Problem set 5

1. Solve the following problem
$$\max x^2 + y^2 + z^2 \quad s.t. \;\; x + y + z = 10$$
Say if the stationary point(s) is (are) global max/min

2. A consumer is characterized by an utility function $U(x, y) = x + \ln y$. He faces prices $p_x = 1$ and $p_y > 1$. Moreover he faces a budget constraints of $B > 0$. Note the function is defined for all $x \geq 0$ and $y > 0$.
   a. Find the values of x and y that maximize the consumer's utility
   b. Compute the value of relaxing constrain B
   c. Compute the effect on the consumer of an increases of p_y

3. Consider the following problem
$$\max_{\{K,L\}} aK + \ln L$$
subject to $K + bL = M$
where a>0, b>0, M >0

   a. Write the Lagrangian
   b. Write the first order conditions
   c. Find the quantities of K and L that satisfy the first order conditions
   d. Write the bordered Hessian matrix
   e. Prove that this result is a local maximum (second order condition)
   f. Prove that this result is a global maximum
   g. Find the marginal effect of a change of parameter b on the maximized value
   h. Find the marginal effect of a change of parameter *a* on the maximized value
   i. Find the marginal effect of a change of parameter M on the maximized value

4. Define the function *f* by $f(x, r) = x^{1/2} - rx$, where $x \geq 0$. On a graph with *r* on the horizontal axis, sketch the function for several values of *x (for example x=o.5, x=1, x=2)*. Sketch, in addition, the value function f^*, where $f^*(r)$ is the maximal value of $f(x, r)$ for each given value of *r*.