LEZIONE 9-10

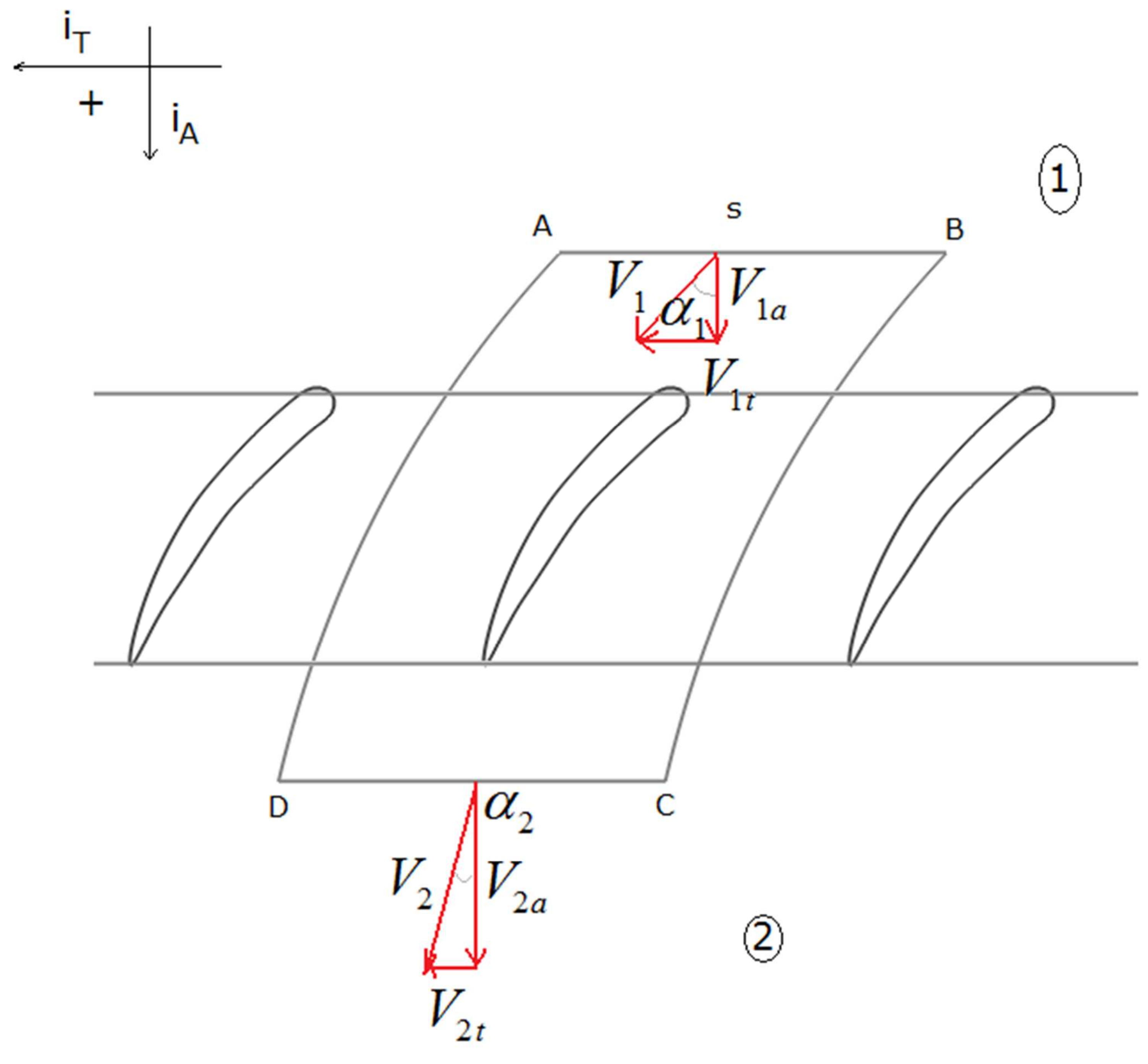
Schiere di pale

a) $F_a = f\left(\Delta\text{velocità}, \Delta p_0\right)$
$F_t = f\left(\Delta\text{velocità}, \Delta p_0\right)$ ($\Delta\text{velocità}$ = triangolo di velocità, Δp_0 perdite di carico)

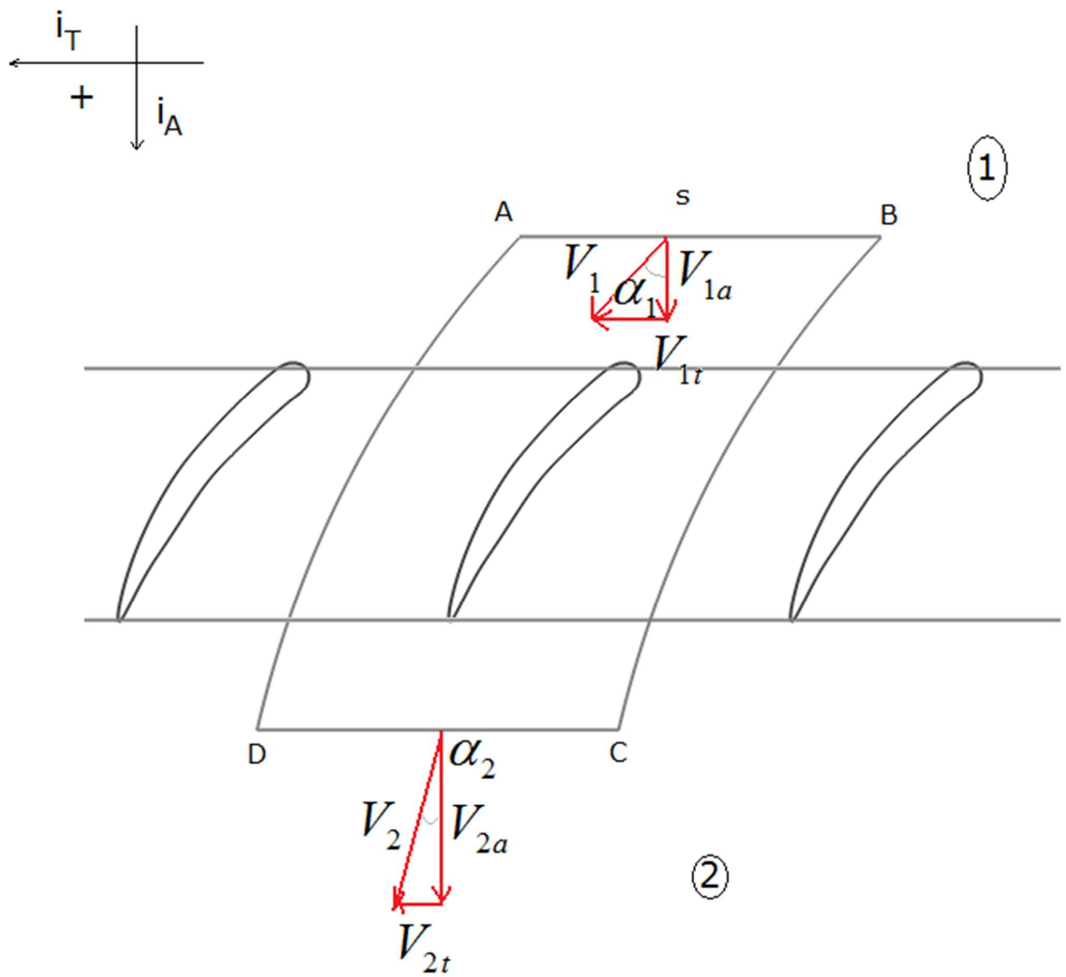
b) $L = f\left(F_a, F_t\right)$
$D = f\left(F_a, F_t\right)$ (portanza e resistenza)

c) Adimensionalizzazione di F_a, F_t, L, D

Schiere di pale



Schiere di pale



Eq. continuità:

$$s \cdot \rho \cdot V_{1a} = s \cdot \rho \cdot V_{2a} \quad \Rightarrow \quad V_{1a} = V_{2a} = V_a$$

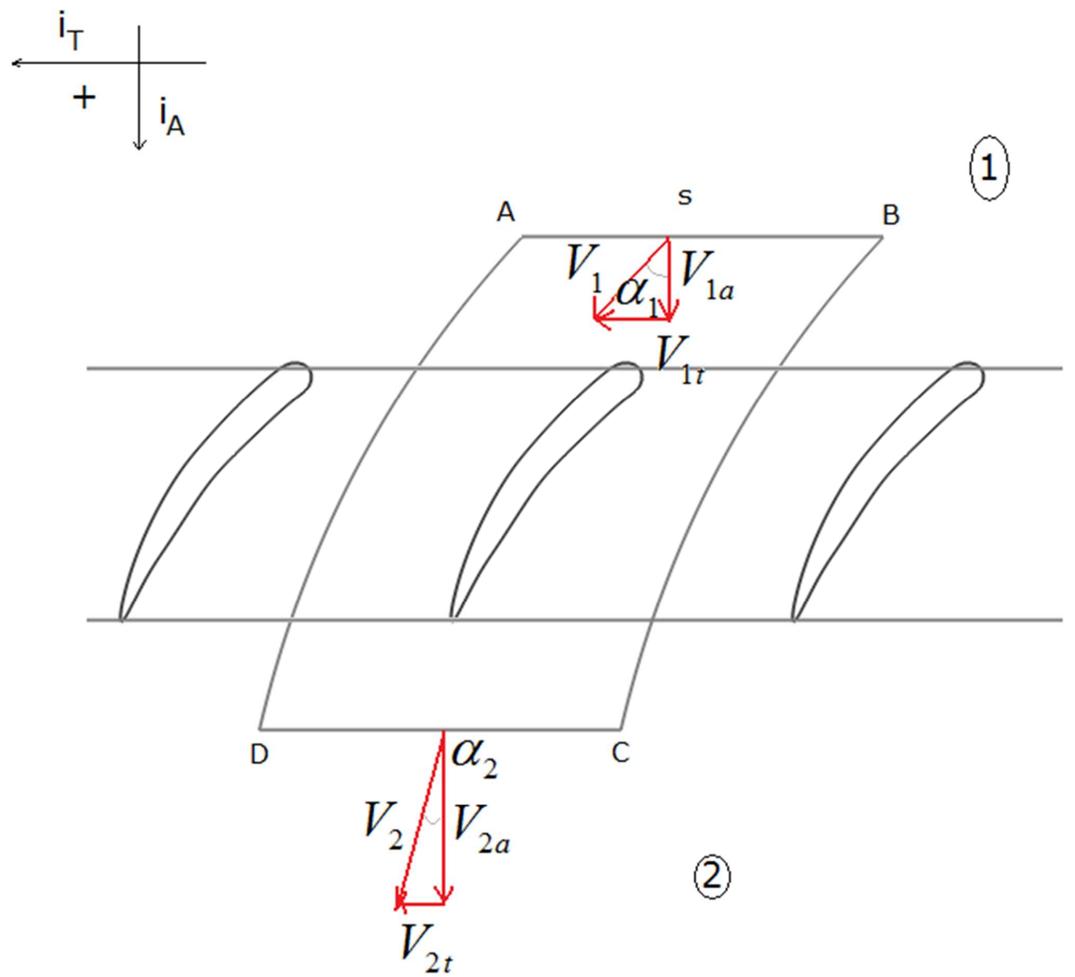
Cons. quantità di moto in dir. t:

$$F_t = \dot{m} \Delta V_t = s \cdot \rho \cdot V_a \cdot \left(V_{1t} - V_{2t} \right)$$

Cons. quantità di moto in dir. a:

$$F_a = s \cdot \left(p_1 - p_2 \right)$$

Schiere di pale



$$F_a = s \cdot \left(p_1 - p_2 \right) =$$

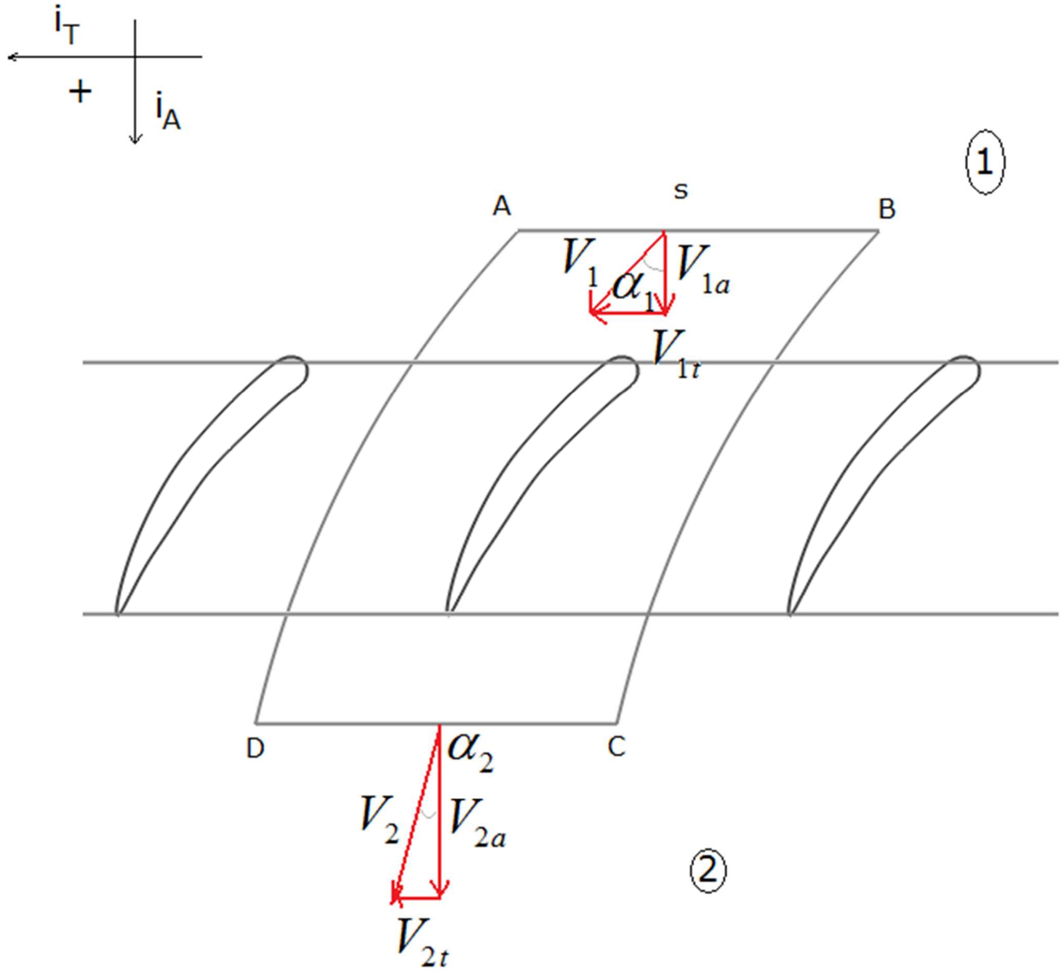
$$= s \cdot \left(p_{01} - \frac{1}{2} \rho V_1^2 - p_{02} + \frac{1}{2} \rho V_2^2 \right) =$$

$$= \frac{1}{2} \rho s \left(V_2^2 - V_1^2 \right) + s \left(p_{01} - p_{02} \right) =$$

$$= \frac{1}{2} \rho s \left(V_{t2}^2 + \cancel{V_a^2} - V_{t1}^2 - \cancel{V_a^2} \right) + s \Delta p_0 =$$

$$= \frac{1}{2} \rho s \left(V_{t2} - V_{t1} \right) \left(V_{t2} + V_{t1} \right) + s \Delta p_0$$

Schiere di pale



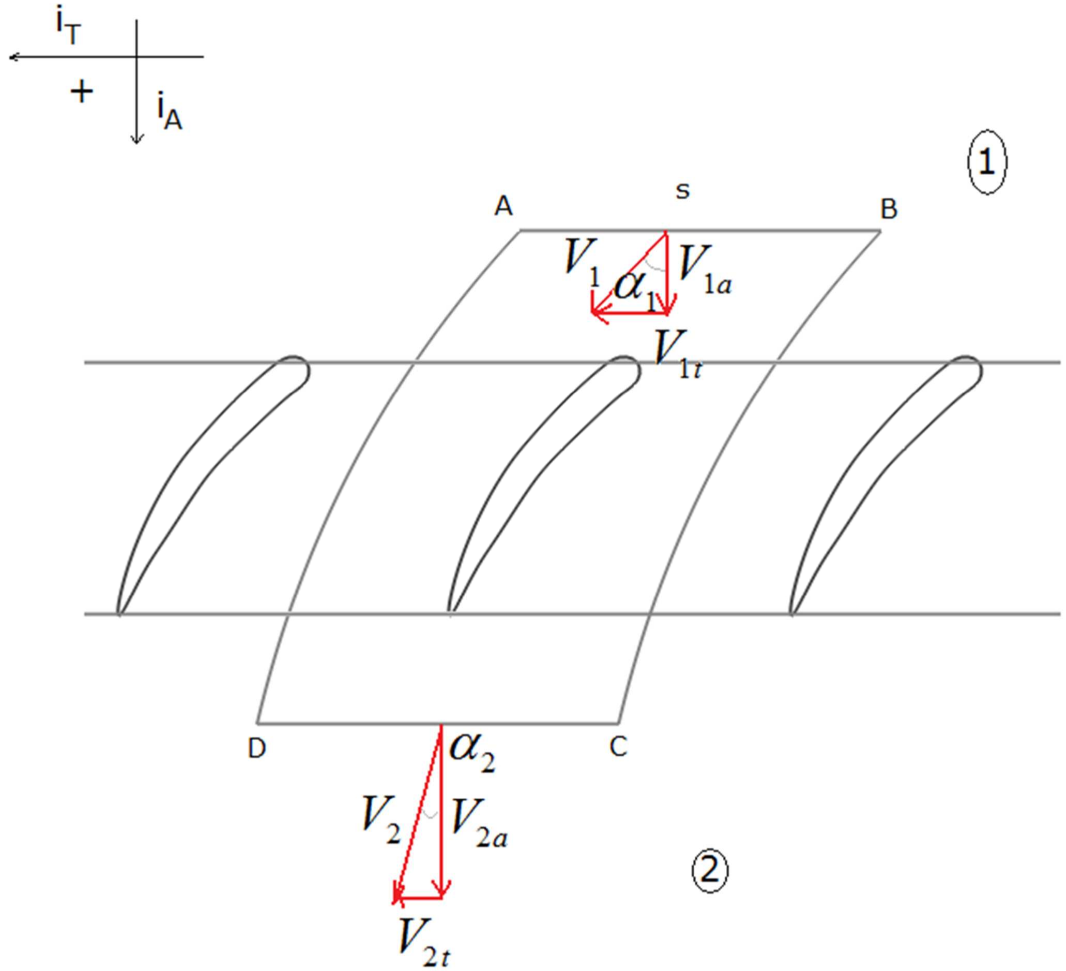
$\left(p_{01} - p_{02}\right)$ è la perdita di carico

$$V_{t\infty} = \frac{V_{t1} + V_{t2}}{2}$$

velocità media indisturbata

$$F_a = \rho s V_{t\infty}\left(V_{t2} - V_{t1}\right) + s\Delta p_0$$

Schiere di pale



Cons. quantità di moto in dir. t:

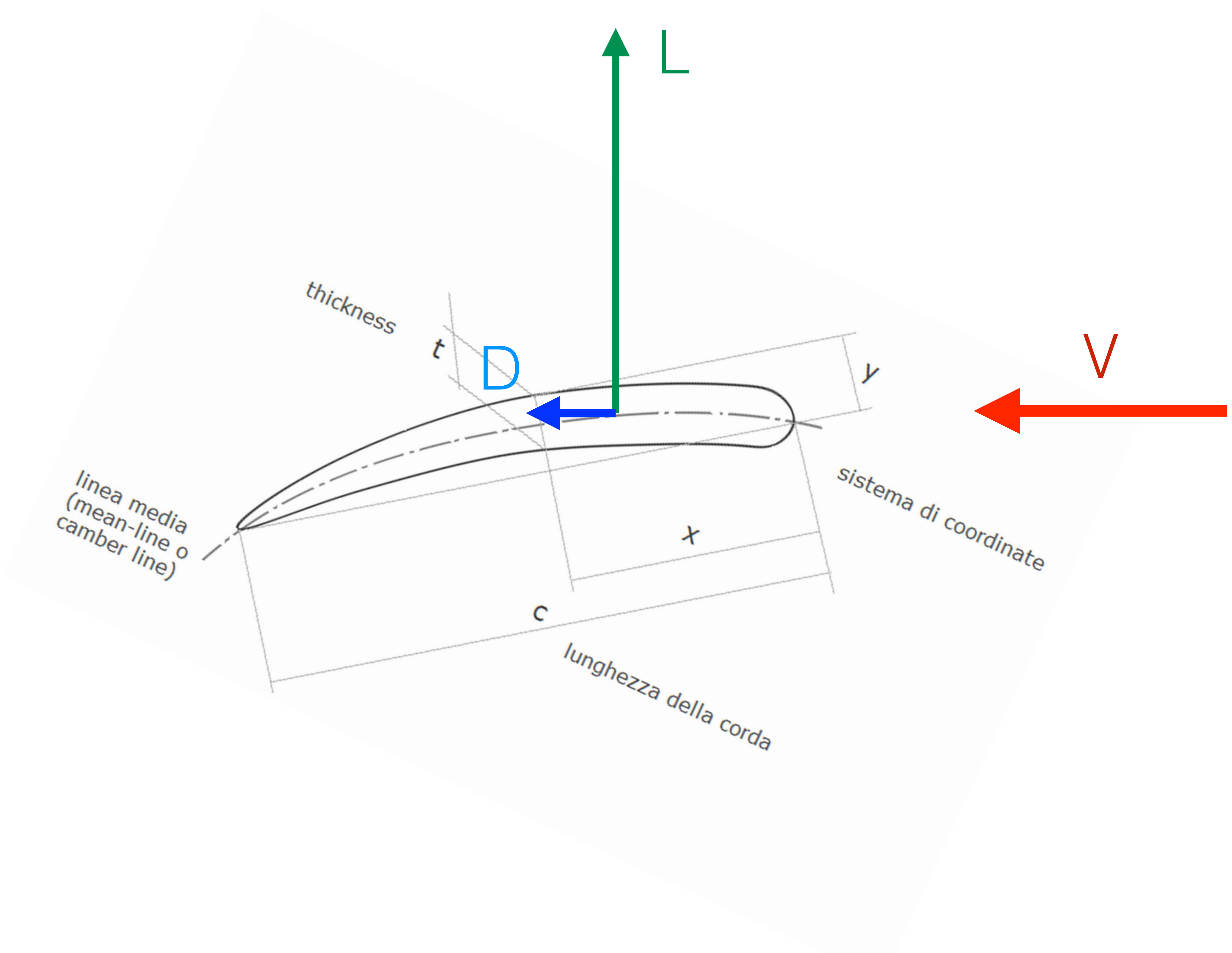
$$F_t = \dot{m}\Delta V_t = s\cdot\rho\cdot V_a \underline{\cdot\left(V_{1t} - V_{2t}\right)}$$

$$F_a = \rho s V_{t\infty} \underline{\left(V_{t2} - V_{t1}\right)} + s\Delta p_0$$

$$F_a = -F_t \frac{V_{t\infty}}{V_a} + s\Delta p_0 = -F_t \tan\alpha_\infty + s\Delta p_0$$

$$y = \frac{\Delta p_0}{p_{02} - p_2} = \frac{\Delta p_0}{\frac{1}{2}\rho V_2^2}$$

Coeff. di perdita



L
D
V
thickness
t
y
x
c
linea media (mean-line o camber line)
sistema di coordinate
lunghezza della corda

Schiere di pale

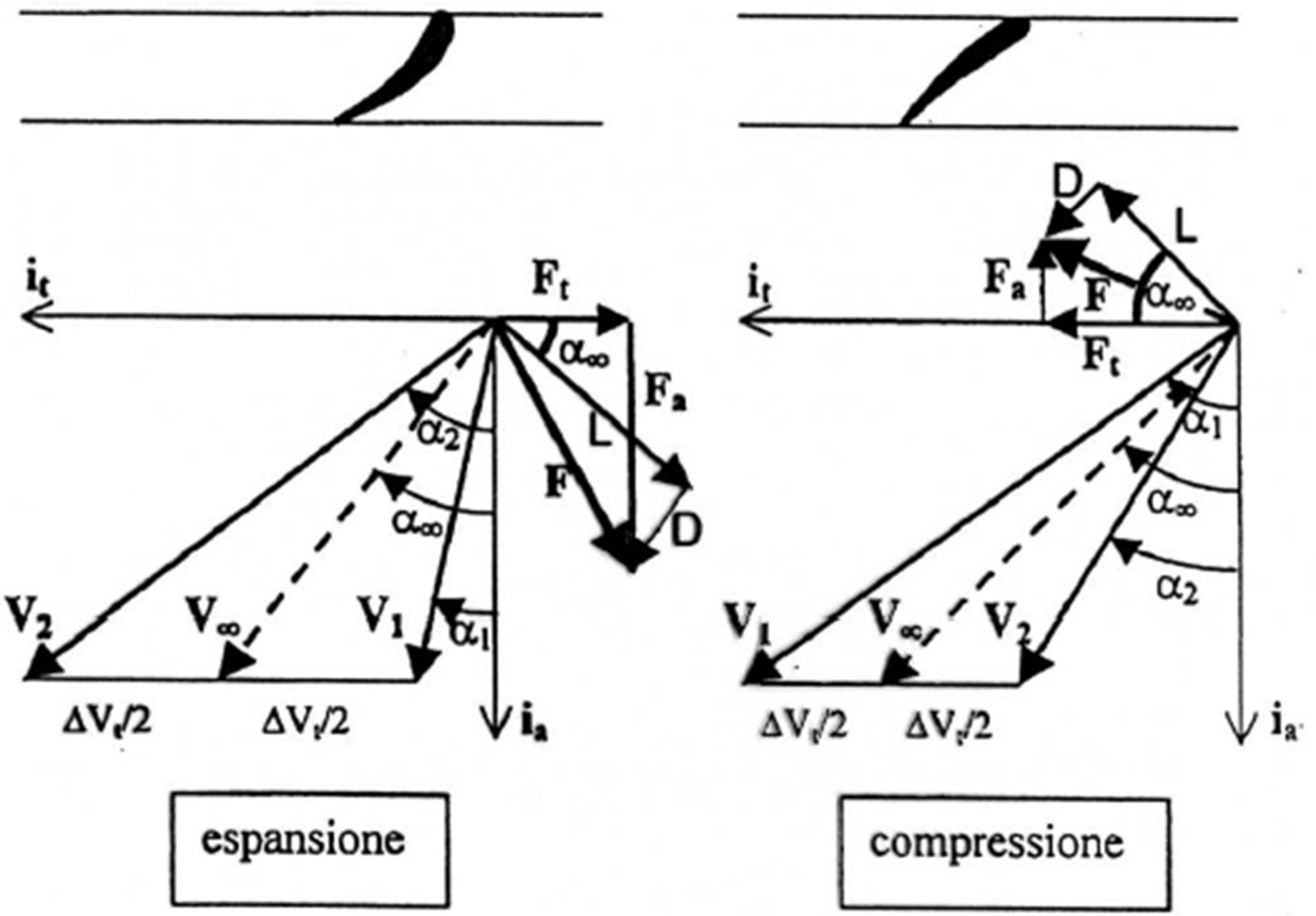


Figura 5.11: *Portanza e resistenza agenti sui profili di schiere piane di pale.*

Schiere di pale

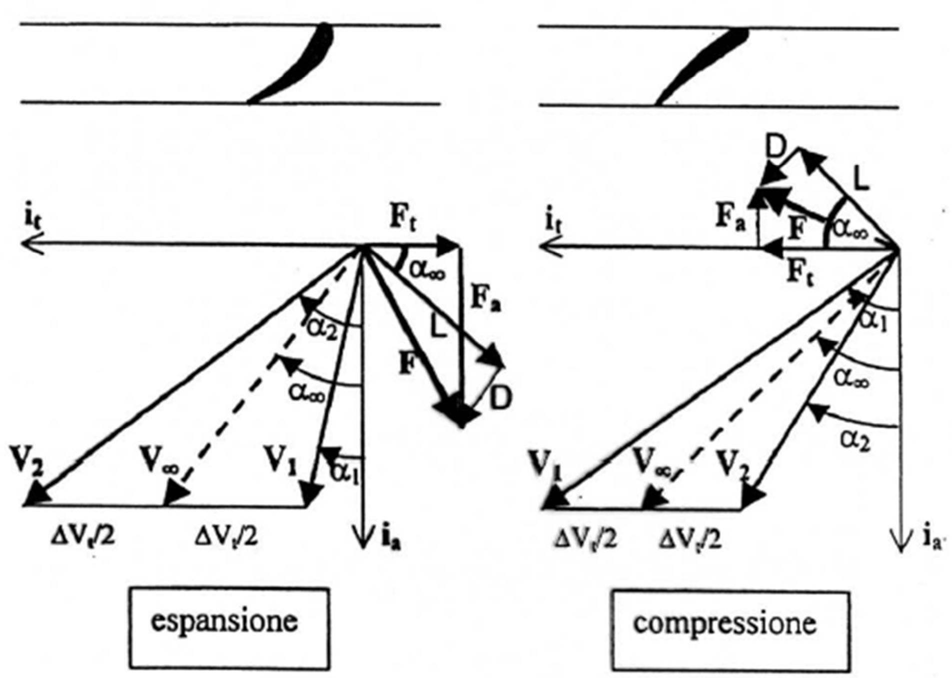


Figura 5.11: *Portanza e resistenza agenti sui profili di schiere piane di pale.*

$$L = F_t \cos\alpha_\infty - F_a sen\alpha_\infty$$

$$D = F_t sen\alpha_\infty + F_a \cos\alpha_\infty$$

$$F_a = -\left(L sen\alpha_\infty - D\cos\alpha_\infty\right)$$

$$F_t = L\cos\alpha_\infty + D sen\alpha_\infty$$

Schiere di pale

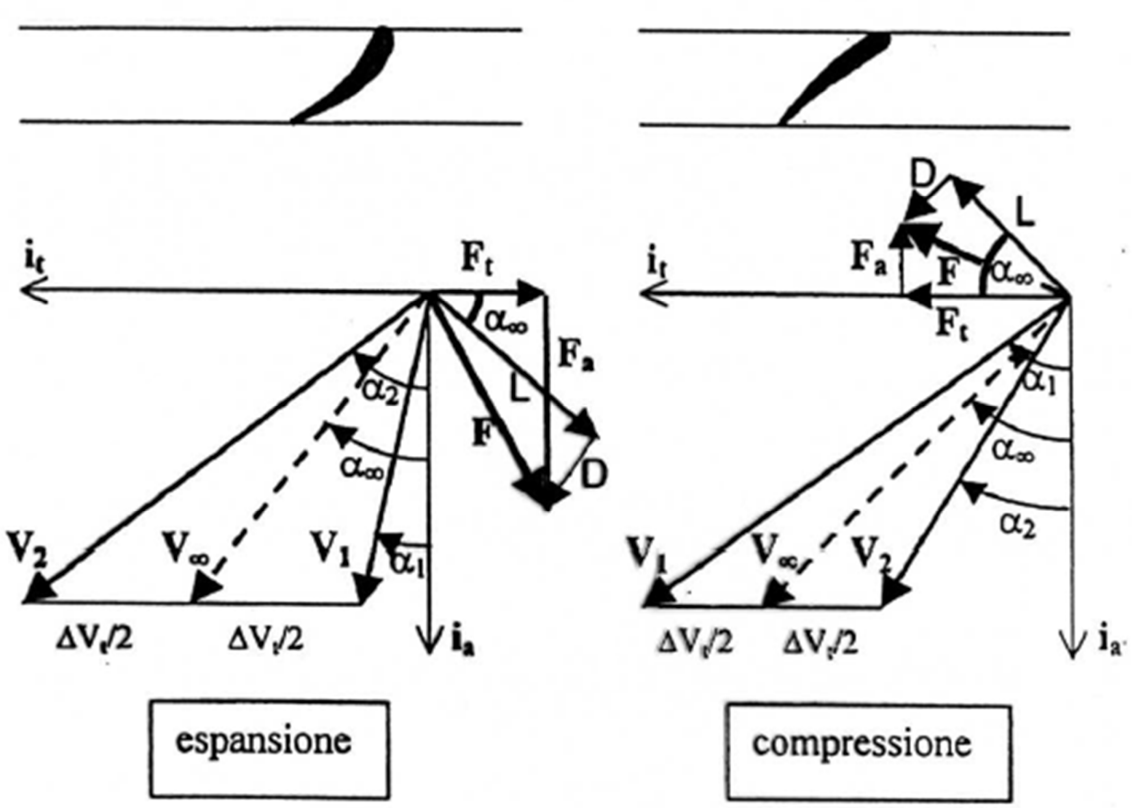


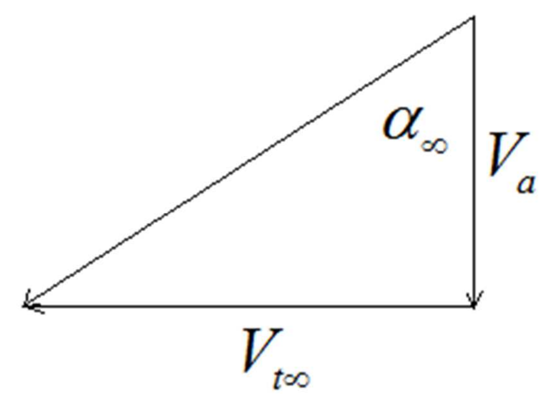
Figura 5.11: *Portanza e resistenza agenti sui profili di schiere piane di pale.*

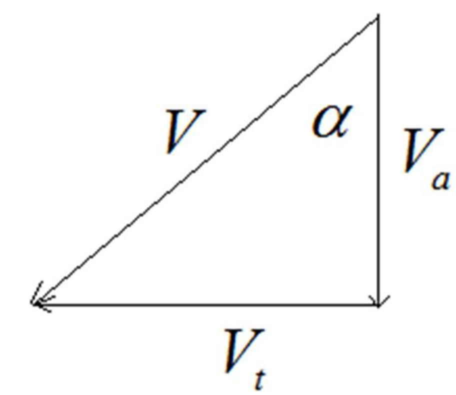
$$c_L = \frac{L}{\frac{1}{2}\rho c \cdot V_\infty^2}$$

$$c_D = \frac{D}{\frac{1}{2}\rho c \cdot V_\infty^2}$$

Schiere di pale

$$c_F = \frac{F_t}{\frac{1}{2}\rho c \cdot V_\infty^2} = \frac{s \cancel{\rho} V_a \cdot \left(V_{t1} - V_{t2}\right)}{\frac{1}{2}\cancel{\rho} c \cdot V_\infty^2}$$





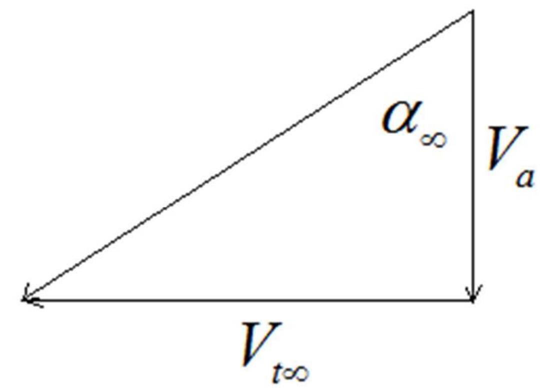
$$V_a = V_\infty \cos \alpha_\alpha$$
$$V_{t1,2} = V_a \tan \alpha_{1,2}$$

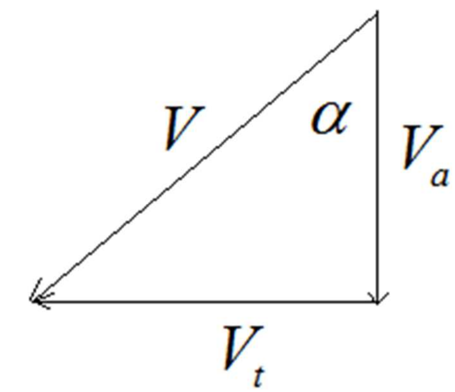
$$c_F = \frac{F_t}{\frac{1}{2}\rho c \cdot V_\infty^2} = \frac{s \cancel{\rho} V_a \cdot \left(V_{t1} - V_{t2}\right)}{\frac{1}{2}\cancel{\rho} c \cdot V_\infty^2} = 2\left(\frac{s}{c}\right)\cos^2 \alpha_\infty \left(\tan \alpha_1 - \tan \alpha_2\right)$$

Schiere di pale

$$c_P = \frac{F_a}{\frac{1}{2}\rho c \cdot V_\infty^2} = \frac{s\Delta p_0 - F_t \tan\alpha_\infty}{\frac{1}{2}\rho c \cdot V_\infty^2} = \frac{s\Delta p_0}{\frac{1}{2}\rho c \cdot V_\infty^2} - c_F \tan\alpha_\infty$$

$$y = \frac{\Delta p_0}{\frac{1}{2}\rho V_2^2}$$ coeff. di perdita





$$c_P = \left(\frac{s}{c}\right) y \left(\frac{V_2}{V_\infty}\right)^2 - c_F \tan\alpha_\infty$$

$$V_a = V_\infty \cos\alpha_\alpha$$
$$V_{t1,2} = V_a \tan\alpha_{1,2}$$

$$c_P = \left(\frac{s}{c}\right) y \frac{\cos^2\alpha_\infty}{\cos^2\alpha_2} - 2\left(\frac{s}{c}\right)\left(\tan\alpha_1 - \tan\alpha_2\right) sen\alpha_\infty \cdot \cos\alpha_\infty$$

Schiere di pale

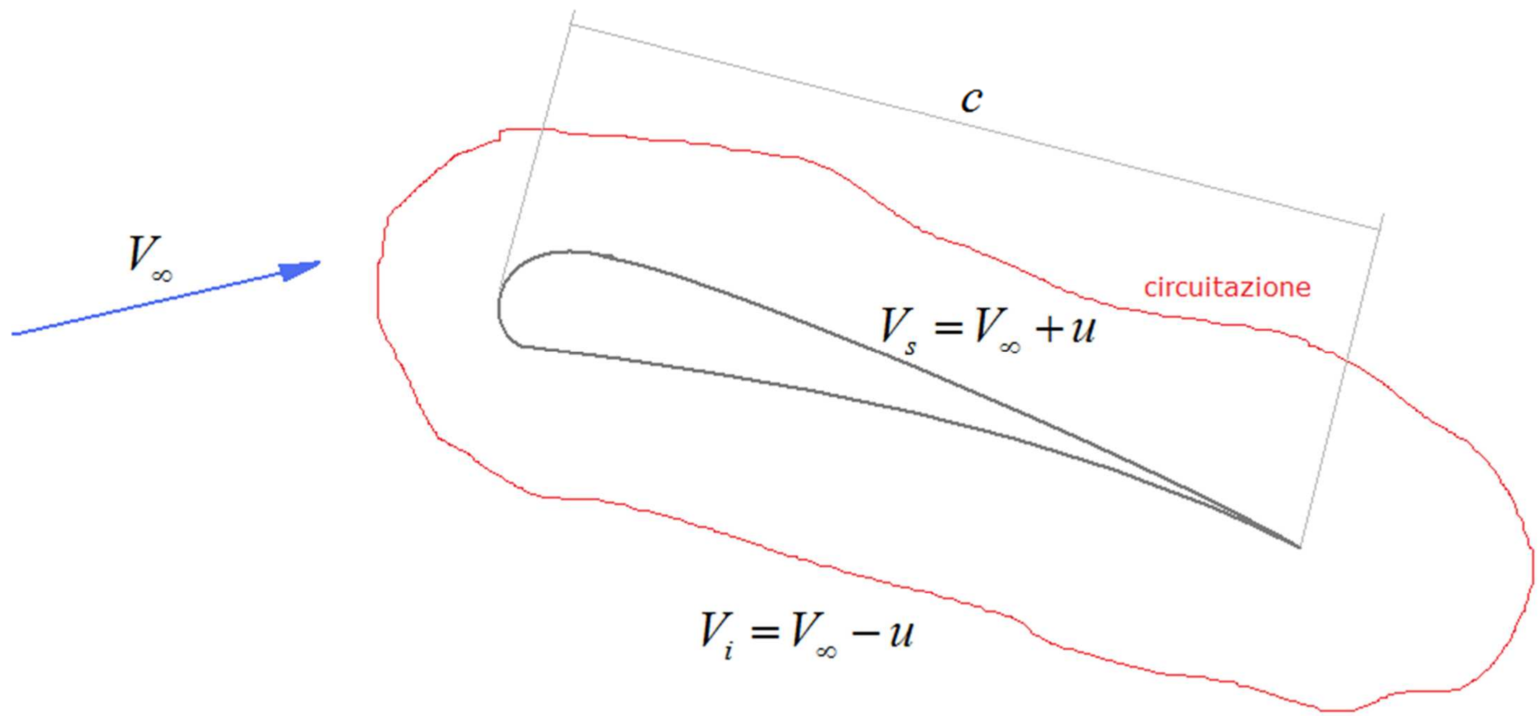
Possiamo trovare allora

$$c_L = c_F \cos\alpha_\infty - c_P sen\alpha_\infty$$
$$c_D = c_F\, sen\alpha_\infty + c_P \cos\alpha_\infty$$

Sostituendo c_F e c_P si ottiene

$$c_L = 2\left(\frac{s}{c}\right)\left(\tan\alpha_1 - \tan\alpha_2\right)\cos\alpha_\infty - c_D \tan\alpha_\infty$$
$$c_D = \left(\frac{s}{c}\right) y \frac{\cos^3\alpha_\infty}{\cos\alpha_2}$$

teorema di Kutta-Jukowsky



$$\Gamma = \oint_\ell \overline{v} \cdot d\,\overline{\ell}$$

$$L = \rho \Gamma V_\infty$$

$$\Delta p = p_i - p_s = \frac{1}{2}\rho\left(V_s^2 - V_i^2\right) = 2\rho u V_\infty$$

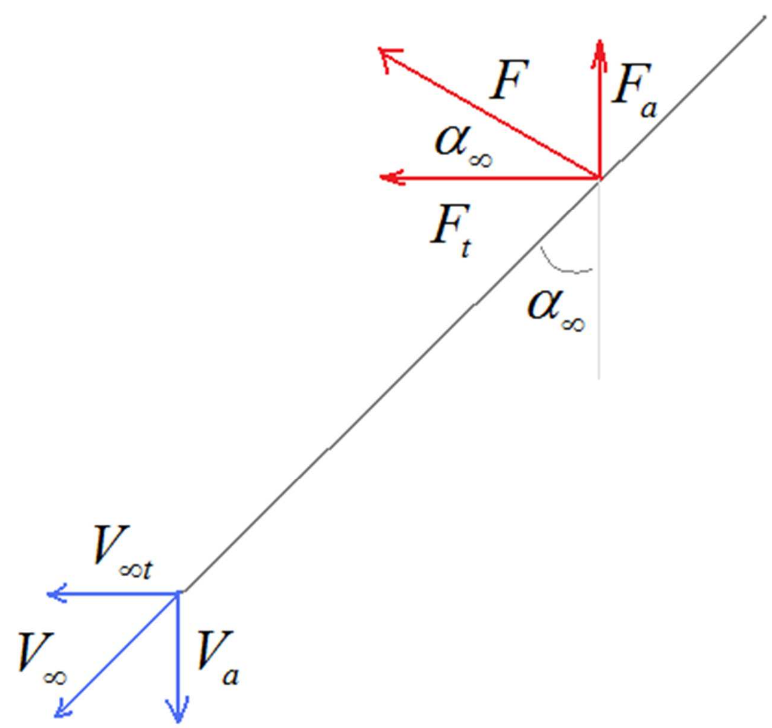
$$\Gamma = 2cu$$

Schiere di pale

$\Delta p_0 = 0$ ipotesi perdite nulle

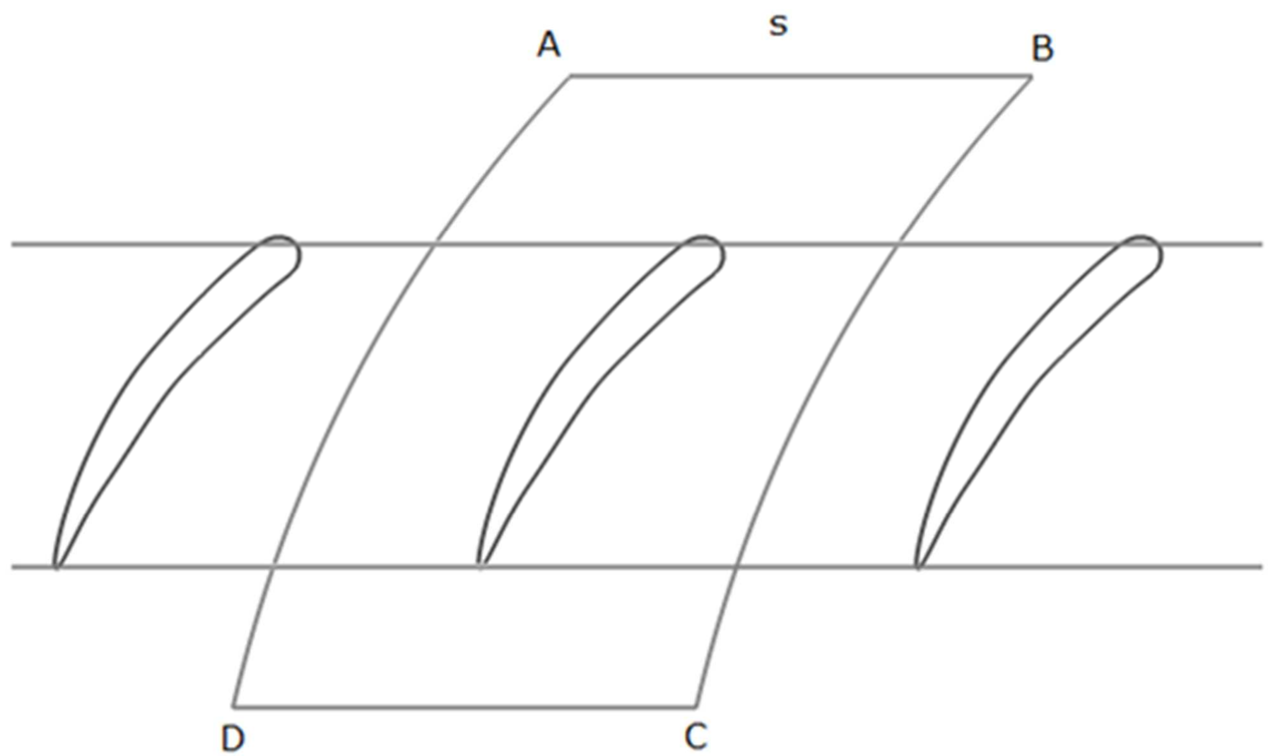
$$F_a = s\rho V_{\infty t}\left(V_{2t} - V_{1t}\right)$$
$$F_t = s\rho V_a\left(V_{1t} - V_{2t}\right)$$

$$\frac{V_{\infty t}}{V_a} = -\frac{F_a}{F_t} = \tan\alpha_\infty$$



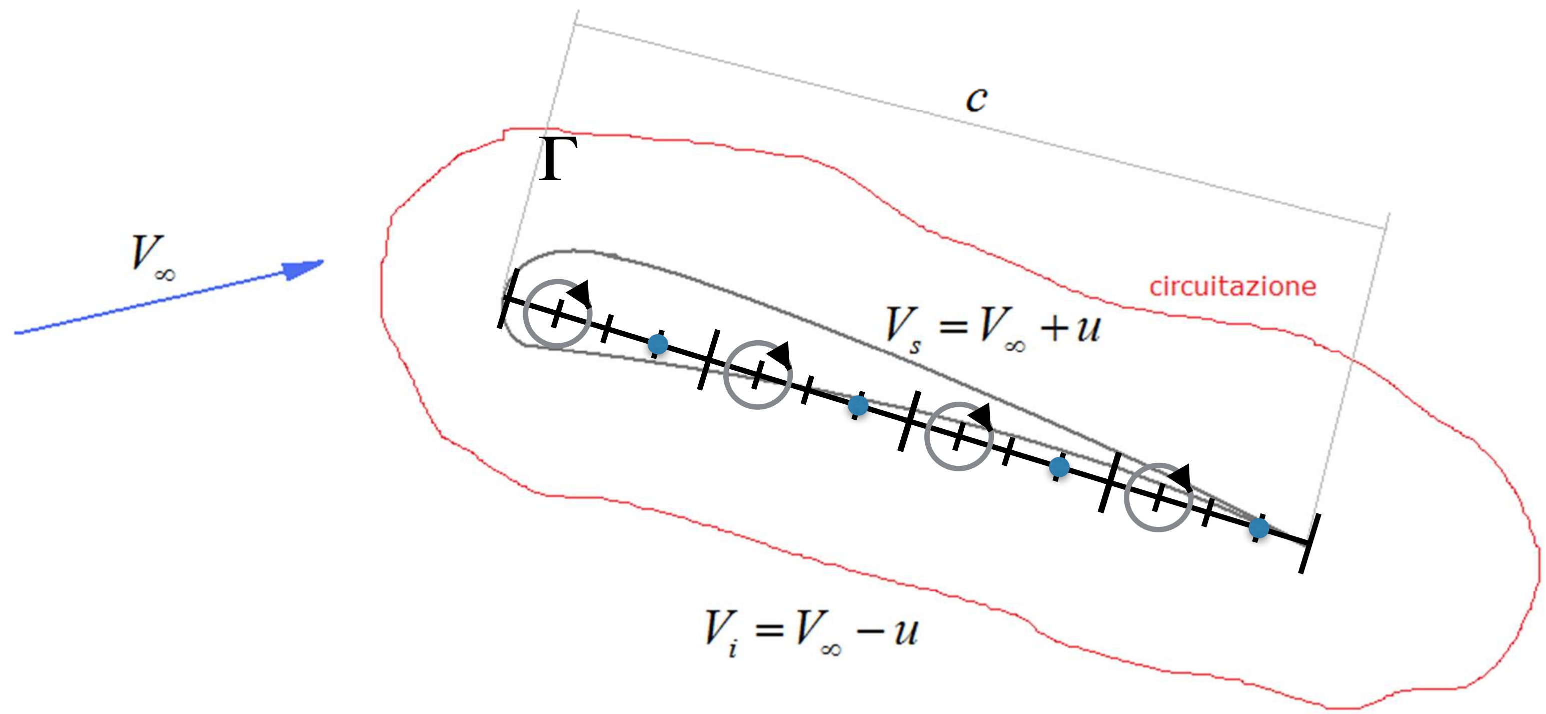
$$F = \frac{F_t}{\cos\alpha_\infty} = \rho\frac{V_a}{\cos\alpha_\infty}s\left(V_{1t} - V_{2t}\right) = \rho V_\infty s\left(V_{1t} - V_{2t}\right) = L$$

Schiere di pale



$$\Gamma = s\left(V_{1t} - V_{2t}\right) \quad \rightarrow \quad L = \rho V_\infty \Gamma$$

Metodo delle singolarità vorticose



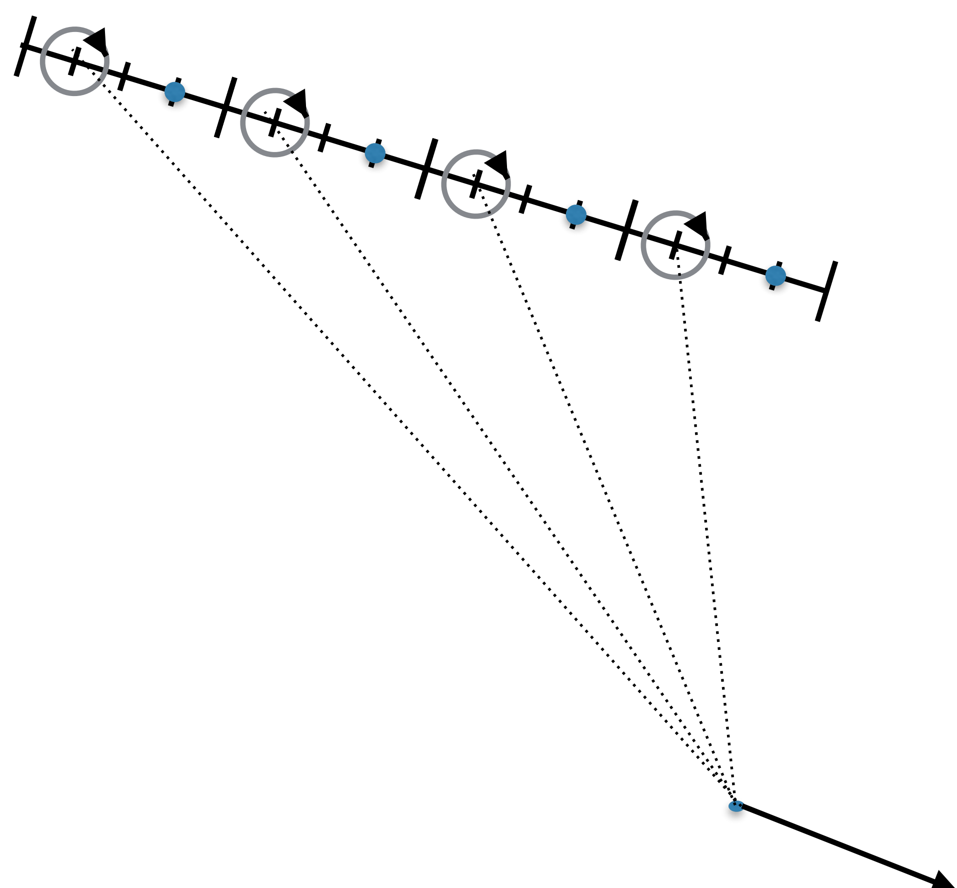
$$\Gamma = \oint_{\ell} \overline{v} \cdot d\overline{\ell}$$

$$L = \rho \Gamma V_\infty$$

$$\Delta p = p_i - p_s = \frac{1}{2}\rho\left(V_s^2 - V_i^2\right) = 2\rho u V_\infty$$

$$\Gamma = 2cu$$

Metodo delle singolarità vorticose



$$V\infty+\sum \frac{\Gamma_i}{2\Pi\ r_i}$$