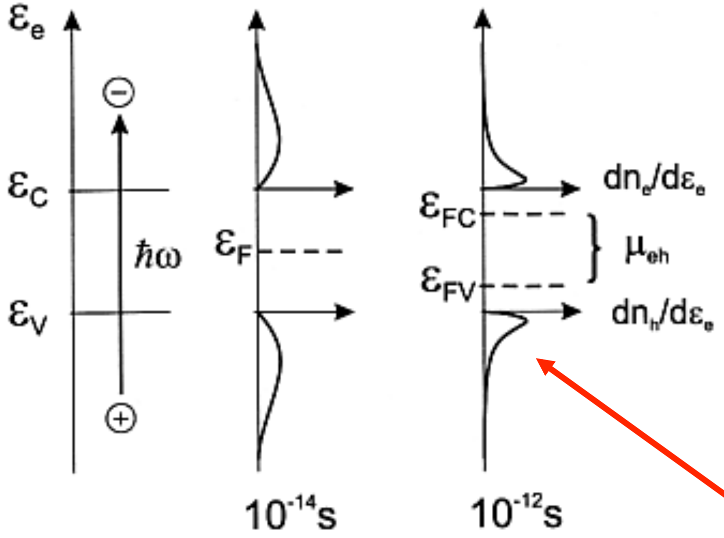
- “carrier injection” = process of introducing “excess” carriers in a semiconductor

- Several methods (optical, etc.)

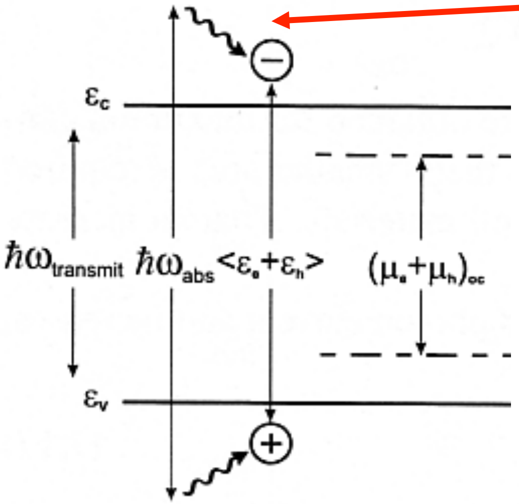
- Low-level injection: relative effect on concentration
  - Negligible on majority carriers
  - Important for minority carriers (also called “minority carriers injection”)

- High-level injection
  - If very high, both concentrations become comparable
  - Sometimes encountered in device operation

Quasi-equilibrium



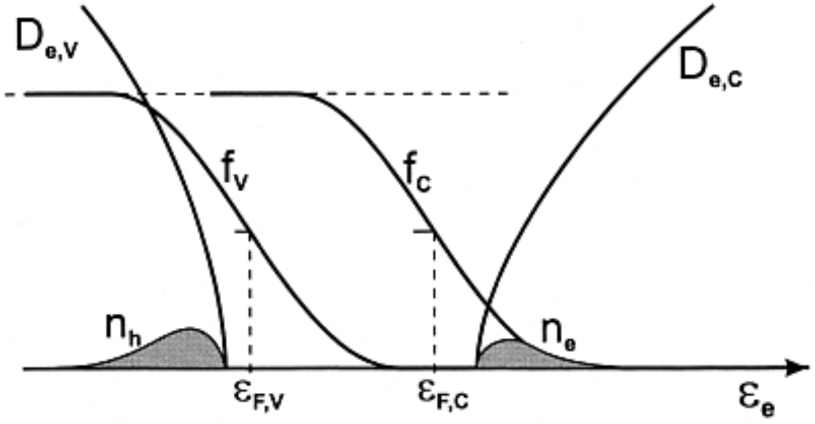
Example: “Injection” or “Generation” of an excess of electrons and holes by absorption of photons in a very short time, *about 10^{-14} s*



Excess electrons and holes relax separately to thermal equilibrium in *about 10^{-12} s* and remain for a much longer time in this quasi-equilibrium state

Recombination of electrons and holes via several possible mechanisms takes typically *about 10^{-6} s*; plenty of time to do something useful with “stable” electrons and holes!

Quasi-equilibrium and quasi-Fermi levels



F_P F_N

- In quasi-equilibrium conditions:
  - T*wo different* "Quasi-Fermi Levels" F_N and F_P, *describe the separate quasi-equilibrium concentrations of electrons and holes,*
  - each population corresponding to a separate Fermi-Dirac pdf (one for electrons, another for holes)

Quasi-Fermi levels: definition

- **In quasi-equilibrium conditions:**
  - Two *different* "Quasi-Fermi Levels" F_N and F_P, *describe the separate quasi-equilibrium concentrations of electrons and holes*:

- **Electrons:**

$$n \equiv n_i e^{(F_N - E_i)/kT} = N_C e^{-(E_C - F_N)/kT} \quad \Leftrightarrow$$

$$F_N \equiv E_i + kT \ln\left(n/n_i\right) = E_C - kT \ln\left(N_C/n\right)$$

- **Holes:**

$$p \equiv n_i e^{(E_i - F_P)/kT} = N_V e^{-(F_N - E_V)/kT} \quad \Leftrightarrow$$

$$F_P \equiv E_i - kT \ln\left(p/n_i\right) = E_V + kT \ln\left(N_V/p\right)$$

Modified mass action law

- **Fermi level E_F at equilibrium:**

$$n_0 = n_i e^{(E_F - E_i)/kT} \qquad p_0 = n_i e^{(E_i - E_F)/kT}$$

$$n_0 p_0 = n_i^2$$

- **Quasi-Fermi levels at quasi-equilibrium:**

$$n \equiv n_i e^{(F_N - E_i)/kT} \qquad p \equiv n_i e^{(E_i - F_P)/kT}$$

$$np = n_i^2 e^{\left(F_N - F_p\right)/kT} > n_i^2$$

Difference in total chemical potentials or quasi-Fermi levels

Quasi-Fermi levels: an example

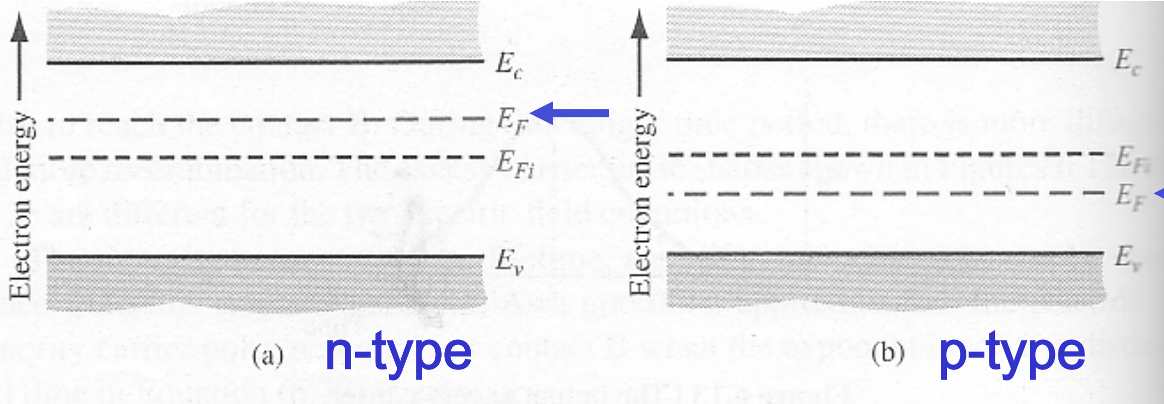


Figure 6.14 | Thermal-equilibrium energy-band diagrams for (*a*) n-type semiconductor and (*b*) p-type semiconductor.

Thermal equilibrium Fermi level E_F

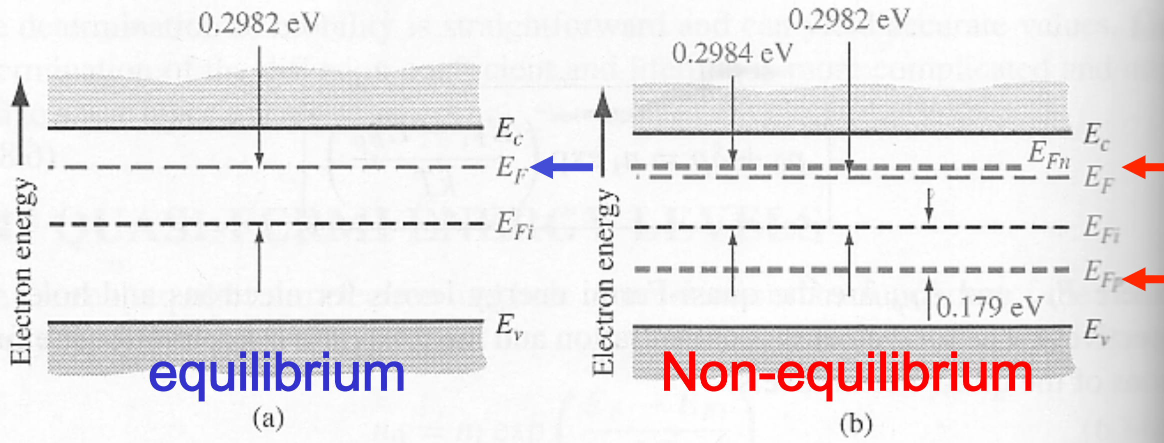


Figure 6.15 | (a) Thermal-equilibrium energy-band diagram for $N_d = 10^{15}$ cm^{-3} and $n_i = 10^{10}$ cm^{-3}. (b) Quasi-Fermi levels for electrons and holes if 10^{13} cm^{-3} excess carriers are present.

Non-equilibrium Quasi-Fermi levels F_N, F_P

Quasi-Fermi levels and currents

- **At (quasi-)equilibrium, (quasi-)Fermi levels are constant**
  - Why? Because from the thermodynamic point of view the **Fermi level** is the **"total" chemical potential**, including the "internal" chemical potential and "external" potential energy contributions, like for instance the electrostatic potential energy.
- **Off-equilibrium, the net movement of carriers is related to the changing *total chemical potential* or *(quasi-)Fermi level* by:**

$$J_{n,x} \equiv \mu_n n \frac{\partial F_N}{\partial x} \qquad\qquad J_{p,x} \equiv \mu_p p \frac{\partial F_P}{\partial x}$$

- **From the definitions of F_N, F_P by substitution one obtains:**

$$J_{n,x} = q\mu_n n E_x + q\boxed{\mu_n \frac{k_B T}{q}} \frac{\partial n}{\partial x} \qquad\qquad J_{p,x} = q\mu_p p E_x - q\boxed{\mu_p \frac{k_B T}{q}} \frac{\partial p}{\partial x}$$

drift **diffusion D_n** **drift** **diffusion D_p**

NB: A quasi-Fermi level that varies with position in a band diagram immediately indicates that current is flowing in the semiconductor! (see exercise 1 for an application)

Generation and Recombination

Charge carriers: electrons and holes

Electrons and holes

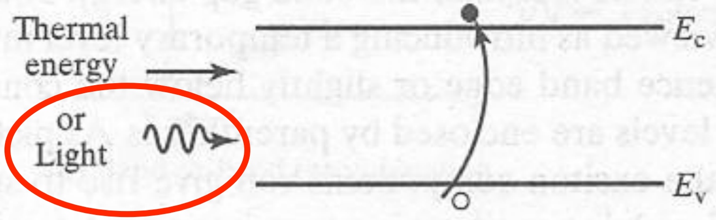
- **Generation rate G :**
  - G = number of free carriers generated (separating electrons from holes) per second and per unit volume
  - G is usually a function of the available energy (temperature, etc.)

- **Recombination rate R:**
  - R = number of free carriers "disappearing" due to recombination per second and per unit volume
  - R is usually proportional to the product of concentrations of "carriers" and "recombination centers" and to a "capture coefficient" defined as $c = v_{th}\sigma$, where v_{th} is the thermal velocity and σ is the recombination process "cross-section"

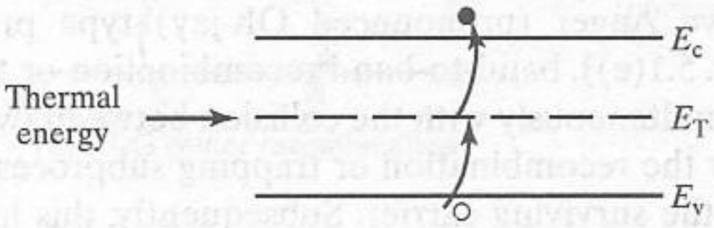
- **Net recombination rate: $U = R - G$**

Generation processes

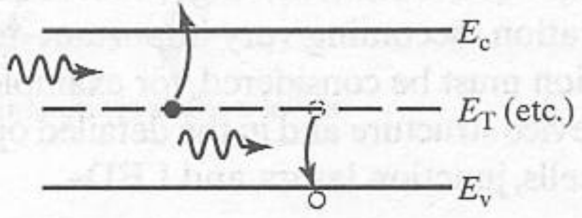
Generation processes



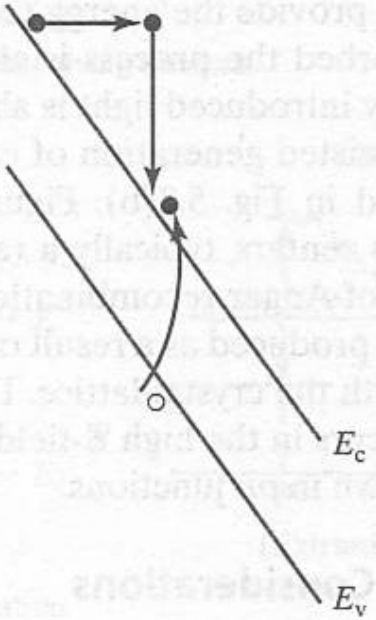
(a) Band-to-band generation



(b) R–G center generation



(c) Photoemission from band gap centers

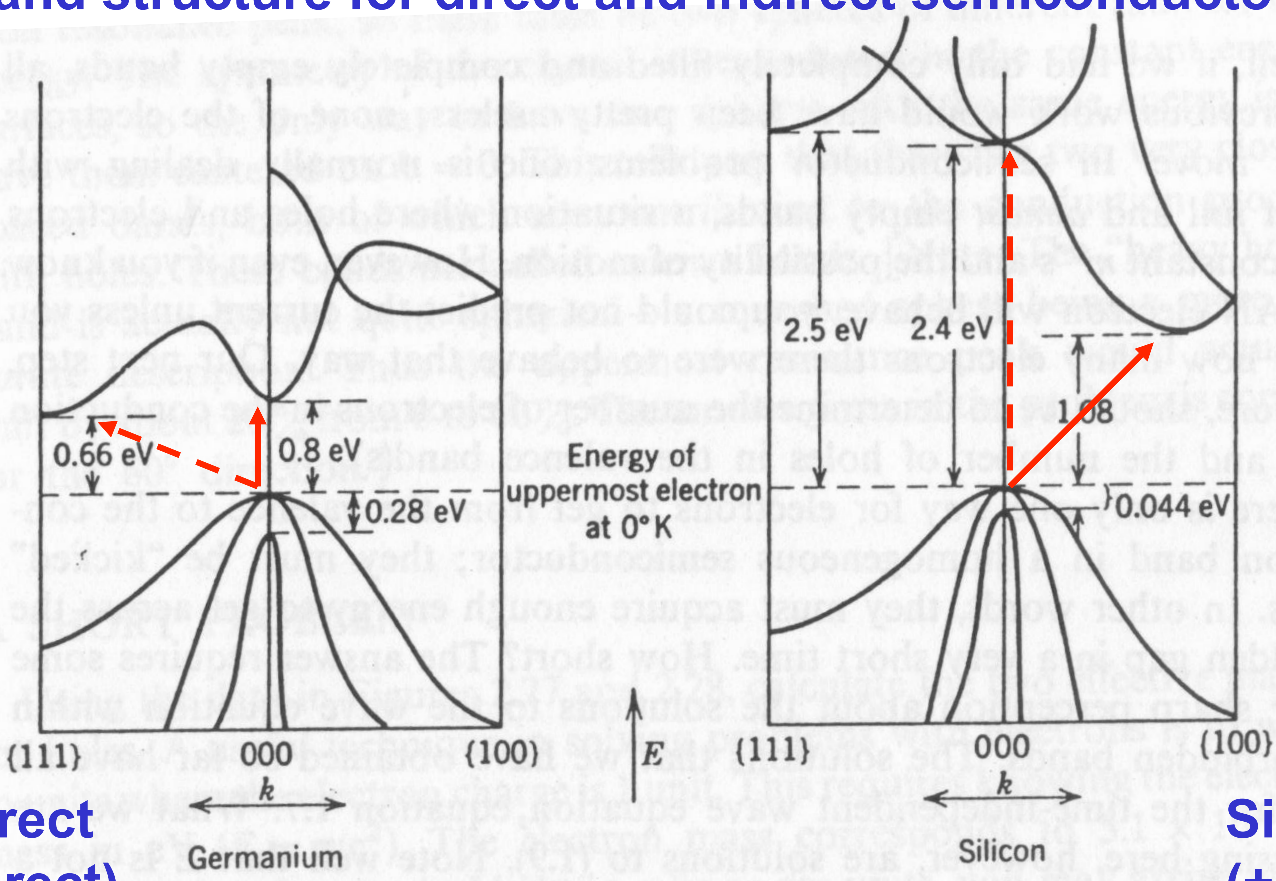


(d) Carrier generation via impact ionization

(for quantitative details: see bibliography)

Radiative (light) generation

Band structure for direct and indirect semiconductors



Ge: direct (+ indirect)

Si: indirect (+ direct)

Figure 2.29 Energy versus wave number along the Cartesian axes and the cube diagonals. Gap energies are for room temperature. (After H. P. R. Frederikse, 9-47 in *The American Institute of Physics Handbook*, second edition, McGraw-Hill, 1963.)

Photon absorption: ingredients

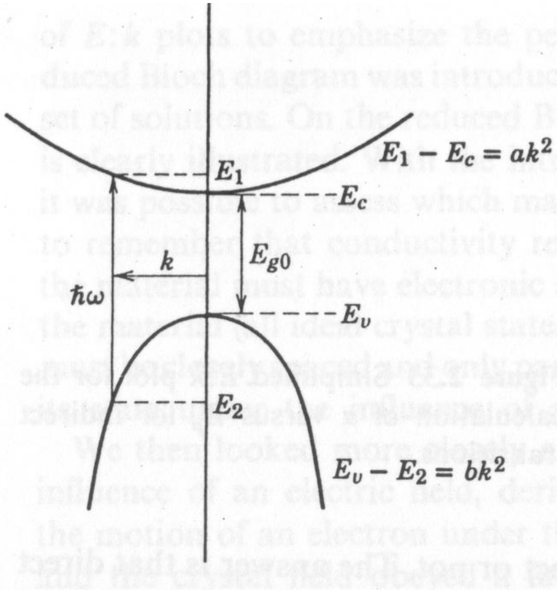


Figure 2.32 Simplified $E{:}k$ plot for the calculation of dN/dE_p for direct transitions.

direct transitions

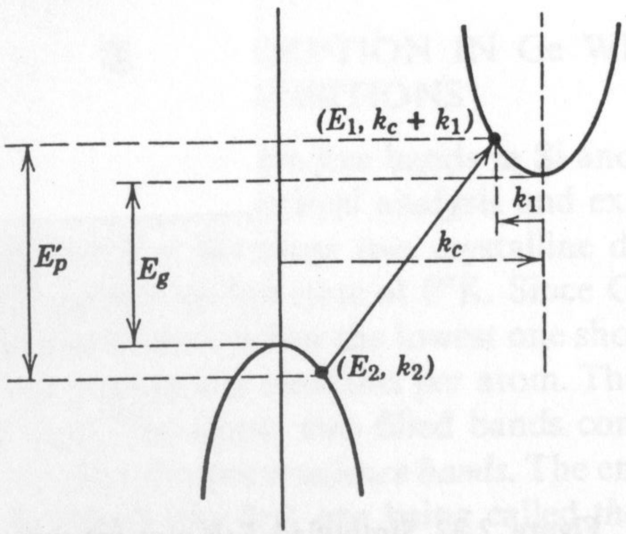


Figure 2.33 Simplified $E{:}k$ plot for the calculation of α versus E_p for indirect transitions.

indirect transitions

Photon absorption coefficient α

92 ELECTRONS AND X RAYS IN CRYSTALS

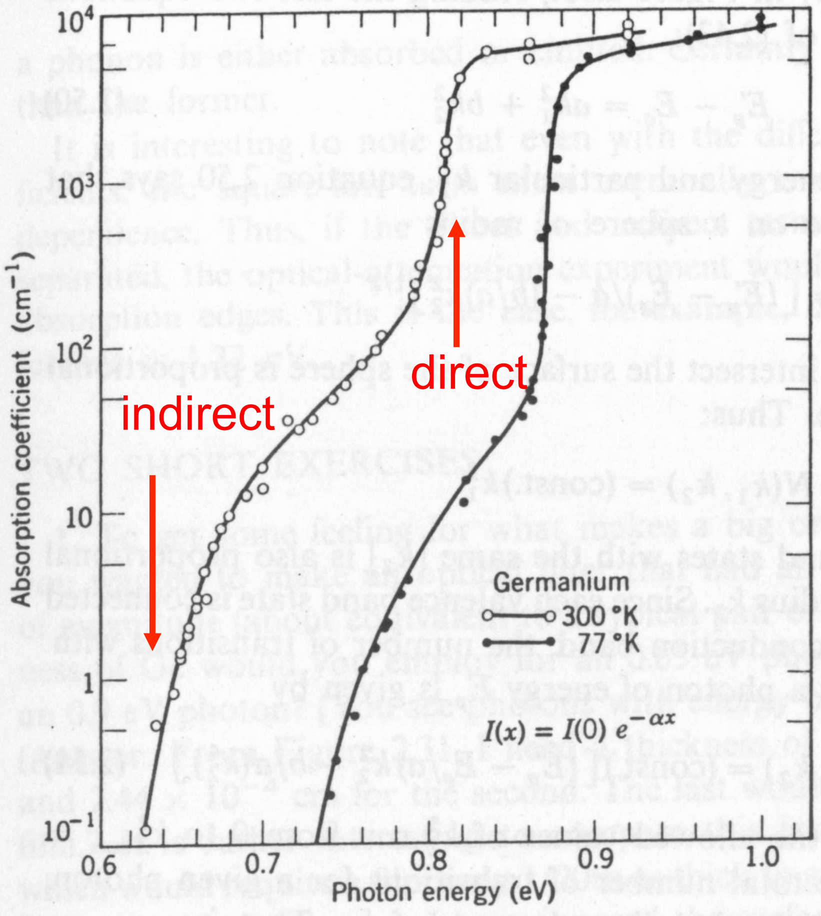
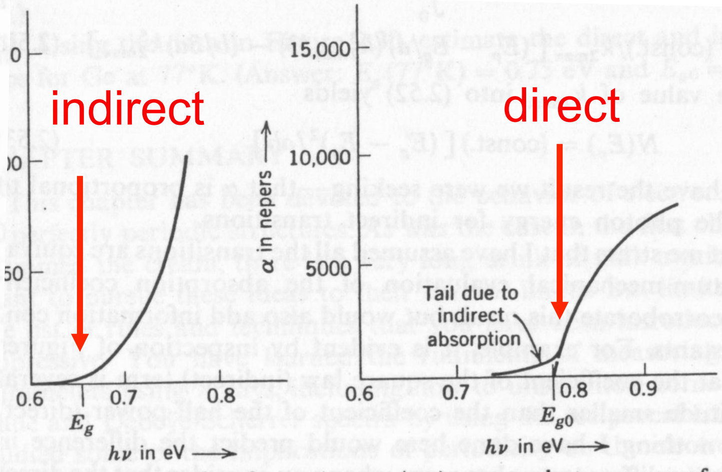
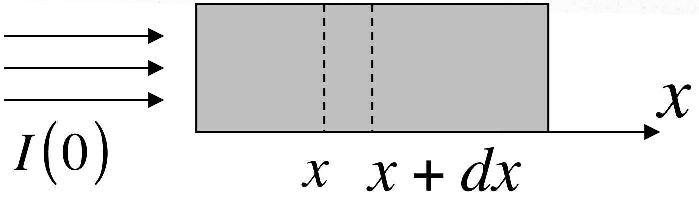


Figure 2.30 Attenuation factor (in nepers/cm) versus photon manium at two temperatures. [After W. C. Dash and R. Ne 1151 (1955).]



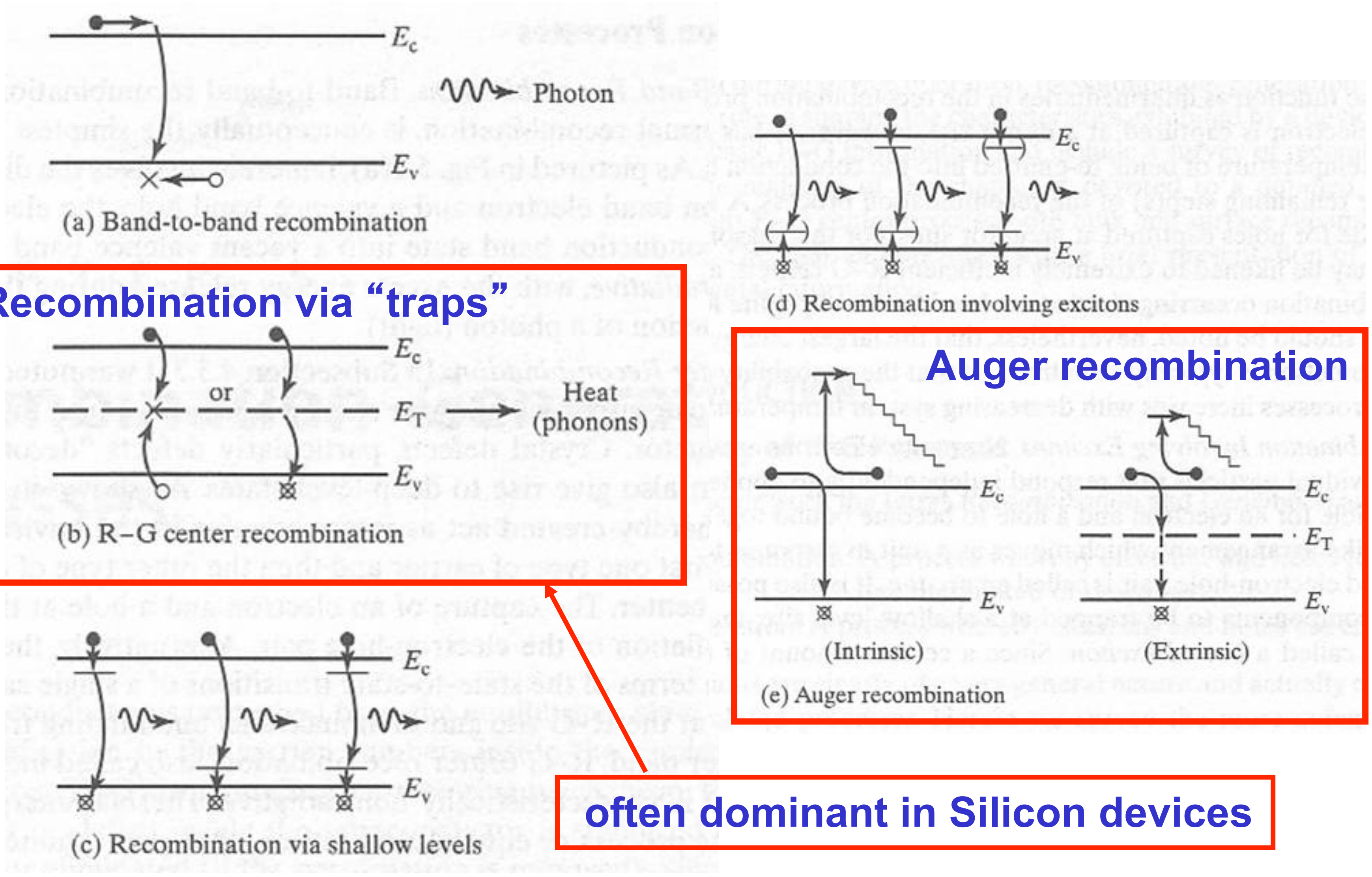
ure 2.31 Attenuation factor (nepers/cm) versus photon energy for Ge (on linear les). Note that for the indirect transition, α is proportional to $(E_p - E_g - E_{\text{phon}})^2$, ile for the direct transition α is proportional to $(E_p - E_{g0})^{1/2}$. Dash et al., loc. cit. previous figure.)



$$\frac{dI}{dx} = -\alpha I(x) \quad \Rightarrow \quad I(x) = I(0)e^{-\alpha x}$$

Recombination processes

Recombination processes



(a) Band-to-band recombination

(b) R–G center recombination

(c) Recombination via shallow levels

(d) Recombination involving excitons

(e) Auger recombination

Recombination via "traps"

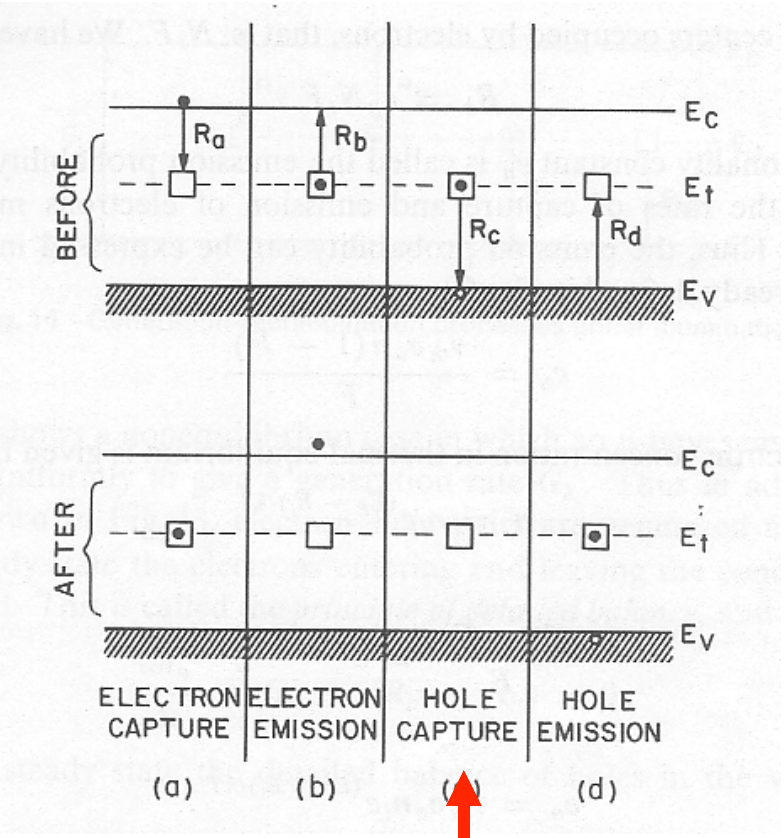
Auger recombination

often dominant in Silicon devices

Recombination via traps

- Recombination: often dominated by indirect processes through "recombination centers" or "traps" (direct recombination is negligible for Si)

- Example: in an n-type semicond., under low-injection conditions:
  - for the minority-carriers (holes !) excess-recombination, the bottleneck is "hole capture", that determines the hole "lifetime" τ_p
  - Once captured, the hole recombines quickly, since there are many electrons available



$$U \approx v_{th}\sigma_p N_t \left(p_n - p_{n0}\right)$$

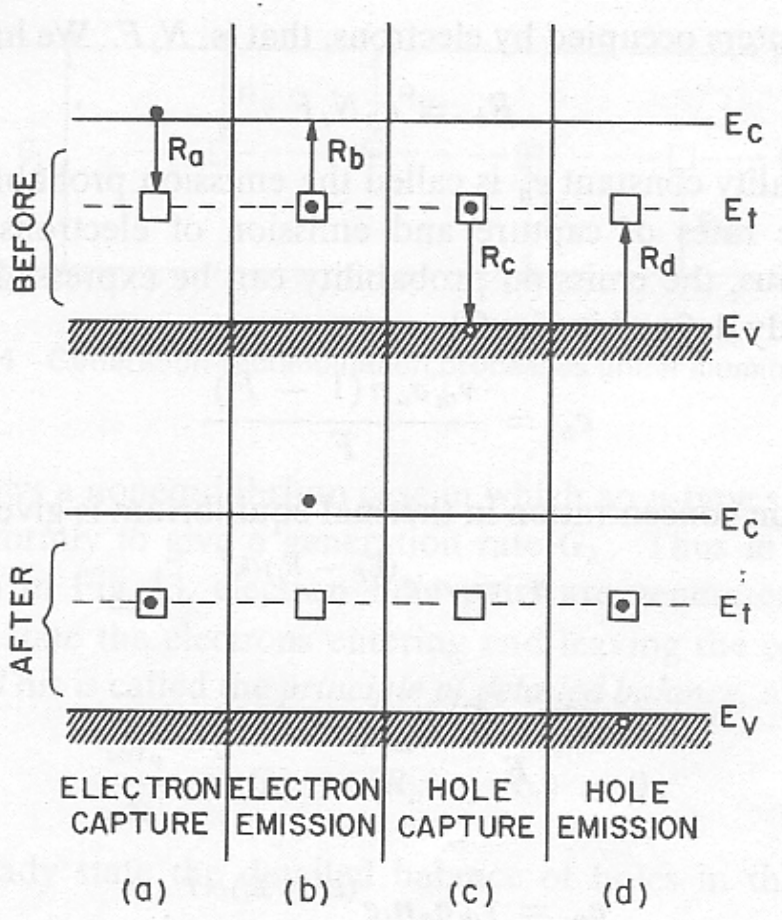
$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t}$$

Recombination via traps: terminology

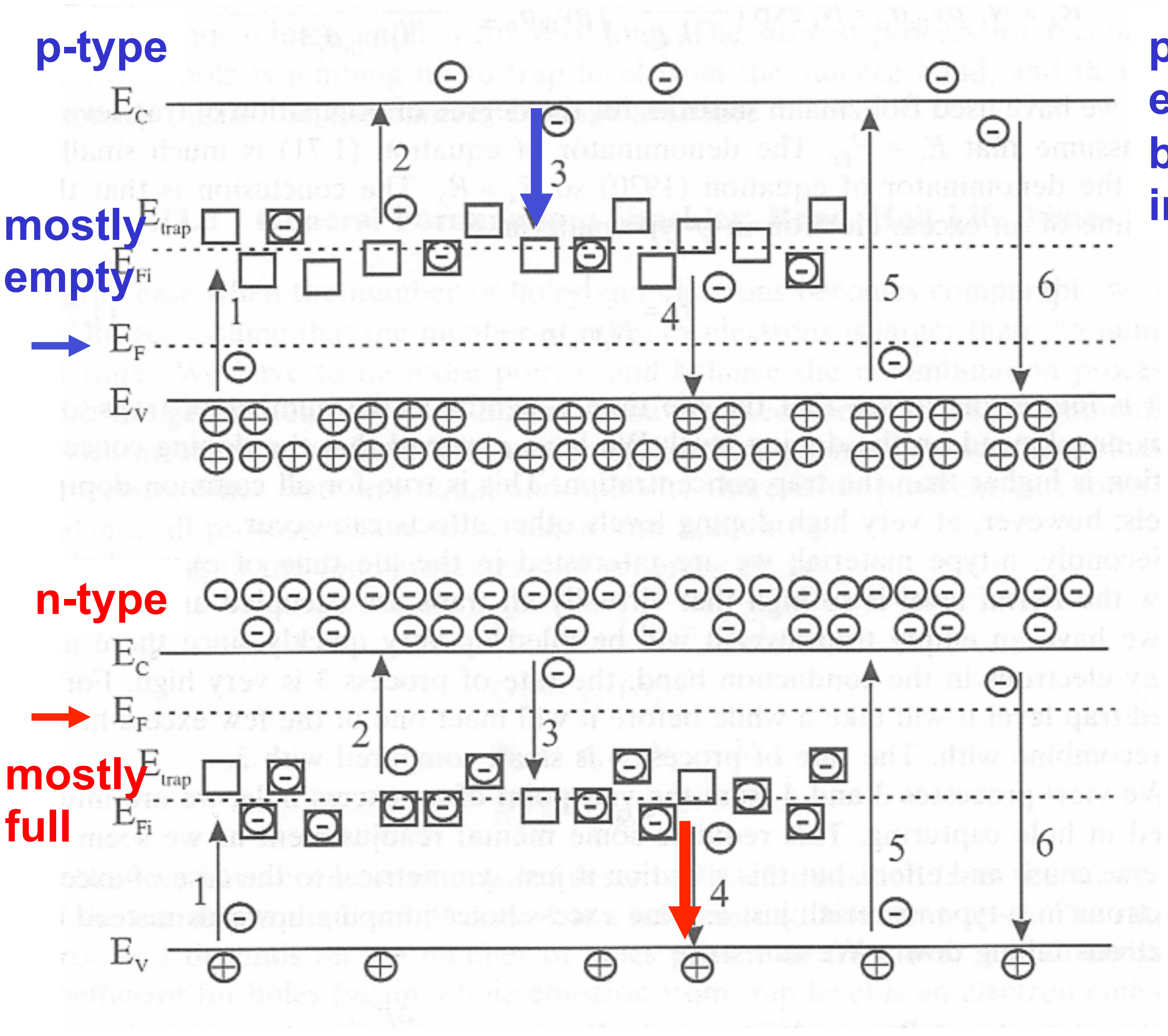
- **“Capture”, “emission”:**
  - **From the point of view of the trap!**

- **In particular (figure, next slide):**
  - **(a) electron capture = (3)**
  - **(b) electron emission = (2)**
  - **(c) hole capture = (4)**
  - **(d) hole emission = (1)**

- **Detailed treatment: beyond our scope!**
  - **Shockley-Read-Hall model**
  - **See back-up slides and reference texts**



Recombination via traps: "lifetime" approximation



p-type semiconductor: electron lifetime dominated by "electron capture" (3) in "empty" RG centers

$$U \approx v_{th}\sigma_n N_t\left(n_p - n_{p0}\right)$$

$$\tau_n \equiv \frac{1}{v_{th}\sigma_n N_t} \approx 1.0\,\mu s \quad (\mathrm{Si})$$

n-type semiconductor: hole lifetime dominated by "hole capture" (4) in "full" RG centers

$$U \approx v_{th}\sigma_p N_t\left(p_n - p_{n0}\right)$$

$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t} \approx 0.3\,\mu s \quad (\mathrm{Si})$$

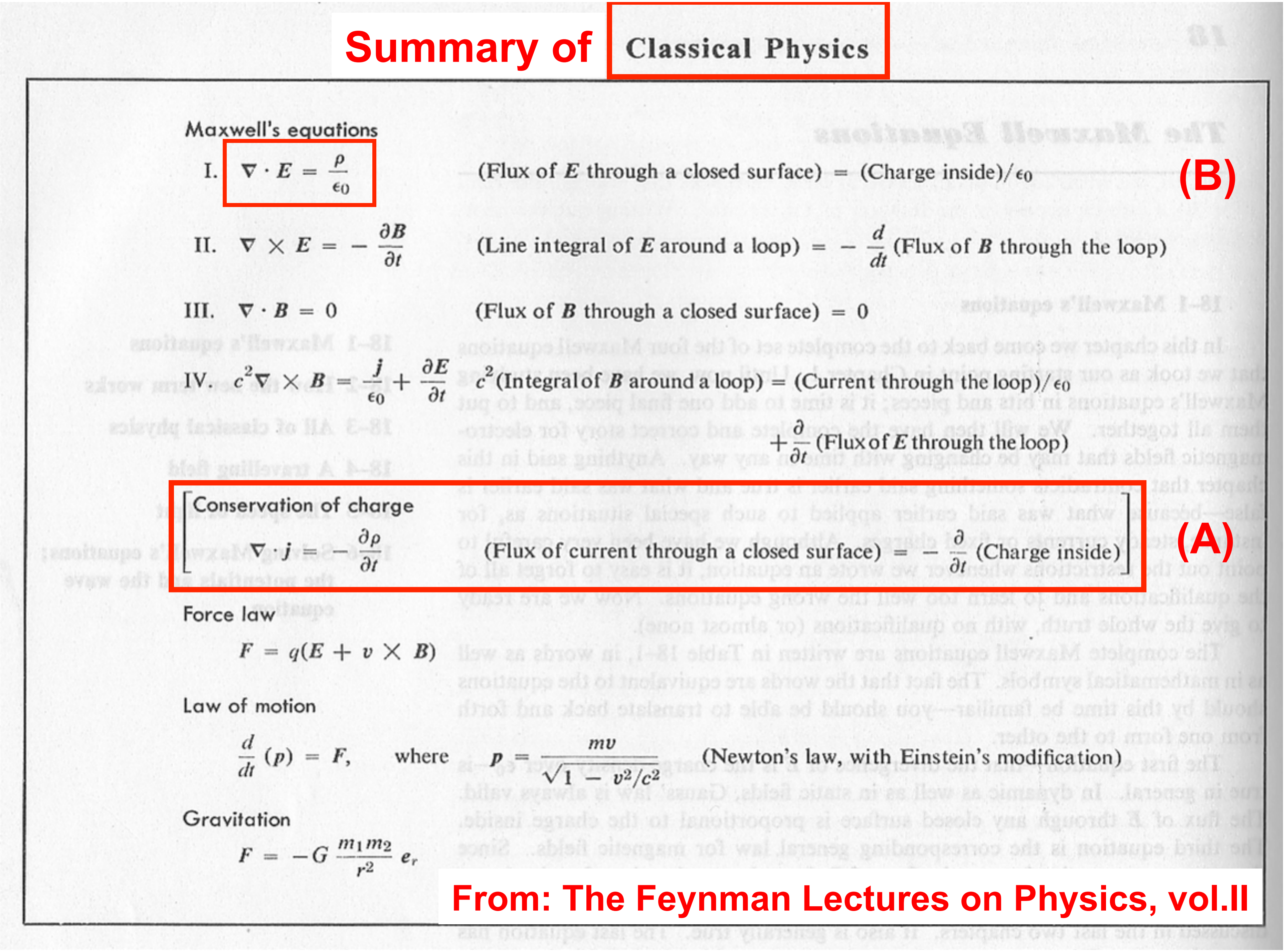
Continuity equations

Overall conservation of charge!

Detailed accounting of local carrier density as a function of time:

Generation, recombination, drift, diffusion

Summary of Classical Physics



Maxwell's equations

I. $\boxed{\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}}$ (Flux of $\boldsymbol{E}$ through a closed surface) = (Charge inside)/ϵ_0 **(B)**

II. $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ (Line integral of $\boldsymbol{E}$ around a loop) = $-\frac{d}{dt}$(Flux of $\boldsymbol{B}$ through the loop)

III. $\nabla \cdot \boldsymbol{B} = 0$ (Flux of $\boldsymbol{B}$ through a closed surface) = 0

IV. $c^2\nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$ c^2(Integral of $\boldsymbol{B}$ around a loop) = (Current through the loop)/ϵ_0 + $\frac{\partial}{\partial t}$(Flux of $\boldsymbol{E}$ through the loop)

Conservation of charge

$\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$ (Flux of current through a closed surface) = $-\frac{\partial}{\partial t}$(Charge inside) **(A)**

Force law

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Law of motion

$\frac{d}{dt}(\boldsymbol{p}) = \boldsymbol{F}$, where $\boldsymbol{p} = \frac{m\boldsymbol{v}}{\sqrt{1 - v^2/c^2}}$ (Newton's law, with Einstein's modification)

Gravitation

$$\boldsymbol{F} = -G\frac{m_1 m_2}{r^2}\boldsymbol{e}_r$$

From: The Feynman Lectures on Physics, vol.II

(A) Conservation of charge: continuity equations

- Any net flow of charge must come from some supply!

$$\oint_S \vec{J} \bullet \hat{n}\, dS = \oint_V \vec{\nabla} \bullet \vec{J} dV = -\frac{d}{dt}\oint_V \rho\, dV = -\frac{dQ}{dt}$$

$$\vec{\nabla} \bullet \vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = -\frac{\partial \rho}{\partial t}$$

  - **The flux of a current from a closed surface is equal to the decrease of the charge inside the surface**
  - **ρ is the net charge density (negative and positive, algebraic sum)**

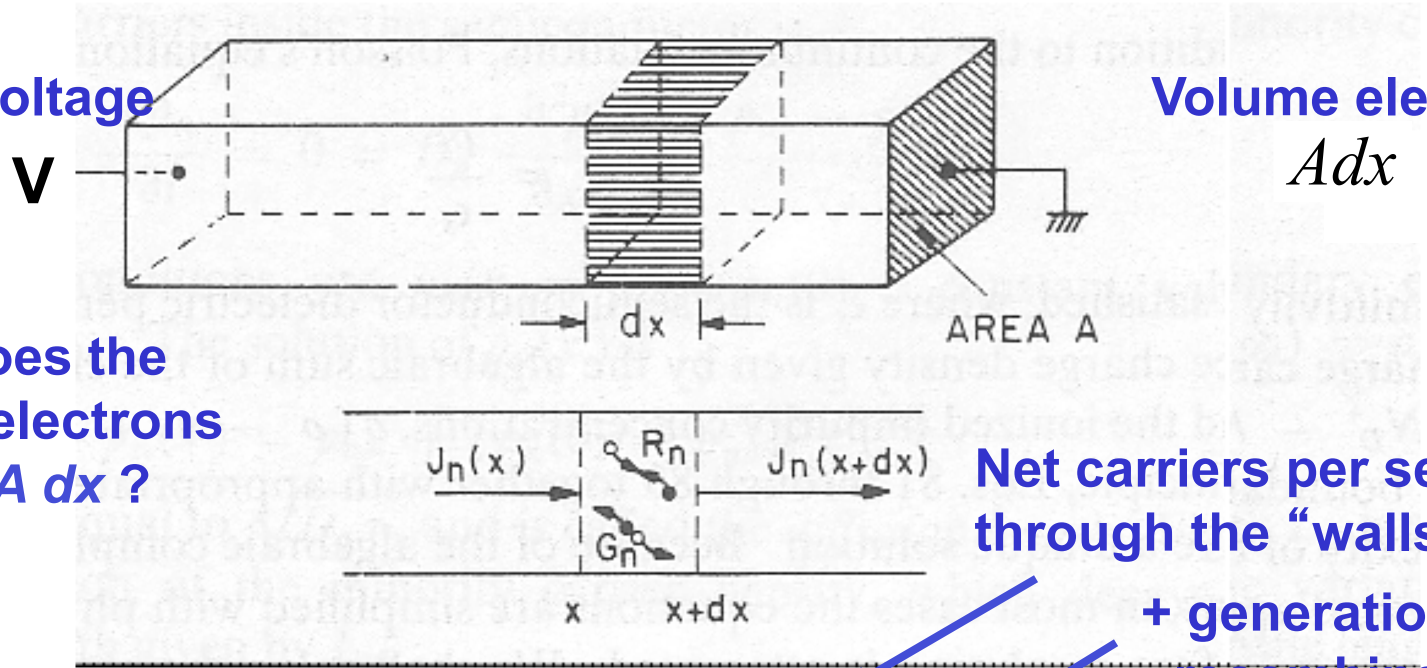
- Let us consider electrons and holes, separately, in a semiconductor, in a simple one-dimensional case

Continuity for electrons

External voltage

Volume element

Adx



How fast does the number of electrons change in *A dx* ?

$$\boxed{\frac{1}{-|q|}\frac{\partial \rho}{\partial t}}$$

Net carriers per second through the "walls"

+ generation

- recombination

$$\frac{\partial n}{\partial t}Adx = \left[\frac{J_n(x)A}{-|q|} - \frac{J_n(x+dx)A}{-|q|}\right] + (G_n - R_n)Adx$$

Substituting:

$$J_n(x+dx) = J_n(x) + \frac{\partial J_n}{\partial x}dx + \ldots$$

and dividing by *A dx*

⇒ see next page…

Continuity for electrons and holes

One-dimensional

$$\frac{\partial n}{\partial t} = \frac{1}{|q|}\frac{\partial J_n}{\partial x} + \left(G_n - R_n\right)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{|q|}\frac{\partial J_p}{\partial x} + \left(G_p - R_p\right)$$

Three-dimensional

$$\frac{\partial n}{\partial t} = \frac{1}{|q|}\vec{\nabla} \bullet \vec{J}_n + \left(G_n - R_n\right)$$

$$\frac{\partial p}{\partial t} = -\frac{1}{|q|}\vec{\nabla} \bullet \vec{J}_p + \left(G_p - R_p\right)$$

Continuity for electrons and holes

Minority carriers:

Electrons in p-type

$$J_{n,x} = n_p |q| \mu_n E_x + D_n \frac{\partial n_p}{\partial x}$$

holes in n-type

$$J_{p,x} = p_n |q| \mu_p E_x - D_p \frac{\partial p_n}{\partial x}$$

One-dimensional, under low-injection conditions, for minority carriers:

(electric field)

Electrons: n_p in p-type

$$\frac{\partial n_p}{\partial t} = n_p \mu_n \frac{\partial E_x}{\partial x} + \mu_n E_x \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}$$

holes: p_n in n-type

$$\frac{\partial p_n}{\partial t} = -p_n \mu_p \frac{\partial E_x}{\partial x} - \mu_p E_x \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{n0}}{\tau_p}$$

minority carrier excess

minority carrier lifetime

Recombination rate R

Simply substitute J=J(drift)+J(diffusion)…

Continuity:
generation, recombination,
drift, diffusion

Summary and real-life applications

Generation, Recombination, Continuity - 1

Net recombination rate (2)

$$U_n = R_n - G_n$$
$$U_p = R_p - G_p$$

For instance Photogener. G_L

minority excess Δn_p (Δn_p); approximations:

1-dimensional, Electric field~0,

Uniform doping $n_0 \neq n_0(x)$, $p_0 \neq p_0(x)$,

Low-injection, photogeneration GL only

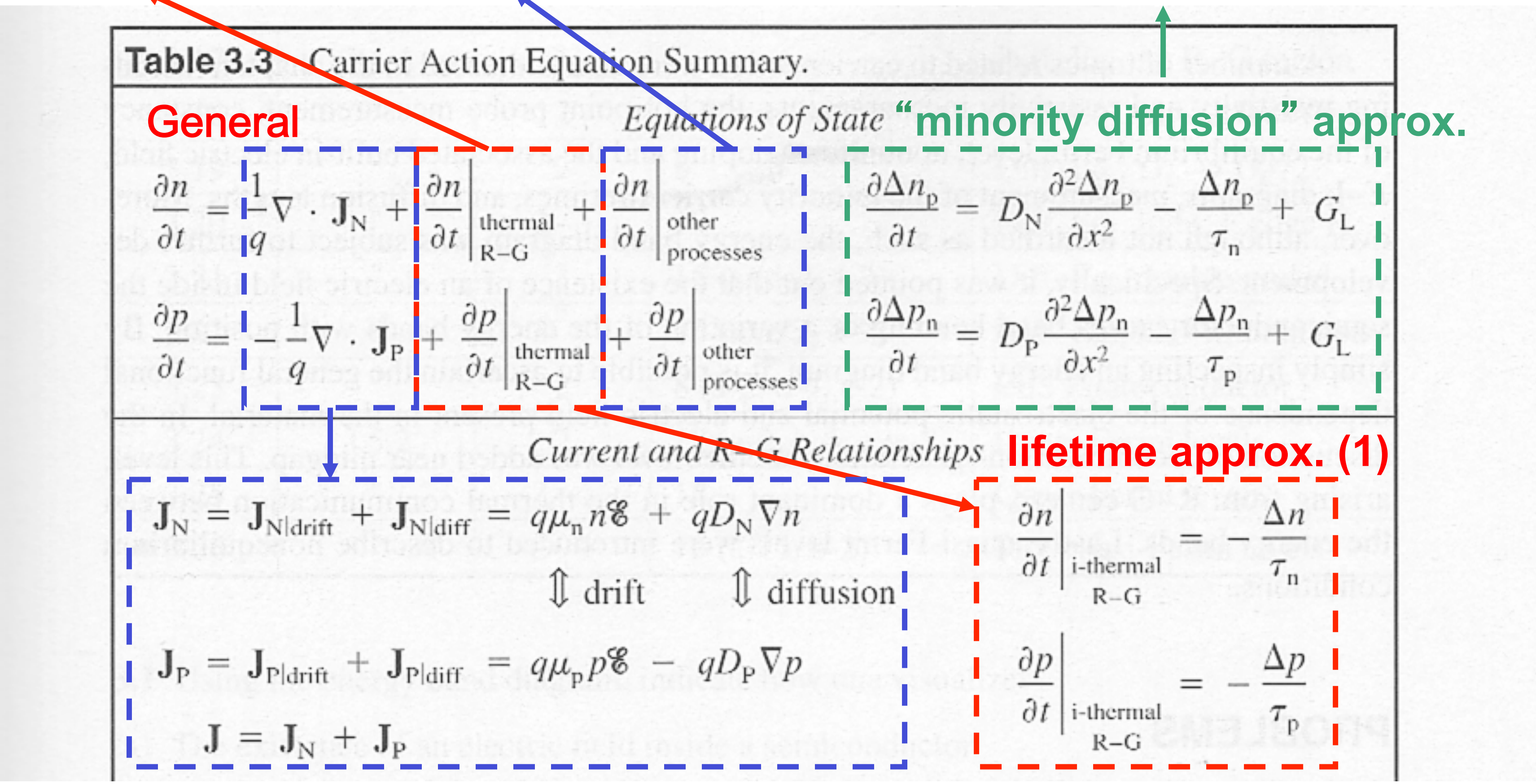


Table 3.3 Carrier Action Equation Summary.

Equations of State

General

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot \mathbf{J}_N + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot \mathbf{J}_P + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

"minority diffusion" approx.

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

Current and R–G Relationships

$$\mathbf{J}_N = \mathbf{J}_{N|\text{drift}} + \mathbf{J}_{N|\text{diff}} = q\mu_n n\mathscr{E} + qD_N \nabla n$$

$$\Updownarrow \text{ drift} \qquad \Updownarrow \text{ diffusion}$$

$$\mathbf{J}_P = \mathbf{J}_{P|\text{drift}} + \mathbf{J}_{P|\text{diff}} = q\mu_p p\mathscr{E} - qD_P \nabla p$$

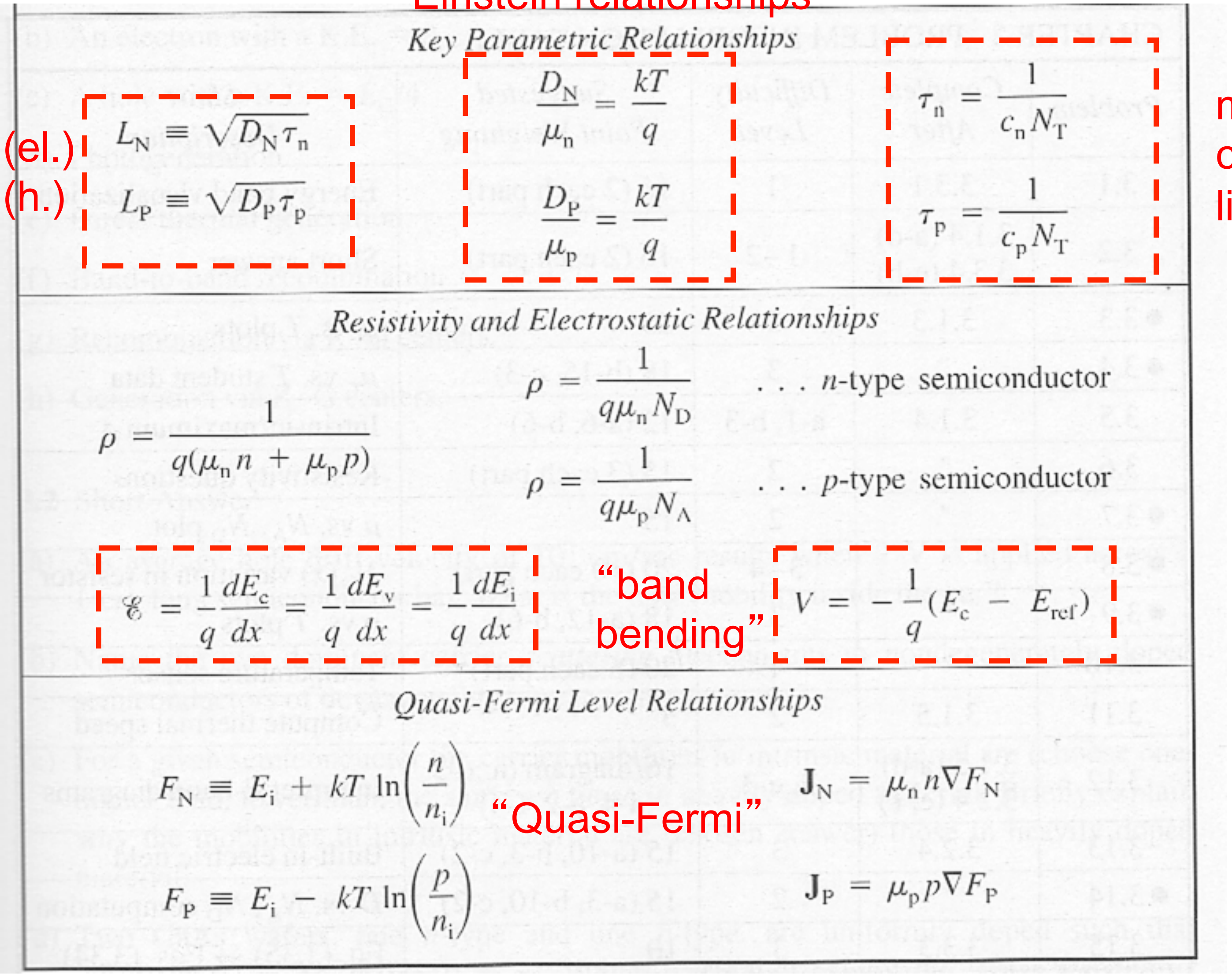
$$\mathbf{J} = \mathbf{J}_N + \mathbf{J}_P$$

lifetime approx. (1)

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_n}$$

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\ \text{R-G}}} = -\frac{\Delta p}{\tau_p}$$

Generation, Recombination, Continuity - 2



Einstein relationships

Key Parametric Relationships

minority carriers diffusion lengths

(el.) $L_N \equiv \sqrt{D_N \tau_n}$

(h.) $L_P \equiv \sqrt{D_P \tau_p}$

$$\frac{D_N}{\mu_n} = \frac{kT}{q}$$

$$\frac{D_P}{\mu_p} = \frac{kT}{q}$$

$$\tau_n = \frac{1}{c_n N_T}$$

$$\tau_p = \frac{1}{c_p N_T}$$

minority carrier lifetimes

Resistivity and Electrostatic Relationships

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$\rho = \dfrac{1}{q\mu_n N_D}$ . . . *n*-type semiconductor

$\rho = \dfrac{1}{q\mu_p N_A}$ . . . *p*-type semiconductor

$$\mathscr{E} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

"band bending"

$$V = -\frac{1}{q}(E_c - E_{ref})$$

Quasi-Fermi Level Relationships

$$F_N \equiv E_i + kT \ln\left(\frac{n}{n_i}\right) \qquad \mathbf{J}_N = \mu_n n \nabla F_N$$

"Quasi-Fermi"

$$F_P \equiv E_i - kT \ln\left(\frac{p}{n_i}\right) \qquad \mathbf{J}_P = \mu_p p \nabla F_P$$

Device simulations

- **In real life, device designers use programs performing numerical integrations in discrete space and time steps, to obtain (*):**

Process simulator

doping N_D, N_A *profiles*

+ boundary conditions, external fields and "excitations"

Device simulator

n, p

el. field

$\vec{J}_n, \vec{J}_p$

(*) carrier concentrations, fields, currents in:
(1) equilibrium,
(2) steady state,
(3) transients

$$\vec{\nabla} \bullet \vec{E} = -\nabla^2 V = \frac{\rho}{\varepsilon}$$

$$\tilde{n} = -n + p - N_A + N_D$$

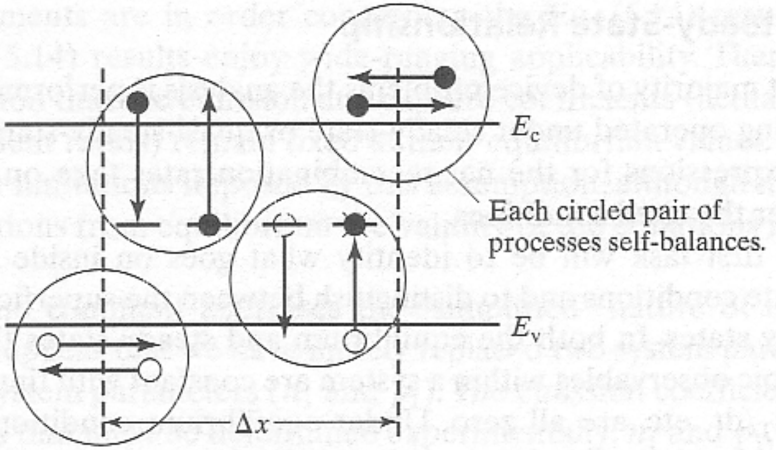
$$\vec{J}_n = -qn\mu_n \vec{\nabla} V + qD_n \vec{\nabla} n$$

$$\vec{J}_p = -qn\mu_n \vec{\nabla} V - qD_n \vec{\nabla} n$$

$$\frac{1}{q} \vec{\nabla} \bullet \vec{J}_n = (R - G) + \frac{\partial n}{\partial t}$$

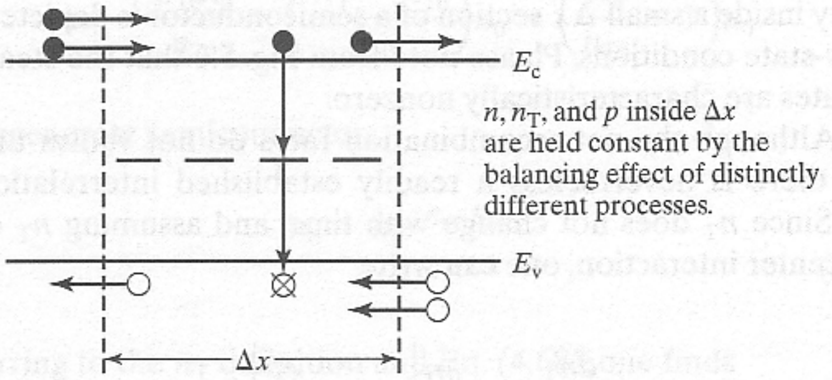
$$\frac{1}{q} \vec{\nabla} \bullet \vec{J}_p = (R - G) + \frac{\partial p}{\partial t}$$

“Equilibrium” vs “steady state”



Each circled pair of processes self-balances.

(a) Equilibrium



n, n_T, and p inside Δx are held constant by the balancing effect of distinctly different processes.

(b) Steady-state

“Equilibrium”: detailed balance, for *each* process

“steady state”: overall balance

Continuity equations: applications

Three examples

(1) Steady state injection from one side
(2) Recombination at the surface
(3) Haynes-Shockley experiment

System of differential equations

(A) Continuity (transport) equations for minority carriers, 1-d case (Sze notations):

$$\frac{\partial n_p}{\partial t} = n_p \mu_n \frac{\partial E_x}{\partial x} + \mu_n E_x \frac{\partial n_p}{\partial x} + D_n \frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{p0}}{\tau_n}$$

$$\frac{\partial p_n}{\partial t} = -p_n \mu_p \frac{\partial E_x}{\partial x} - \mu_p E_x \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{n0}}{\tau_p}$$

(B) Gauss' law, relating the divergence of the electric field with the local charge density, 1-d case:

$$\frac{\partial E_x}{\partial x} = \frac{\rho}{\varepsilon}$$

Globally neutral, locally can be unbalanced!

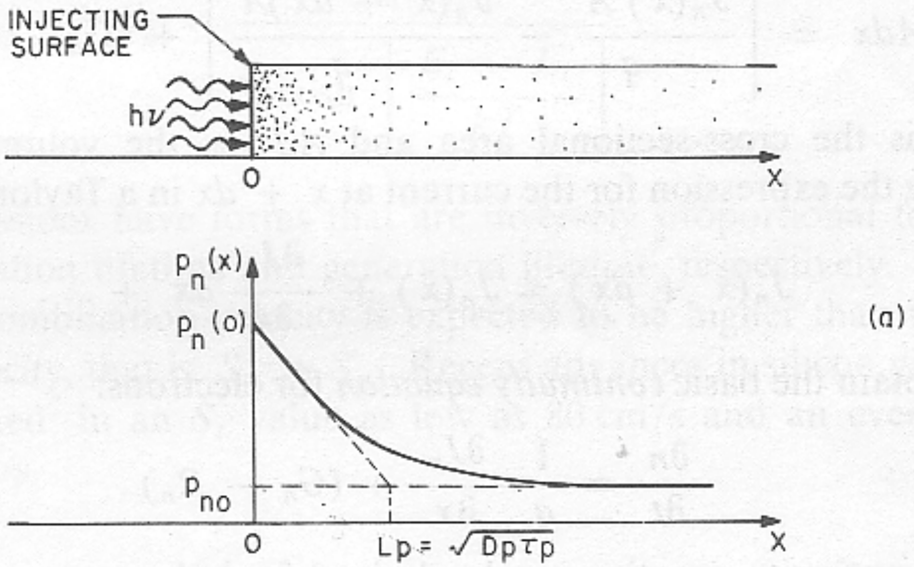
$$\rho = |q|\left(p - n + N_D^+ - N_A^-\right) \approx |q|\left(p - n + N_D - N_A\right)$$

$$N = N_D - N_A$$

To be solved with given boundary conditions!

(ex.1) Steady-state injection from one side

n-type semiconductor
minority carriers: holes
concentration $p_n(x) = ?$



$\frac{\partial p_n}{\partial t} = 0$ steady state

$E_x = 0$ no applied field

$G_p = 0$ no generation in the bulk

p_{n0} at thermal equilibrium

$p_n(0) - p_{n0}$ excess injected at x = 0 (boundary condition)

Continuity equation in this case:

$$\frac{\partial^2 (p_n - p_{n0})}{\partial x^2} = \frac{1}{D_p \tau_p}(p_n - p_{n0})$$

"Diffusion length"

Solution: $p_n(x) = p_{n0} + (p_n(0) - p_{n0})e^{-x/L_p}$

$$\boxed{L_p = \sqrt{D_p \tau_p}}$$

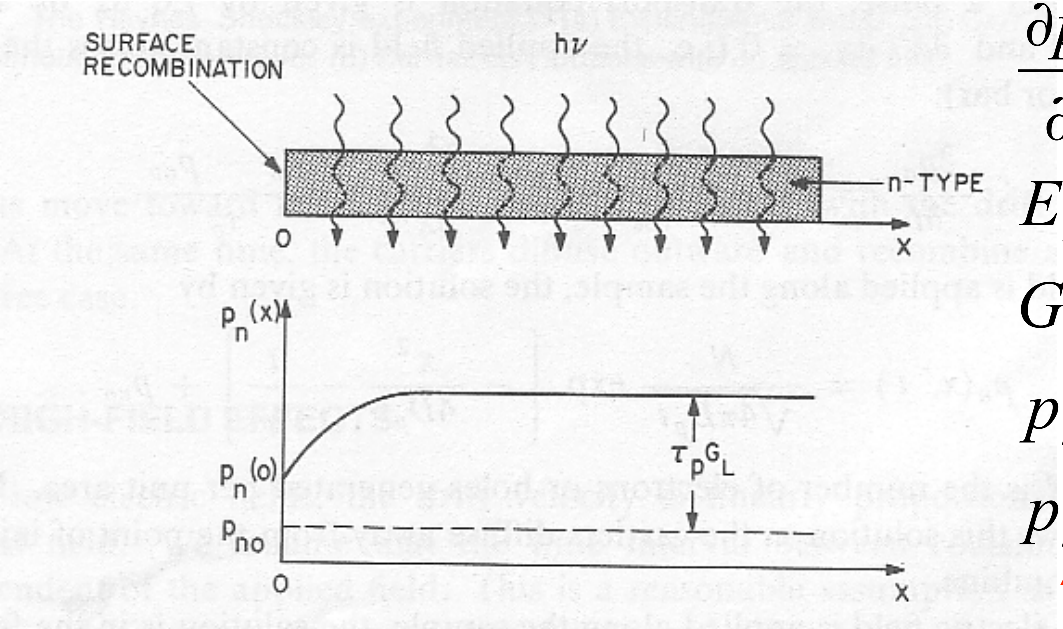
(ex.1) Diffusion length - typical values

“Diffusion length” $L_p = \sqrt{D_p \tau_p}$ $L_n = \sqrt{D_n \tau_n}$

	μ_n [cm²/Vs]	D_n [cm²/s]	μ_p [cm²/Vs]	D_p [cm²/s]
Si	**1350**	**35**	**480**	**12.4**
GaAs	**8500**	**220**	**400**	**10.4**
Ge	**3900**	**101**	**1900**	**49.2**

example $L_p = \sqrt{D_p \tau_p} = \sqrt{(12.4)(5 \times 10^{-7})} = 25\ \mu\text{m}$

(ex.2) Recombination at the surface



$\frac{\partial p_n}{\partial t} = 0$ **steady state**

$E_x = 0$ **no applied field**

$G_L \neq 0$ **Uniform generation in the bulk !!!**

p_{n0} **at thermal equilibrium**

$p_n(0) - p_{n0}$ **boundary condition**

Here the boundary condition is fixed by the rate at which carriers disappear with "surface recombination velocity" depending on the "surface trap density" N_{st}

$$S_{lr} = v_{th} \sigma_p N_{st}$$

cm s^{-1} cm s^{-1} cm^2 cm^{-2}

Equation to be solved (x > 0, bulk):

$$\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{D_p \tau_p} + \frac{G_L}{D_p} = 0 \qquad \Delta p_n = p_n(x) - p_{n0}$$

(ex.2) Solution with boundary conditions

General solution: $\Delta p_n(x) = \boxed{A e^{x/L_p} + B e^{-x/L_p}} + \boxed{G_L \tau_p}$

"complementary" (homegeneous)

"particular"

$$L_p = \sqrt{D_p \tau_p}$$

Boundary conditions:

$$\Delta p_n(x) \xrightarrow[x \to +\infty]{} G_L \tau_p \quad \Rightarrow \quad A = 0$$

$$\Delta p_n(x) \xrightarrow[x \to 0]{} \Delta p_n(0) \quad \Rightarrow \quad \Delta p_n(0) = B + G_L \tau_p$$

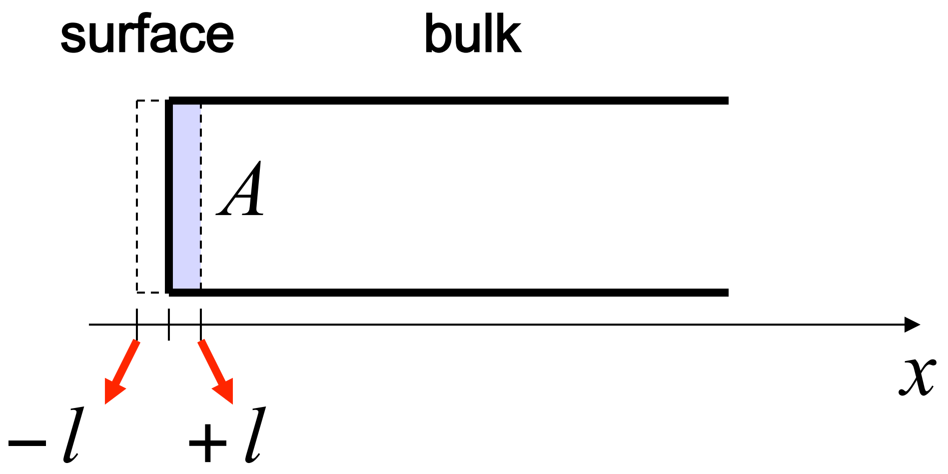
$$B = \Delta p_n(0) - G_L \tau_p$$

after some algebra, substituting A and B, our solution:

$$p_n(x) = p_{n0} + G_L \tau_p \left(1 + \frac{\boxed{\Delta p_n(0)} - G_L \tau_p}{G_L \tau_p} e^{-x/L_p} \right)$$

$$\boxed{\Delta p_n(0) = ???} \rightarrow \boxed{S_{lr}}$$

(ex.2) Surface boundary condition



Consider a thin volume $(A \times 2l)$ enclosing the surface:

$$-J_x(x=l)\,A = \left[G_L - (v_{th}\sigma_p N_t)\Delta p_n(0)\right] \underset{\uparrow\ \boxed{l \to 0}}{Al} - (v_{th}\sigma_p N_{st})\Delta p_n(0)\,A$$

diffusion current

Gener. – recomb. (bulk)

Recombination (surface)

In the limit $l \to 0$: $\quad -J_x(0) = -(v_{th}\sigma_p N_{st})\Delta p_n(0)$

(ex.2) Solution with surface recomb. velocity

$$-J_x(0) = -\left(v_{th}\sigma_p N_{st}\right)\Delta p_n(0) \quad \Rightarrow \quad D_p\left(\frac{d\Delta p_n}{dx}\right)_{x=0} = S_{lr}\,\Delta p_n(0)$$

cm^2 s^{-1} cm^{-4} cm s^{-1} cm^{-3}

from the general solution and boundary conditions:

$$\left(\frac{d\Delta p_n}{dx}\right)_{x=0} = -\frac{B}{L_p} \quad \Rightarrow \quad D_p\left(-\frac{B}{L_p}\right) = S_{lr}\left(G_L\tau_p + B\right)$$

$$\Delta p_n(0) = G_L\tau_p + B \quad \Rightarrow \quad B = \frac{-S_{lr}G_L\tau_p}{D_p/L_p + S_{lr}}$$

Solution expressed in terms of the surface recombination velocity:

$$p_n(x) = p_{n0} + G_L\tau_p\left(1 - \frac{S_{lr}\tau_p}{L_p + S_{lr}\tau_p}e^{-x/L_p}\right)$$

(ex.2) Minority carriers at the surface

$$\frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{D_p \tau_p} + \frac{G_L}{D_p} = 0$$

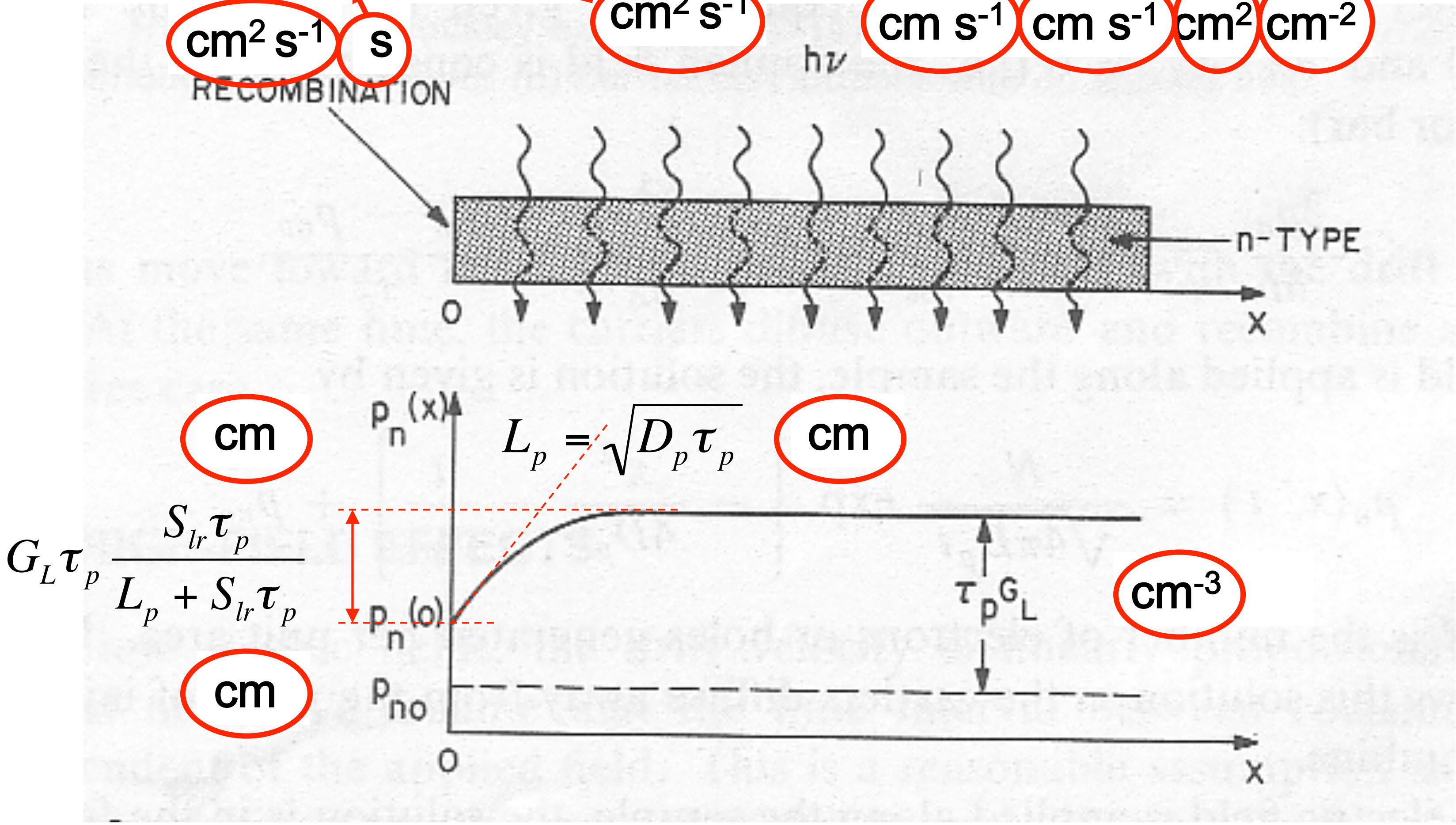
cm^{-3} s^{-1}

cm^2 s^{-1} s

cm^2 s^{-1}

$$S_{lr} = v_{th} \sigma_p N_{st}$$

cm s^{-1} cm s^{-1} cm^2 cm^{-2}



$$L_p = \sqrt{D_p \tau_p}$$

cm

$$G_L \tau_p \frac{S_{lr}\tau_p}{L_p + S_{lr}\tau_p}$$

cm

cm

cm^{-3}

(ex.2) Limiting cases

Neglecting surface recombination:

$$S_{lr}\tau_p << L_p \quad \Rightarrow \quad p_n(x) = p_{n0} + G_L\tau_p$$

$$p_n(0) = p_{n0} + G_L\tau_p$$

as expected!

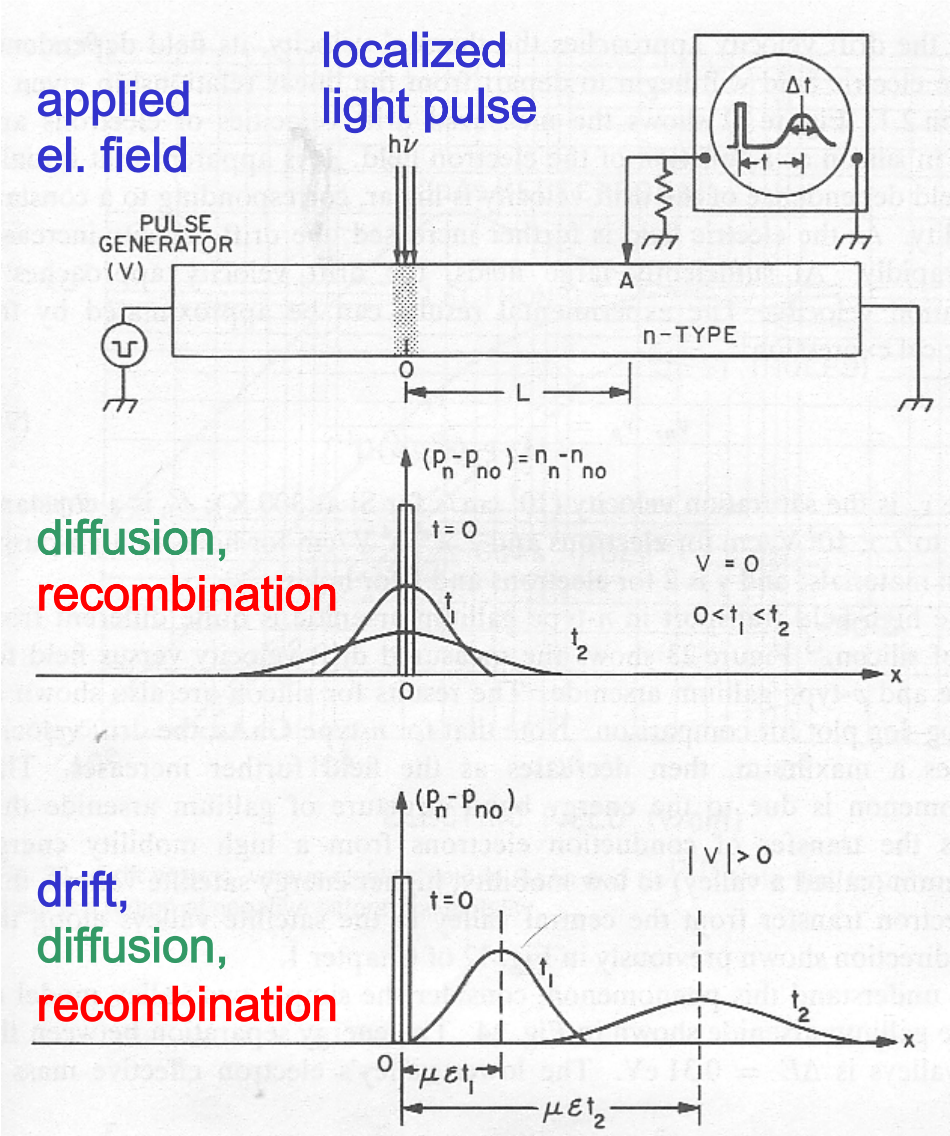
Large ("immediate") surface recombination:

$$S_{lr}\tau_p >> L_p \quad \Rightarrow \quad p_n(x) = p_{n0} + G_L\tau_p\left(1 - e^{-x/L_p}\right)$$

$$p_n(0) = p_{n0}$$

as expected!

(ex.3) The Haynes-Shockley experiment



applied
el. field

localized
light pulse

diffusion,
recombination

drift,
diffusion,
recombination

Experimental set-up

excess carrier distributions
at successive times t_1 and t_2,
no applied field

excess carrier distributions
at successive times t_1 and t_2,
with a constant applied field

(ex.3) The Haynes-Shockley experiment

After a light pulse: $G_L = 0$ **no bulk generation**

$\frac{\partial E_x}{\partial x} = 0$ **constant applied field**

Transport equation for excess minority carriers (n-type semiconductor):

$$\frac{\partial \Delta p_n}{\partial t} = \mu_p E_x \frac{\partial \Delta p_n}{\partial x} + D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} \qquad \Delta p_n = p_n - p_{n0}$$

Solution, no applied field:

$$\Delta p_n(x,t) = \frac{N}{\sqrt{4\pi \boxed{D_p t}}} \exp\left(-\frac{x^2}{4\boxed{D_p t}} - \frac{t}{\boxed{\tau_p}} \right)$$

diffusion,
recombination

Solution, with applied field:

$$\Delta p_n(x,t) = \frac{N}{\sqrt{4\pi \boxed{D_p t}}} \exp\left(-\frac{\left(x - \boxed{u_p E_x t}\right)^2}{4\boxed{D_p t}} - \frac{t}{\boxed{\tau_p}} \right)$$

drift,
diffusion,
recombination

The role of Gauss’ law

Dielectric relaxation
Ambipolar transport

The role of Gauss' law

Divergence of the electric field, 1-d case:

$$\frac{\partial E_x}{\partial x} = \frac{\rho}{\varepsilon}$$

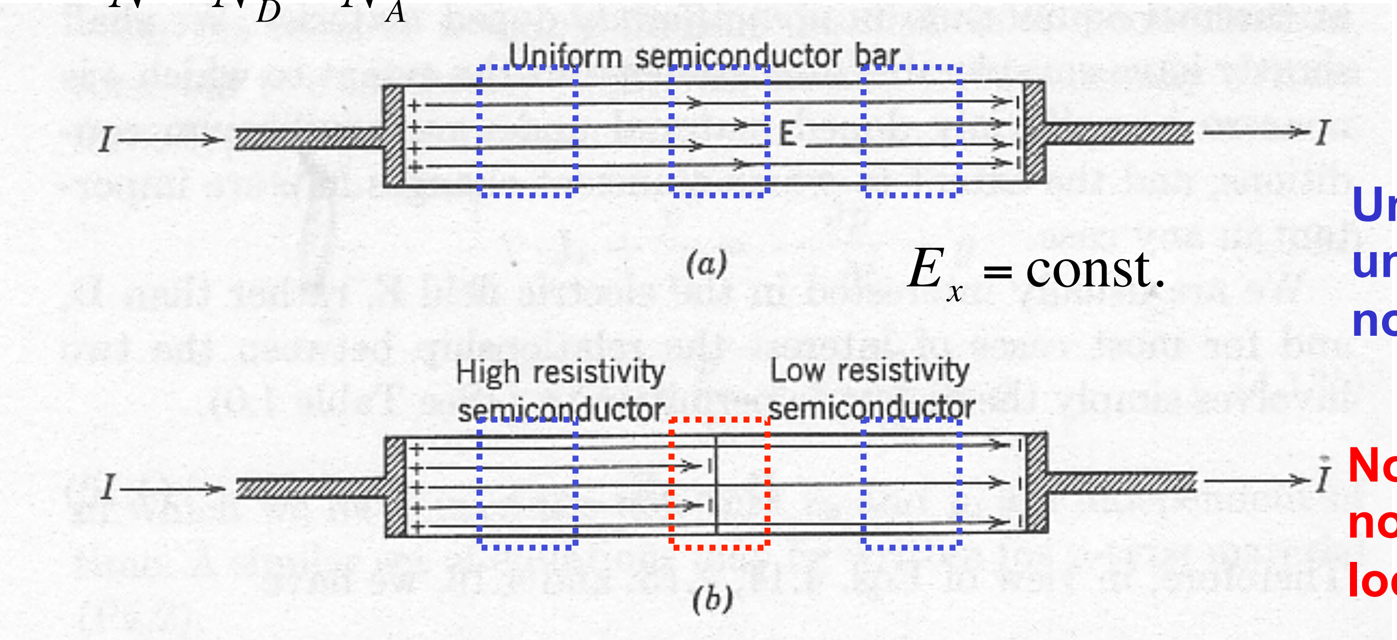
~ complete ionization

$$\rho = |q|\left(p - n + N_D^+ - N_A^-\right) \approx |q|\left(p - n + N_D - N_A\right)$$

$$N = N_D - N_A$$

divergence-less fields can be non-zero ! (due to "external" charges)

$$\frac{\partial E_x}{\partial x} = 0 \quad \not\Rightarrow \quad E_x = 0$$



$$E_x = \text{const.}$$

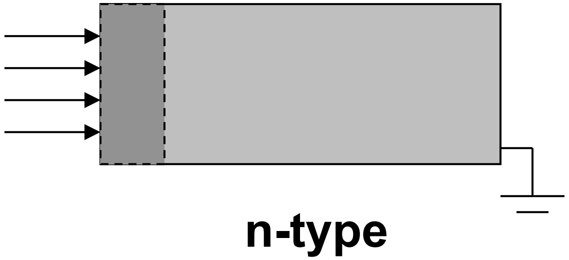
Example: current in a semiconductor

Uniform resistivity: uniform el.field, no local charge

Non-uniform resistivity: non-uniform field, local charge $\neq 0$!

$$E_{x1}/\rho_1 = J_x = E_{x2}/\rho_2 \quad \Rightarrow \quad E_{x1} > E_{x2}$$

Dielectric relaxation time constant



Example: in a short time inject an excess of holes Δp in a small region of a semiconductor crystal, that will experience a local unbalance of electric charge. How fast will be electrical neutrality restored?

Poisson (Gauss) $$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

Ohm $$\vec{J} = \sigma \vec{E}$$

Continuity $$\vec{\nabla} \bullet \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\vec{\nabla} \bullet \vec{J} = \sigma \vec{\nabla} \bullet \vec{E} = \frac{\sigma \rho}{\varepsilon} = -\frac{\partial \rho}{\partial t} \quad \Rightarrow \quad \frac{d\rho}{dt} + \left(\frac{\sigma}{\varepsilon}\right) \rho = 0$$

$$\rho(t) = \rho(0)\, e^{-t/\tau_d} \qquad \boxed{\tau_d = \frac{\varepsilon}{\sigma}}$$ **Dielectric relaxation time constant**

Dielectric relaxation: Debye length

"Debye length" L_D (~ 10^{-5} cm):

$$L_D \approx \sqrt{D_p\left(\frac{\varepsilon}{\sigma}\right)} = \sqrt{D_p \tau_d} \qquad \tau_d \equiv \varepsilon / \sigma$$

"dielectric relaxation time" τ_d (~ 10^{-12} s)

Expect no significant departures from electrical neutrality, over distances greater than about 4 L_D to 5 L_D in *uniformly doped extrinsic* material, at thermal equilibrium (also true off-equilibrium!); this process is much faster than the typical excess carrier lifetime (10^{-7} s)

Numerical example for n-type Si, doped with donor concentration $N_D = 10^{16}$ cm^{-3}

$$\tau_d = \frac{\varepsilon}{\sigma} \approx \frac{\varepsilon_r \varepsilon_0}{q_e \mu_n N_D} = \frac{(11.7)\left(8.85 \times 10^{-14}\right)}{\left(1.6 \times 10^{-19}\right)(1200)\left(10^{16}\right)} \frac{\mathrm{F} \cdot \mathrm{cm}}{(\Omega \cdot \mathrm{cm})^{-1}}$$

$$= 5.4 \times 10^{-13}\ \mathrm{s} = 0.54\ \mathrm{ps}$$

“ambipolar” transport

Two examples:

(1) Bipolar diffusion

(2) Shockley experiment

"Ambipolar transport" - equations

Combining the *transport equations* for electrons and holes with *Gauss' law*, under some simplifying assumptions, (see back-up slides and reference texts):

$$p' \equiv p - p_0 \qquad n' \equiv n - n_0 \qquad n' \approx p' \qquad |E_{\text{int}}| << |E_{\text{app}}|$$

⇒ equations of *coupled* continuity for excess concentrations

$$\boxed{D' \frac{\partial^2 n'}{\partial x^2} + \mu' E_x \frac{\partial n'}{\partial x} + g - R = \frac{\partial n'}{\partial t}}$$

"ambipolar transport equation"
Non-linear!

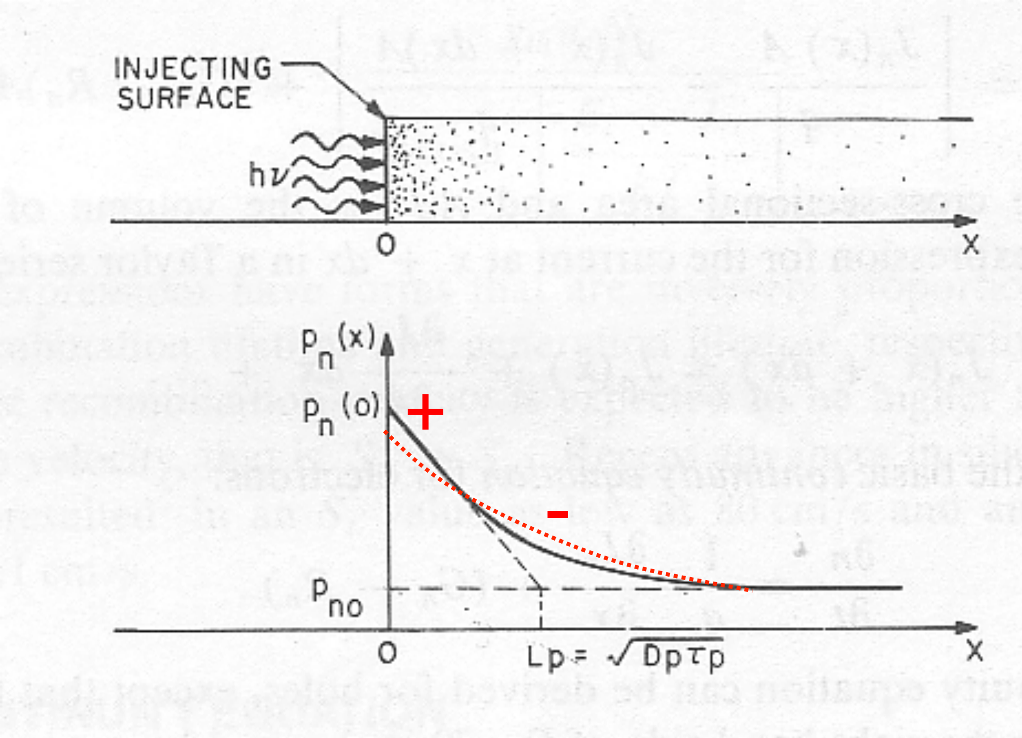
With "ambipolar diffusion coefficient" and "ambipolar mobility":

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \qquad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

Example 1: “Ambipolar diffusion”

Excess electrons and holes produced by light close to the surface, in large concentrations compared to the equilibrium (dark) ones.

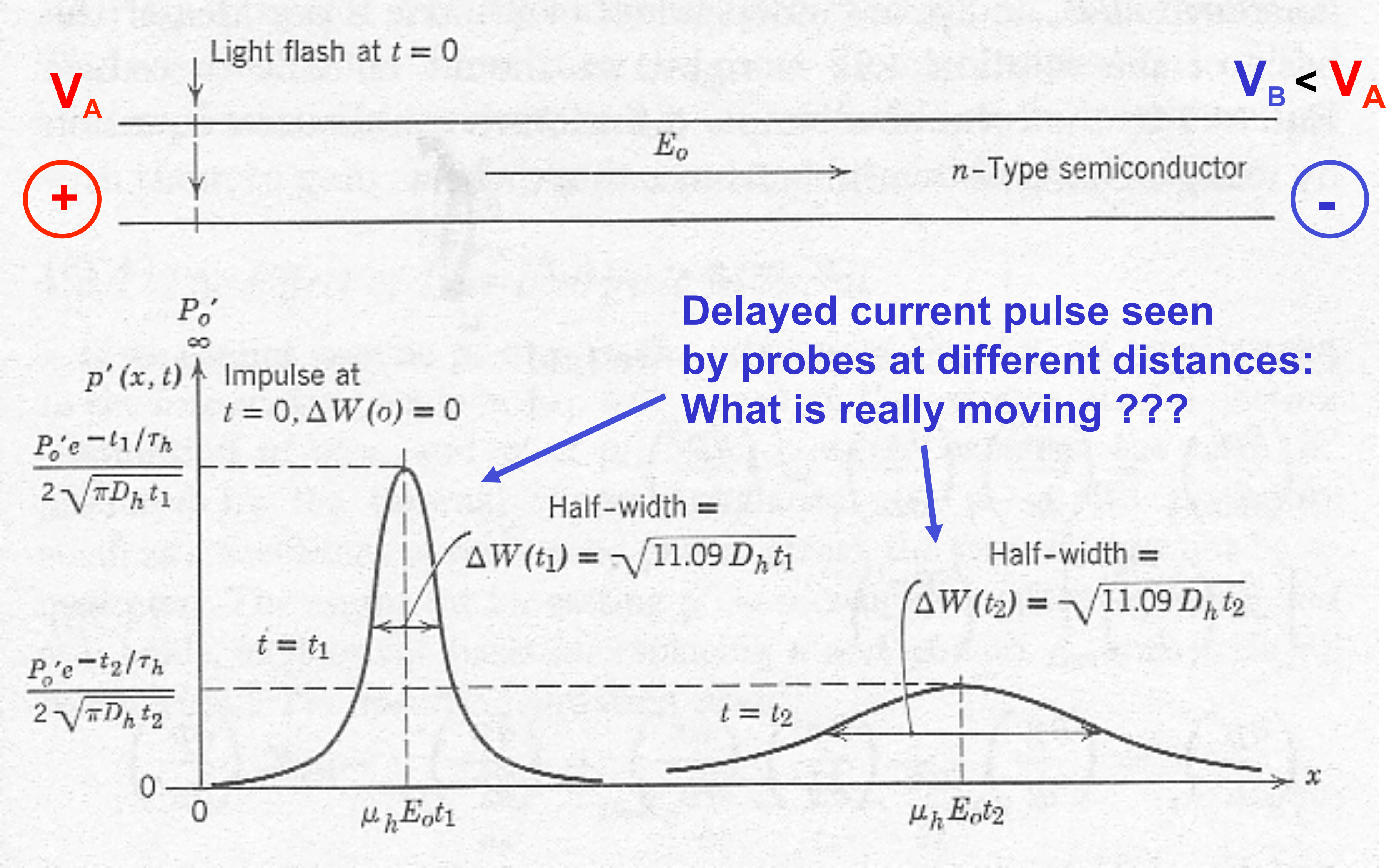
Electrons have larger mobility and move faster: electrons and holes partly separate (net charge positive close to the surface, negative inside)



The resulting electric field is directed so as to compensate for the different mobilities (electrons slowed down, holes accelerated)

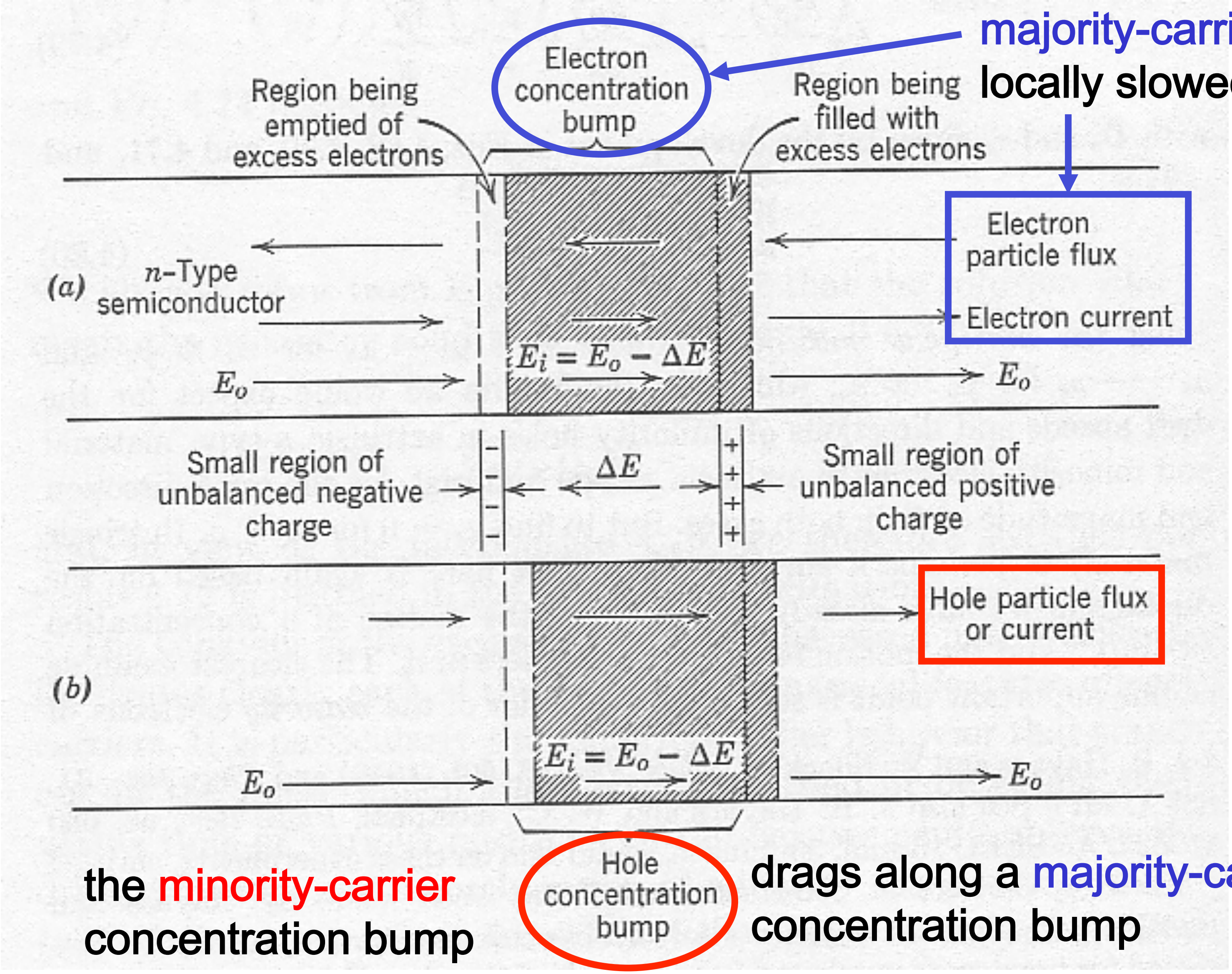
The coupled motion is called ambipolar diffusion. Since electrons and holes move with the same velocity in the same direction, there is no net charge current associated with this motion !

Example 2: Heynes-Shockley experiment



Delayed current pulse seen by probes at different distances: What is really moving ???

Example 2 - qualitative interpretation



majority-carriers, locally slowed down

the minority-carrier concentration bump drags along a majority-carrier concentration bump

Lecture 32 - exercises

- Exercise 1: **An intrinsic Si sample is doped with donors from one side such that N_D=N_0exp(-ax). (a) Find an expression for the built-in electric field E(x) at equilibrium over the range for which N_D>>n_i. (b) Evaluate E(x) when a = 1μm^{-1}.**
- Exercise 2: **An n-type Si slice of thickness L is inhomogeneusly doped with phosphorous donor whose concentration profile is given by N_D(x) = N_0 +(N_L – N_0)(x/L). What is the formula for the electric potential difference between the front and the back surfaces when the sample is at thermal and electric equilibria regardless of how the mobility and diffusivity vary with position? What is the formula for the equilibrium electric field at a plane x from the front surface for a constant diffusivity and mobility?**

Lecture 33 - exercises

- Exercise 1: **Calculate the electron and hole concentration under steady-state illumination in an n-type silicon with $G_L=10^{16}cm^{-3}s^{-1}$, $N_D=10^{15}cm^{-3}$, and $\tau_n=\tau_p=10\ \mu s$.**
- Exercise 2: **An n-type silicon sample has $2x10^{16}$ arsenic atoms/cm^3, $2x10^{15}$ bulk recombination centers/cm^3, and 10^{10} surface recombination centers/cm^2. (a) Find the bulk minority carrier lifetime, the diffusion length, and the surface recombination velocity under low-injection conditions. The values of σ_p and σ_s are $5x10^{-15}$ and $2x10^{-16}$ cm^2, respectively. (b) If the sample is illuminated with uniformly absorbed light that creates 10^{17} electron-hole pairs/(cm^2s), what is the hole concentration at the surface?**
- Exercise 3: **The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to 10^{16} cm^{-3}. The hole concentration is given by $p(x)=10^{15} \exp(-x/L)$ cm^{-3} ($x>0$), where $L = 12\mu m$. The hole diffusion coefficient is D_p=12cm2/s and the electron mobility is $\mu_n=1000cm^2/(Vs)$. The total current density is $J = 4.8\ A/cm^2$. Calculate (a) the hole diffusion current density as a function of x, (b) the electron current density versus x, and (c) the electric field versus x.**

Lecture 34 - exercises

- Exercise 1: **Excess electrons have been generated in a semiconductor so that at $t = 0$ the excess concentration is $\Delta n(0) = 10^{15}\text{cm}^{-3}$. Assuming an excess-carrier lifetime $\tau_n = 10^{-6}$ s, calculate the excess electron concentration and the recombination rate for $t = 4\mu\text{s}$.**
- Exercise 2: **Excess electrons and holes are generated at the end of a silicon bar (at $x = 0$); the silicon bar is doped with phosphorus atoms to a concentration $N_D = 10^{17}\ \text{cm}^{-3}$. The minority lifetime is 10^{-6} s, the electron diffusion coefficient is $D_n = 25\ \text{cm}^2/\text{s}$, and the hole diffusion current is $D_p = 10\ \text{cm}^2/\text{s}$. Determine the steady-state electron and hole concentrations as a function of x (for $x > 0$) and their diffusion currents at $x = 10\mu\text{m}$.**

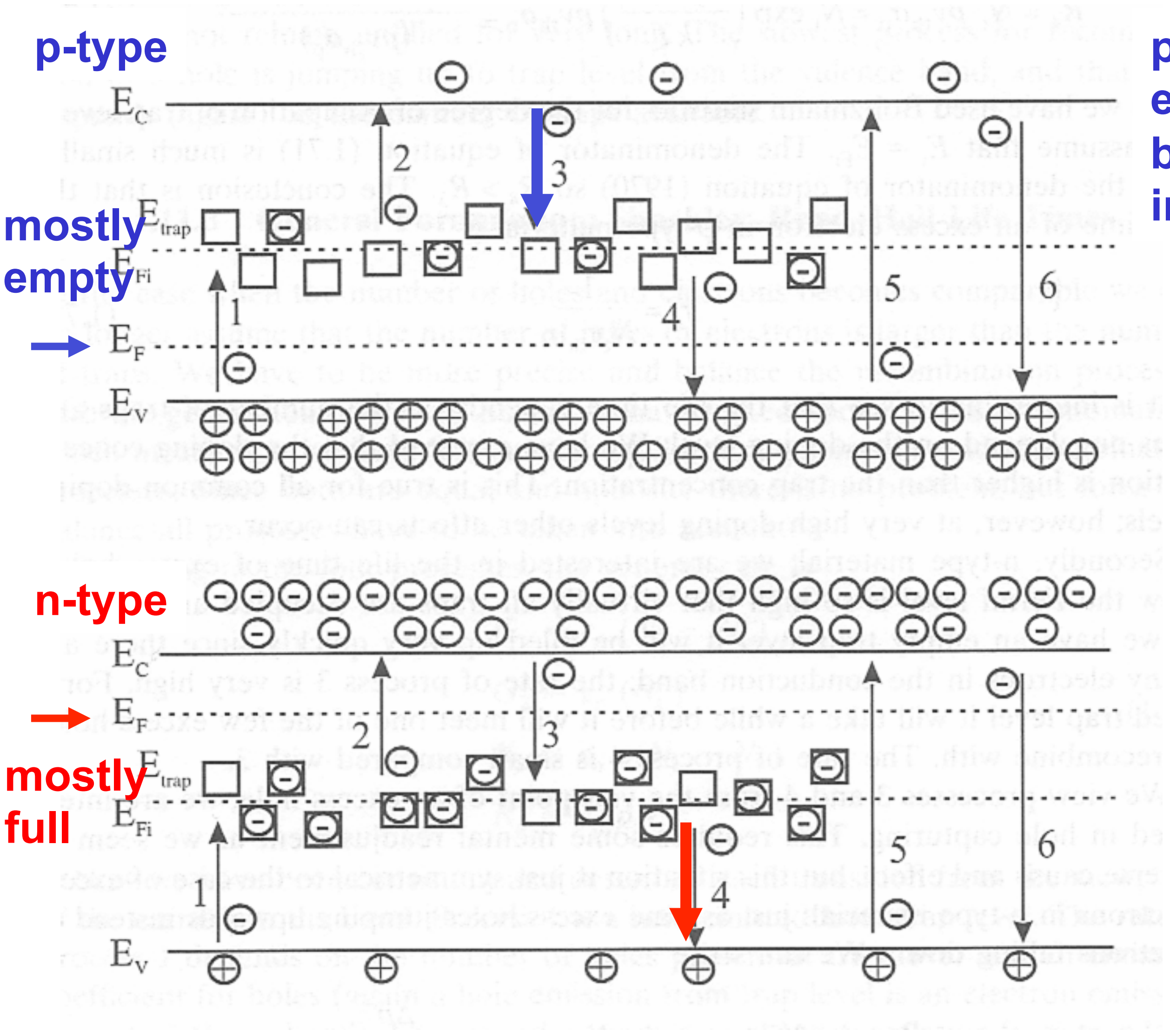
Back-up slides

(topics ***not*** included in the standard program!)

Recombination via traps

Shockley-Read-Hall model

“Low-injection minority lifetime” approximation



p-type

mostly empty

n-type

mostly full

p-type semiconductor: electron lifetime dominated by “electron capture” (3) in “empty” RG centers

$$U \approx v_{th}\sigma_n N_t\left(n_p - n_{p0}\right)$$

$$\tau_n \equiv \frac{1}{v_{th}\sigma_n N_t} \approx 1.0\,\mu s \;\; (\mathrm{Si})$$

n-type semiconductor: hole lifetime dominated by “hole capture” (4) in “full” RG centers

$$U \approx v_{th}\sigma_p N_t\left(p_n - p_{n0}\right)$$

$$\tau_p \equiv \frac{1}{v_{th}\sigma_p N_t} \approx 0.3\,\mu s \; (\mathrm{Si})$$

"Shockley-Read-Hall" lifetimes - 1

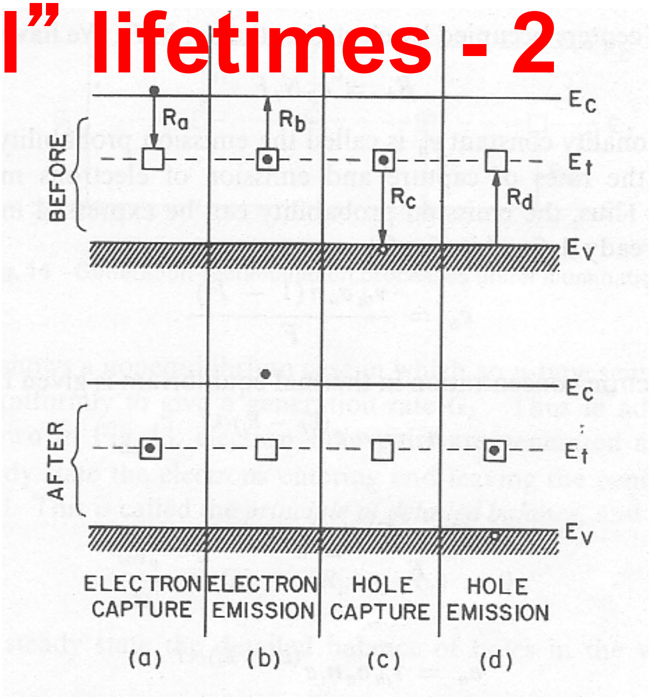
- **What happens if these approximations are not valid?**
  - ***n, p*** **may be comparable (no longer true that** ***n*** **>>** ***p*** **or** ***p*** **>>** ***n*****)**
  - **carrier lifetime no longer dominated by availability of:**

p-type: "empty" traps for "electron capture" ($N_t^0 = N_t(1-F) \approx N_t$)
n-type: "full" or "ionized" traps for "hole capture" ($N_t^- = N_tF \approx N_t$)

“Shockley-Read-Hall” lifetimes - 2

All four “indirect” processes must be taken into account

(see also SZE 2.4.2, “indirect recombination”, or Neamen 6.5.1)



1=d “hole emission” (from a trap)
2=b “electron emission” (from a trap)
3=a “electron capture” (in a trap)
4=c “hole capture” (in a trap)

$$R_d = e_p N_t (1 - F)$$

$$R_b = e_n N_t F$$

$$R_a = c_n N_t (1 - F)$$

$$R_c = c_p N_t F$$

Net recombination rates for electrons and holes separately:

$$U_n = R_a - R_b$$

$$U_p = R_c - R_d$$

"Shockley-Read-Hall" lifetimes - 3

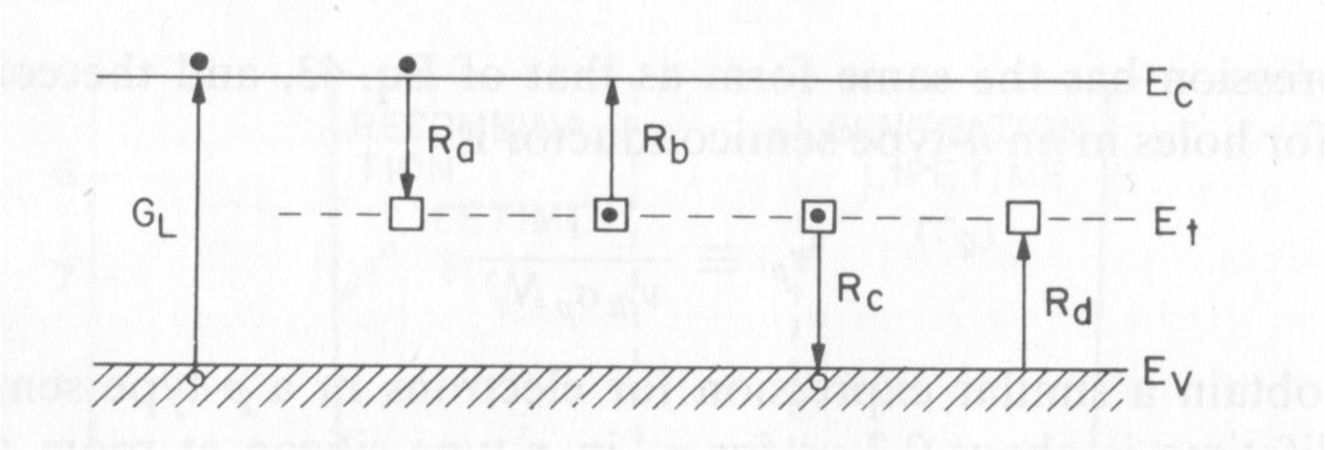
From equilibrium conditions

$G_L = 0$; detailed balance: $R_a - R_b = R_c - R_d = 0$)

- **emission coefficients (e_n, e_p) in terms of:**
- **capture coeff. ($c_n = v_{th}\sigma_n$, $c_p = v_{th}\sigma_p$)**

$$e_n = c_n n_1 \qquad n_1 = n_i e^{(E_i - E_t)/kT}$$

$$e_p = c_p p_1 \qquad p_1 = n_i e^{(E_t - E_i)/kT}$$

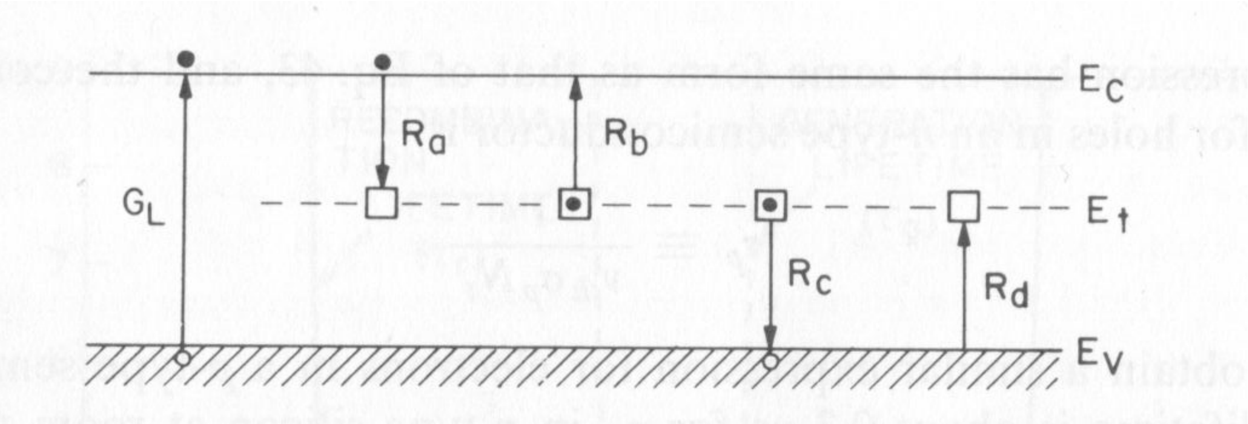


“Shockley-Read-Hall” lifetimes - 4

non-equilibrium steady-state

(G_L = *constant* $\neq$ *0*, and: $U_n = R_a - R_b = U_p = R_c - R_d \neq 0$): see SZE eq. (63)

$$\boxed{U = U_n = U_p = \frac{np - n_i^2}{\frac{1}{c_n N_t}(n + n_1) + \frac{1}{c_p N_t}(p + p_1)} = \frac{np - n_i^2}{\tau_n(n + n_1) + \tau_p(p + p_1)}}$$



This is a general result, usually implemented in device simulations

- **A special case: the previous “Low-injection minority lifetime” result**
- **For very high doping concentrations, direct transitions become likely: this can be modeled by making τ_n and τ_p concentration-dependent**

“ambipolar” transport

Two examples:

(1) Bipolar diffusion

(2) Shockley experiment

“Ambipolar transport” - equations

Special case: homogeneous semiconductor ⇒
Thermal equilibrium concentrations n_0, p_0 constant (time and space)

$$D_p \frac{\partial^2 p'}{\partial x^2} - \mu_p \left(E_x \frac{\partial p'}{\partial x} + p \frac{\partial E_x}{\partial x} \right) + g_p - \frac{p}{\tau_p} = \frac{\partial n'}{\partial x}$$

$$D_n \frac{\partial^2 n'}{\partial x^2} + \mu_n \left(E_x \frac{\partial n'}{\partial x} + n \frac{\partial E_x}{\partial x} \right) + g_n - \frac{n}{\tau_n} = \frac{\partial n'}{\partial x}$$

$$\vec{\nabla} \bullet \vec{E} = \frac{\partial E_x}{\partial x} = \frac{q}{\varepsilon}\left(p' - n'\right) \qquad p' \equiv p - p_0 \quad n' \equiv n - n_0$$

Assume:

- Small internal electric field, with respect to the applied field $\quad \left|E_{\text{int}}\right| << \left|E_{\text{app}}\right|$
- Almost complete balance of electron and hole concentrations $\quad n' \approx p'$
- Generation, recombination $\quad g_n = g_p \equiv g \qquad \dfrac{n}{\tau_{nt}} = \dfrac{p}{\tau_{pt}} \equiv R$

"Ambipolar transport" - equations

We get then :

$$D_p \frac{\partial^2 n'}{\partial x^2} - \mu_p \left(E_x \frac{\partial n'}{\partial x} + p \frac{\partial E_x}{\partial x} \right) + g - R = \frac{\partial n'}{\partial t} \qquad \times \mu_p p$$

$$D_n \frac{\partial^2 n'}{\partial x^2} + \mu_n \left(E_x \frac{\partial n'}{\partial x} + n \frac{\partial E_x}{\partial x} \right) + g - R = \frac{\partial n'}{\partial t} \qquad \times \mu_n n$$

Multiply (see above), add and divide by $\mu_n n + \mu_p p$

$$\boxed{D' \frac{\partial^2 n'}{\partial x^2} + \mu' E_x \frac{\partial n'}{\partial x} + g - R = \frac{\partial n'}{\partial t}}$$

"ambipolar transport equation"
Non-linear!

With "ambipolar diffusion coefficient" and "ambipolar mobility":

$$D' = \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \qquad \mu' = \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}$$

“Ambipolar transport”

In an extrinsic semiconductor under low injection, the ambipolar mobility coefficients reduce to the minority-carrier parameter values, that are constant

p-type
minority: electrons

$$D_n \frac{\partial^2 n'}{\partial x^2} + \mu_n E_x \frac{\partial n'}{\partial x} + g' - \frac{n'}{\tau_{n0}} = \frac{\partial n'}{\partial t}$$

n-type
minority: holes

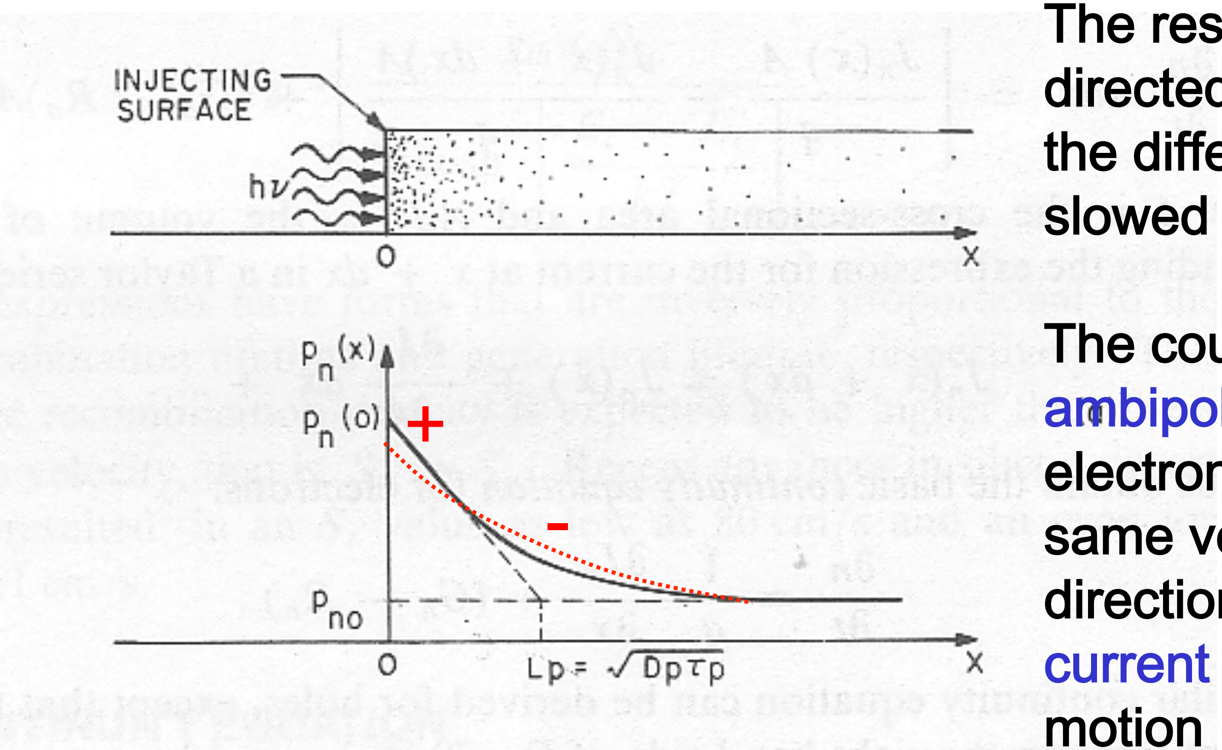
$$D_p \frac{\partial^2 p'}{\partial x^2} - \mu_p E_x \frac{\partial p'}{\partial x} + g' - \frac{p'}{\tau_{p0}} = \frac{\partial p'}{\partial t}$$

The behaviour of excess majority carriers follows that of minority!!!

Example 1: “Ambipolar diffusion”

Excess electrons and holes produced by light close to the surface, in large concentrations compared to the equilibrium (dark) ones.

Electrons have larger mobility and move faster: electrons and holes partly separate (net charge positive close to the surface, negative inside)



The resulting electric field is directed so as to compensate for the different mobilities (electrons slowed down, holes accelerated)

The coupled motion is called ambipolar diffusion. Since electrons and holes move with the same velocity in the same direction, there is no net charge current associated with this motion !

Example 1: “Ambipolar diffusion”

Charge currents
for electrons and holes:

$$j_{x,n} = |q| D_n \frac{\partial n}{\partial x} + |q| n \mu_n E_x$$

$$j_{x,p} = -|q| D_p \frac{\partial p}{\partial x} + |q| p \mu_p E_x$$

The net current density vanishes! Associated electric field:

$$j_x = j_{x,n} + j_{x,p} = 0 \quad \Rightarrow \quad E_x = \frac{D_n \, \partial n / \partial x - D_p \, \partial p / \partial x}{n\mu_n + p\mu_p}$$

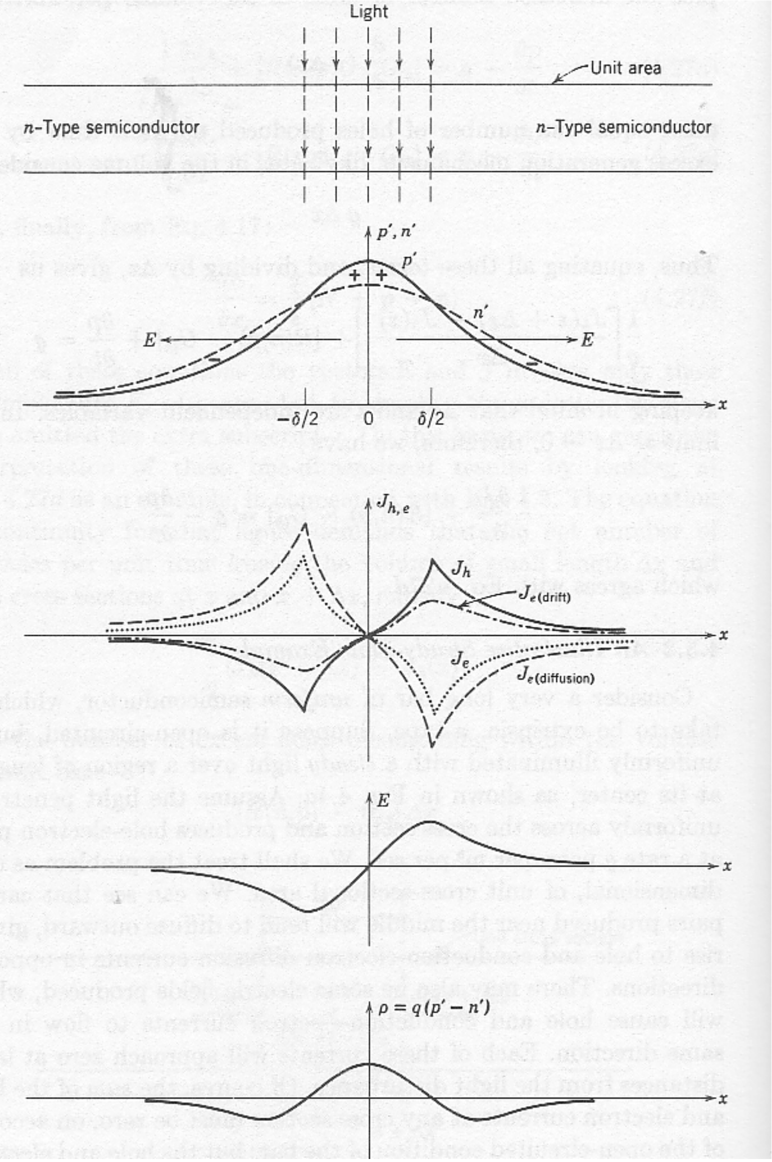
The particle currents are therefore equal for electrons and holes:

$$j_e = j_h = \frac{D_n n\mu_n + D_p p\mu_p}{n\mu_n + p\mu_p} \frac{\partial n}{\partial x} = D_{amb} \frac{\partial n}{\partial x}$$

$$D_{amb} = \frac{D_n n\mu_n + D_p p\mu_p}{n\mu_n + p\mu_p}$$ is the “ambipolar diffusion coefficient”

A “steady-state” example:
locally illuminated semiconductor bar

Ingredients and qualitative expectations



n-type; non-equilibrium; open-circuit;
Local steady illumination

Diffusion of excess carriers (*p'*, *n'*)

$$p' \equiv p - p_0 = \Delta p \qquad n' \equiv n - n_0 = \Delta n$$

Diffusion currents, but also
drift currents due to the electric field E_x

$$J_h = q\mu_h p E_x - qD_h \frac{dp'}{dx} \qquad J_e = q\mu_e n E_x + qD_e \frac{dn'}{dx}$$

$$J = J_e + J_h = 0$$

Electric field E_x (charge unbalance!)

$$\frac{dE_x}{dx} = \frac{q}{\varepsilon}(p' - n') \neq 0$$

The local charge unbalance is small!

$$\left|\frac{p' - n'}{p'}\right| \approx \left|\frac{p' - n'}{n'}\right| << 1$$

Ingredients and qualitative expectations

holes (h): minority
electrons (e): majority

n-type; non-equilibrium; open-circuit;
Local steady illumination

drift (e,h): n >> p

$$\boxed{q\mu_e n E_x >> q\mu_h p E_x}$$

$$J_h = q\mu_h p E_x - qD_h \frac{dp'}{dx}$$

$$J_e = q\mu_e n E_x + qD_e \frac{dn'}{dx}$$

$$\boxed{J = J_e + J_h = 0}$$

$$\Rightarrow \left|q\mu_h p E_x\right| << \left|qD_h \frac{dp'}{dx}\right|$$

$$\Rightarrow \boxed{J_h \approx -qD_h \frac{dp'}{dx}}$$

in this case
minority carriers flow
mainly by diffusion

Diffusion of excess carriers (*p'*, *n'*)

$$p' \equiv p - p_0 = \Delta p \qquad n' \equiv n - n_0 = \Delta n$$

Diffusion currents, but also
drift currents due to the electric field E_x

Diffusion (e,h): opposite currents,
comparable sizes

Electric field E_x (charge unbalance!)

$$\frac{dE_x}{dx} = \frac{q}{\varepsilon}\left(p' - n'\right) \neq 0$$

The local charge unbalance is small!

$$\left|\frac{p' - n'}{p'}\right| \approx \left|\frac{p' - n'}{n'}\right| << 1$$

Under conditions of:
- comparable mobilities
- small injection

in uniform extrinsic material

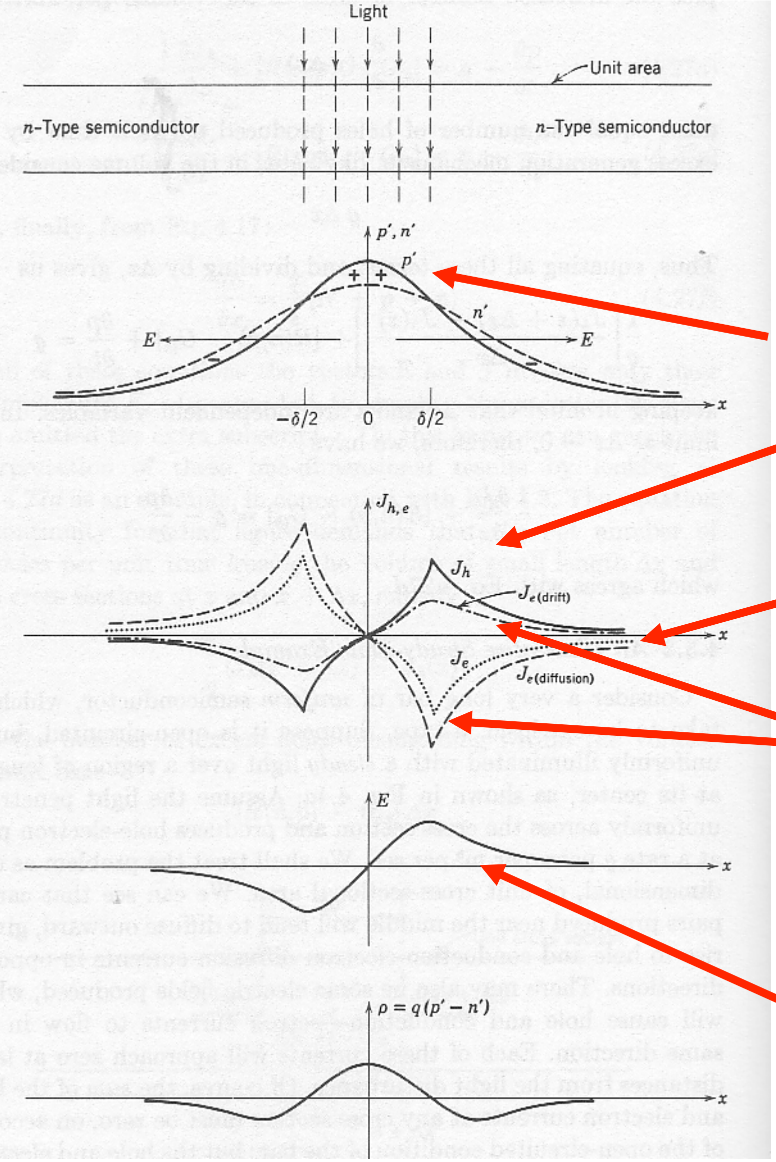
→

the minority-carrier current
will be comparable to
the majority-carrier current
only if
minority carriers
flow mainly by diffusion

Qualitative results

$$\frac{d^2 p'}{dx^2} - \frac{p'}{D_h \tau_h} = -\frac{g_L}{D_h}\,; \qquad 0 < x < \delta/2$$

$$= 0\,; \qquad x > \delta/2$$



The continuity equations above can be solved analytically to obtain p' (x) and $J_h(x) \Rightarrow J_e(x) = - J_h(x)$

$$J_e + J_h = 0 \quad \Rightarrow \quad J_e = -J_h$$

If $D_e = D_h \quad \Rightarrow \quad E_x = 0, \quad p' = n'$

If $D_e > D_h \quad \Rightarrow$ Majority diffusion larger

$\Rightarrow$ Majority drift current: same direction as $J_h(x)$

Electric field E_x

$$J_{e(\mathrm{drift})} = q\mu_e (n_0 + n')E_x \approx q\mu_e n_0 E_x$$

Under conditions of:
- comparable mobilities
- small injection

in uniform extrinsic material

→

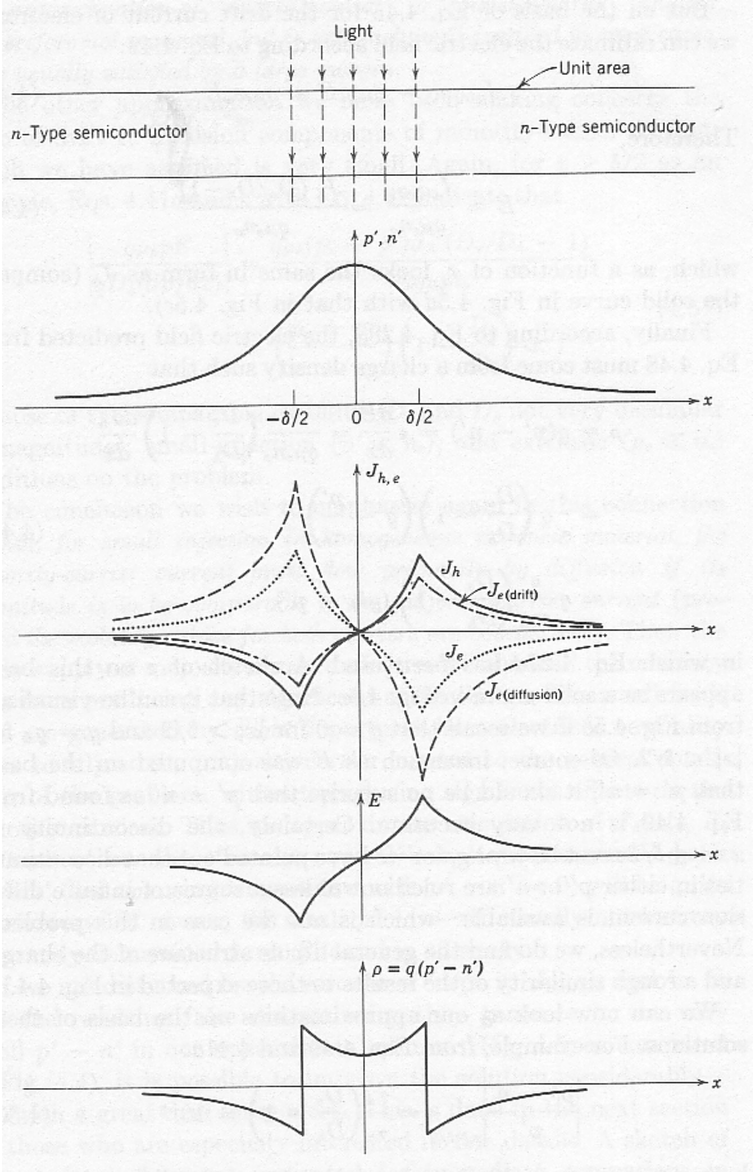
the minority-carrier current will be comparable to the majority-carrier current only if minority carriers flow mainly by diffusion

The large supply of majority carriers effectively "shields" the minority ones from producing any significant space charge.

The small fields that are generated by slight departures from neutrality serve to adjust the majority-carrier current to the general conditions of the problem, without producing significant effects on minority carriers.

An approximate calculation of J_h, J_e, E_x, p', n' in the "quasi-neutral" $n' \approx p'$ approximation (*without* enforcing Gauss' law with $p' - n' = 0$) will be quite satisfactory; of course, the small $p' - n'$ will not be very accurately determined from E_x found in this way

Approximate quantitative solution



Assuming “quasi-neutral” behaviour:

$$p' \approx n' \qquad \frac{dp'}{dx} \approx \frac{dn'}{dx}$$

(well justified in most cases)

$$J_e = J_{e(\text{drift})} + J_{e(\text{diffusion})} = -J_h$$

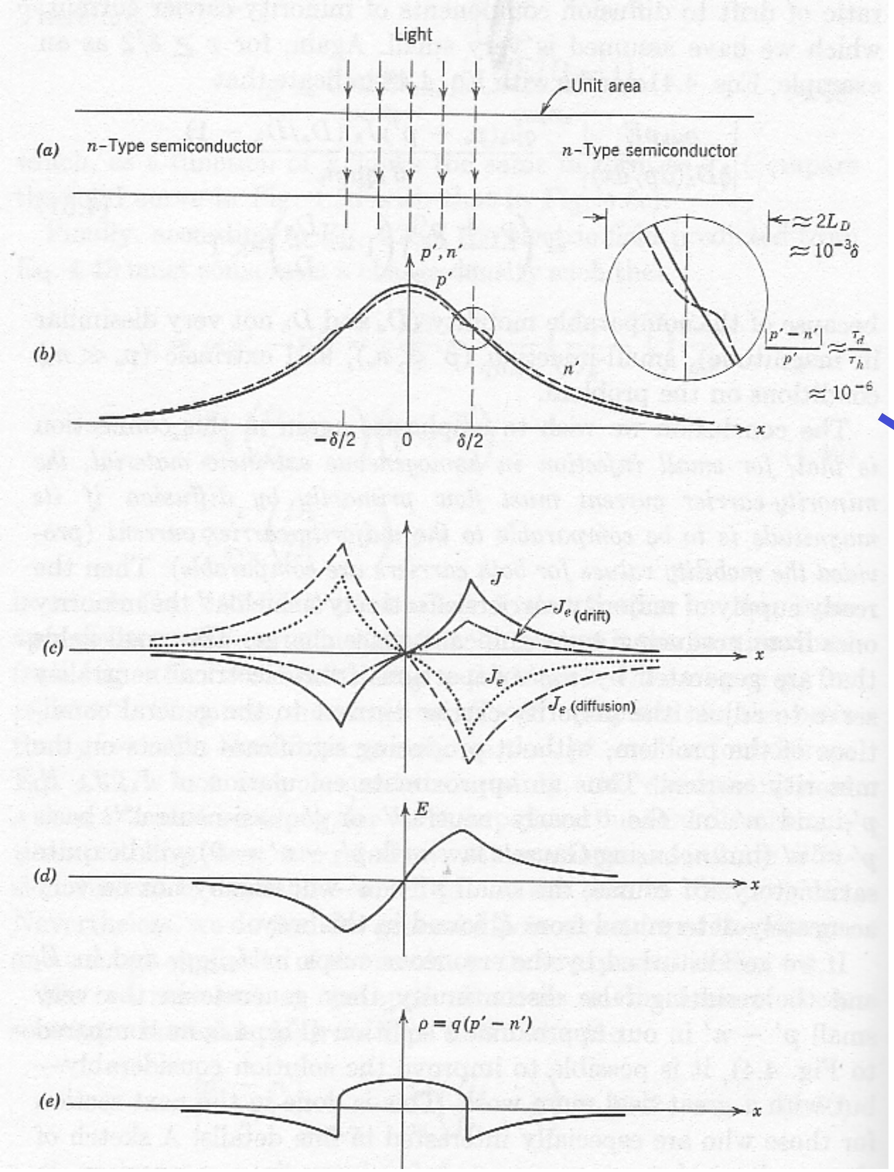
$$J_{e(\text{diffusion})} = qD_e \frac{dn'}{dx} \approx qD_e \frac{dp'}{dx} = -\frac{D_e}{D_h} J_h$$

$$J_{e(\text{drift})} = -J_{e(\text{diffusion})} - J_h \approx J_h \left(\frac{D_e}{D_h} - 1 \right)$$

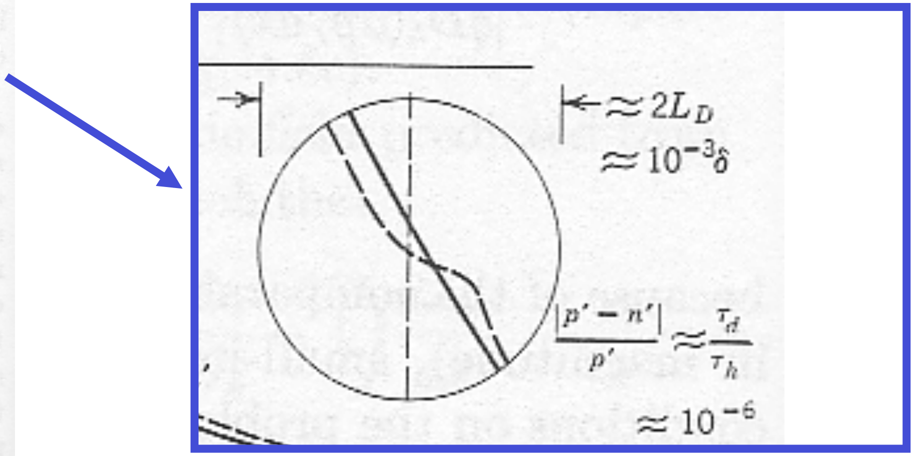
$$E_x = \frac{J_{e(\text{drift})}}{q\mu_e n} \approx \frac{J_{e(\text{drift})}}{q\mu_e n_0} \approx \frac{J_h \left(D_e / D_h - 1 \right)}{q\mu_e n_0}$$

Approximate charge unbalance (dE_x/dx)

Nearly exact solution

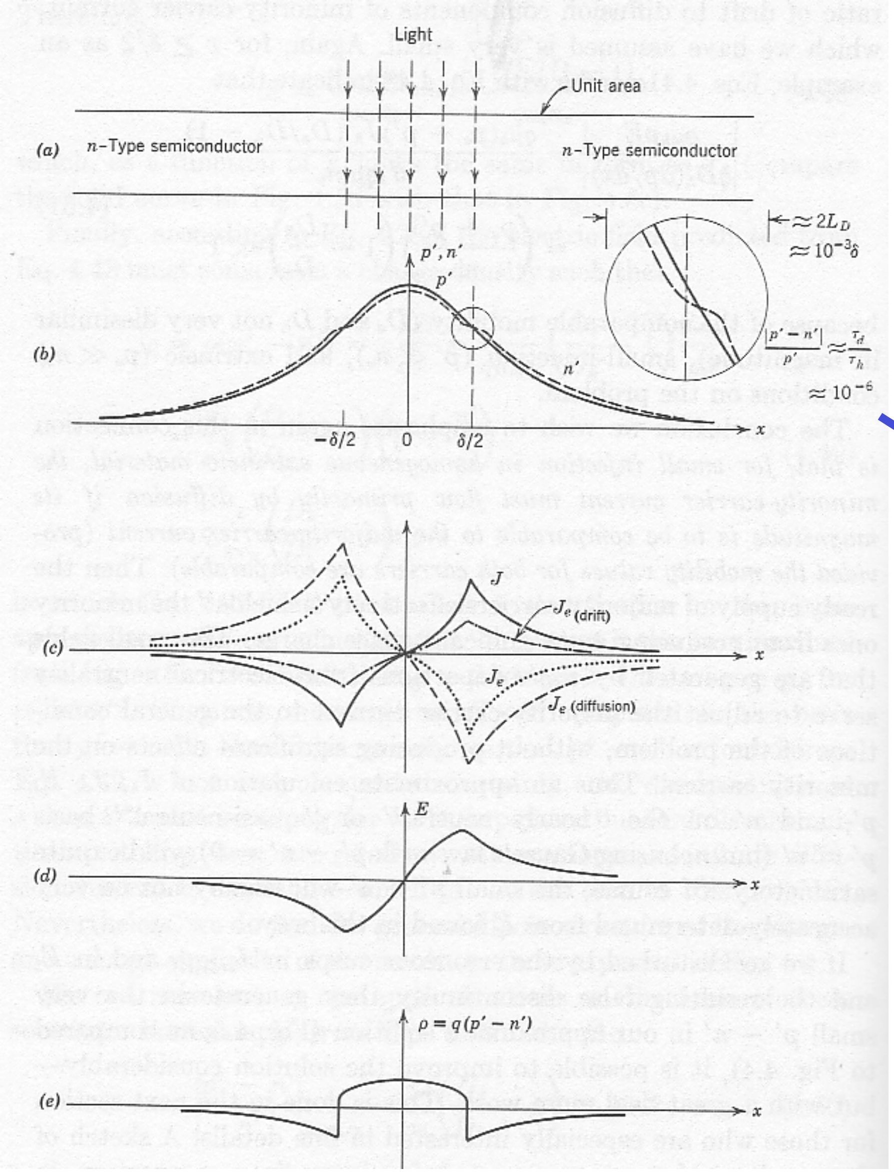


A more accurate solution, not using the p’ ≈ n’ approx. to evaluate $J_{e\ (\text{diffusion})}$

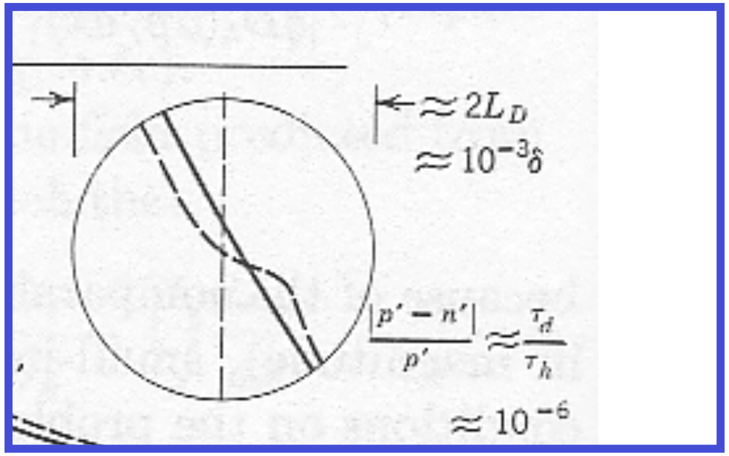


In this example:
$\delta \approx L_h >> L_D$
light beam width δ
hole diffusion length L_h
Debye length L_D

Nearly exact solution



A more accurate solution, not using the p’ ≈ n’ approx. to evaluate $J_{e\ (\mathrm{diffusion})}$



In this example:

$\delta \approx L_h >> L_D$

light beam width δ

hole diffusion length L_h

Debye length L_D