Problem set constrained optimization

1)

a) Write the KT conditions of the following problem:

$$\max_{\{x,y\}} a \cdot (x \cdot y)^b$$
$$s.t. \quad 100 - m \cdot x - n \cdot y \geq 0$$
$$0 \leq x \leq 4$$
$$y \geq 0$$
$$\text{where } 0 < b < 0.5 \quad and \quad a < 0$$

b) Write the KT conditions of the modified Lagrangean

c) Check if these conditions are both necessary and sufficient

2)

a) Solve the following problem:

$$\max_{\{x,y\}} (100 - x) \cdot y$$
$$s.t. \quad x \cdot y \geq 10$$
$$x \leq 2$$
$$y \geq 0$$

b) Check if KT conditions are both necessary and sufficient

3) Solve the following problems:

a) $\max_{\{x,y\}} (x - 1)^2 + (y - 1)^2$ s.t. $0 \leq x \leq 2$ and $0 \leq y \leq 2$

b) $\min_{\{x,y\}} (x - 1)^2 + (y - 1)^2$ s.t. $0 \leq x \leq 2$ and $0 \leq y \leq 2$

4) For each possible value of the constant a, solve the problem

$$max_{\{x,y)} \; x + ay \text{ subject to } x^2 + y^2 \leq 1 \text{ and } x + y \geq 0.$$

5) Consider the following problem.

$$max_{\{x\}} - x_1^2 - x_1 x_2 - x_2^2 \quad subject \text{ to } x_1 - 2x_2 \leq -1 \text{ and } 2x_1 + x_2 \leq 2$$

    a.   Are the Kuhn-Tucker conditions necessary for a solution of this problem?
    b.   Are the Kuhn-Tucker conditions sufficient for a solution of this problem?
    c.   If possible, use the Kuhn-Tucker conditions to find the solution(s) of the problem.