1) ~~To reduce the number of cases we restrict our attention to~~ ~~$0<b<0.5$~~

~~Moreover, given the range of $b$, we have to use the additional condition $x\geq 0$ (so $(xy)^b$ is a real number)~~

a)

$$L = a(xy)^b - \lambda_1(mx+my-100) - \lambda_2(x-4) - \lambda_3(-x) - \lambda_4(-y)$$

KT CONDITIONS ARE:

$$aby^b x^{b-1} - \lambda_1 m - \lambda_2 + \lambda_3 = 0$$

$$abx^b y^{b-1} - \lambda_1 m + \lambda_4 = 0$$

$$mx+my \leq 100 \qquad \lambda_1 \geq 0 \qquad \lambda_1(mx+my-100)=0$$

$$x \leq 4 \qquad \lambda_2 \geq 0 \qquad \lambda_2(x-4)=0$$

$$-x \leq 0 \qquad \lambda_3 \geq 0 \qquad \lambda_3 x = 0$$

$$-y \leq 0 \qquad \lambda_4 \geq 0 \qquad \lambda_4 y = 0$$

b)

$$M = a(xy)^b - \lambda_1(mx+my-100) - \lambda_2(x-4)$$

KT CONDITIONS ARE:

$$x\geq 0 \qquad aby^b x^{b-1} - \lambda_1 m - \lambda_2 \leq 0 \qquad (aby^b x^{b-1} - \lambda_1 m - \lambda_2)x = 0$$

$$y \geq 0 \qquad abx^b y^{b-1} - \lambda_1 m \leq 0 \qquad (abx^b y^{b-1} - \lambda_1 m)y = 0$$

c) GIVEN THE CONSTRAINTS ARE LINEAR IT IS ENOUGH TO CHECK THE CONCAVITY OF THE OBJECTIVE FUNCTION

THE HESSIAN IS

$$H = \begin{pmatrix} ab(b-1)y^{b}x^{b-2} & ab^2(yx)^{b-1} \\ ab^2(yx)^{b-1} & ab(b-1)x^{b}y^{b-2} \end{pmatrix}$$

THE PRINCIPAL MINORS OF ORDER 1 ARE NONEGATIVE $(\geq 0)$

THE PRINCIPAL MINOR OF ORDER 2 IS:

$$a^2b^2(b-1)^2 y^{2b-2}x^{2b-2} - a^2b^4 y^{2b-2}x^{2b-2} \geq 0 \quad \forall x, y \geq 0$$

INDEED THE INEQUALITY REDUCES TO

$$(b-1)^2 - b^2 \geq 0 \quad (\text{easy to verify})$$

THEN THE OBJECTIVE FUNCTION IS NOT CONCAVE (IT IS CONVEX)

THEREFORE KT CONDITIONS ARE ONLY <u>NECESSARY</u>

2)

a) $\max (100-x) y$

s.t $-xy \leq -10$

$x \leq 2$

$y \geq 0$

$M = (100-x)y - \lambda_1(-xy+10) - \lambda_2(x-2)$ KT CONDITIONS ARE:

$$\begin{cases} -y + \lambda_1 y - \lambda_2 = 0 \\ y \geq 0 \quad (100-x) + \lambda_1 x \leq 0 \quad y(100-x+\lambda_1 x) = 0 \\ \lambda_1 \geq 0 \quad xy \geq 10 \quad \lambda_1(xy-10) = 0 \\ \lambda_2 \geq 0 \quad x \leq 2 \quad \lambda_2(x-2) = 0 \end{cases}$$

– SOLUTION

SUPPOSE $y=0$ CONDITION $xy \geq 10$ IS NOT SATISFIED. THEN IN THE SOLUTION $y>0$ THEREFORE THE SECOND LINE OF KT CONDITION REDUCES TO

$$100 - x + \lambda_1 x = 0$$

WE HAVE TO CONSIDER THE FOLLOWING CASES

1) $\lambda_1 = 0 \quad \lambda_2 = 0$
2) $\lambda_1 > 0 \quad \lambda_2 = 0$
3) $\lambda_1 = 0 \quad \lambda_2 > 0$
4) $\lambda_1 > 0 \quad \lambda_2 > 0$

1) $\lambda_1 = 0 \quad \lambda_2 = 0$

FROM THE FIRST CONDITION WE HAVE $y = 0$

THEN THIS CASE CANNOT BE A SOLUTION

(SEE ABOVE)

2) $\lambda_1 > 0 \quad \lambda_2 = 0$ KT CONDITIONS ARE

$$\begin{cases} -y + \lambda_1 y = 0 \\ 100 - x + \lambda_1 x = 0 \\ xy = 10 \\ x \leq 2 \end{cases}$$

FROM THE FIRST WE HAVE $\lambda_1 = 1$. REPLACING $\lambda_1$
IN THE SECOND EQUATION WE GET $100 = 0$
THAT IS NO TRUE
THEN THIS CASE CANNOT BE A SOLUTION

3) $\lambda_1 = 0 \quad \lambda_2 > 0$

THE FIRST EQUATION OF KT CONDITION IS

$$\lambda_2 = -y$$

GIVEN THAT $y > 0$ IT IMPLIES THAT $\lambda_2 < 0$

A CONTRADDICTION WITH THE ASSUMPTION OF $\lambda_2 > 0$

THEN THIS CASE IS NOT A SOLUTION

a) $\lambda_1 > 0 \quad \lambda_2 > 0$ KT CONDITIONS ARE

$$\begin{cases} -y + \lambda_1 y - \lambda_2 = 0 \\ 100 - x + \lambda_1 x = 0 \\ xy = 10 \\ x = 2 \end{cases}$$

FROM THE LAST TWO EQUATION WE GET

$$x = 2 \qquad y = 5$$

REPLACING IN THE FIRST TWO EQUATIONS

$$\begin{cases} -5 + \lambda_1 5 - \lambda_2 = 0 \\ 98 + 2\lambda_1 = 0 \end{cases}$$

$$\lambda_1 = 49 \qquad \lambda_2 = 5\lambda_1 - 5 = 5 \cdot 49 - 5 = 240$$

SOLUTION OF KT CONDITION IS

$$x = 2 \quad y = 5 \quad \lambda_1 = 49 \quad \lambda_2 = 240$$

b) WE CHECK QUASI CONCAVITY OF THE OBJECTIVE FUNCTION

THE BORDERED HESSIAN IS

$$H_b = \begin{bmatrix} 0 & -y & 100-x \\ -y & 0 & -1 \\ 100-x & -1 & 0 \end{bmatrix} \qquad |H_b| = 2(100-x)y \geq 0$$

$$B_1 = -y^2 \leq 0 \quad B_2 = |H_b| = 2(100-x)y \geq 0 \quad \forall x, y.$$

quasi convex

THEN THE FUNCTION IS QUASICONCAVE. GIVEN THAT CONSTRAINTS ARE XXXXXX XXX THE FIRST DERIVATIVES OF THE OBJECTIVE FUNCTION EVALUATED IN THE SOLUTION ARE DIFFERENT FROM ZERO KT CONDITIONS ARE NECESSARY AND SUFFICIENT

5

there exist a vector satisfying strictly all constraint

the derivatives of the constraints evaluated in the solution are not all equal tozero

3) a)

$$M = (x-1)^2 + (y-1)^2 - \lambda_1(x-2) - \lambda_2(y-2)$$

KT CONDITIONS ARE

$2(x-1) - \lambda_1 \leq 0 \quad x \geq 0 \quad x(2(x-1)-\lambda_1) = 0$

$2(y-1) - \lambda_2 \leq 0 \quad y \geq 0 \quad y(2(y-1)-\lambda_2) = 0$

$\lambda_1 \geq 0 \quad x-2 \leq 0 \quad \lambda_1(x-2) = 0$

$\lambda_2 \geq 0 \quad y-2 \leq 0 \quad \lambda_2(y-2) = 0$

WE CONSIDER THE FOLLOWING CASES

1) $x=0 \quad y=0$
2) $x>0 \quad y=0$
3) $x=0 \quad y>0$
4) $x>0 \quad y>0$

NOTE CONSTRAINTS ARE LINEAR BUT OBJECTIVE FUNCTION IS CONVEX
THEN KT CONDITION ARE ONLY NECESSARY

1) $x=0 \quad y=0$

KT CONDITIONS ARE

$-2-\lambda_1 \leq 0$

$-2-\lambda_2 \leq 0$

note that $\lambda_1 = \lambda_2 = 0$

THEN KT CONDITIONS ARE SATISFIED

2) $x > 0 \quad y = 0$ Then $\lambda_2 = 0$

KT CONDITIONS ARE

$2(x-1) - \lambda_1 = 0$

$-2 \leq 0$

$\lambda_1 \geq 0 \quad x - 2 \leq 0 \quad \lambda_1(x-2) = 0$

$-2 \leq 0$

IF $\lambda_1 = 0$ FROM THE FIRST CONDITION WE HAVE $x = 1$

THIRD CONDITION IS SATISFIED $-1 \leq 0$

THEN $x = 1 \ y = 0 \ \lambda_1 = 0 \ \lambda_2 = 0$ COULD BE A SOLUTION

IF $\lambda_2 > 0$ THEN $x = 2$ REPLACING IN THE FIRST CONDITION WE GET $\lambda_1 = 2$

THEN $x = 2 \ y = 0 \ \lambda_1 = 2 \ \lambda_2 = 0$ COULD BE A SOLUTION

3) $x = 0 \quad y > 0$ then $\lambda_1 = 0$

KT CONDITIONS ARE

$-2 \leq 0$

$2(y-1) - \lambda_2 = 0$

$-2 \leq 0$

$\lambda_2 \geq 0 \quad y - 2 \leq 0 \quad \lambda_2(y-2) = 0$

IF $\lambda_2 = 0 \longrightarrow y = 1$ LAST CONDITION IS SATISFIED

$\lambda_1 = 0 \ \lambda_2 = 0 \ x = 0 \ y = 1$ COULD BE A SOLUTION

IF $\lambda_2 > 0 \longrightarrow y = 2 \longrightarrow \lambda_2 = 2$

$x = 0 \ y = 2 \ \lambda_1 = 0 \ \lambda_2 = 2$ COULD BE A SOLUTION

4) $x > 0 \quad y > 0$ KT CONDITIONS ARE:

$$2(x-1) - \lambda_1 = 0$$
$$2(y-1) - \lambda_2 = 0$$
$$\lambda_1 \geq 0 \quad x-2 \leq 0 \quad \lambda_1(x-2) = 0$$
$$\lambda_2 \geq 0 \quad y-2 \leq 0 \quad \lambda_2(y-2) = 0$$

a) $\lambda_1 = \lambda_2 = 0 \quad x = y = 1$ FROM FIRST AND SECOND CONDITIONS

THIRD AND FOURTH CONDITIONS ARE SATISFIED

THEN IT COULD BE A SOLUTION

b) $\lambda_1 > 0 \quad \lambda_2 = 0$. FROM THE SECOND CONDITION $y = 1$

FROM THIRD CONDITION $x = 2$. REPLACING IN THE FIRST

WE GET $\lambda_1 = 2$. FOURTH CONDITION IS SATISFIED

THEN $x = 2 \; y = 1 \; \lambda_1 = 2 \; \lambda_2 = 0$ COULD BE A SOLUTION

c) $\lambda_1 = 0 \quad \lambda_2 > 0$ Then $y = 2$ FROM FIRST CONDITION $x = 1$.

FROM THE SECOND $\lambda_2 = 2$. THIRD CONDITION IS SATISFIED.

d) $\lambda_1 > 0 \quad \lambda_2 > 0 \rightarrow x = y = 2$

FROM THE FIRST AND SECOND CONDITION WE GET $\lambda_1 = \lambda_2 = 2$.

IT COULD BE ~~A~~ SOLUTION.

TO GET THE SOLUTION WE HAVE TO EVALUATE

~~EACH~~ THE OBJECTIVE FUNCTION AT EACH SOLUTION

OF KT CONDITIONS

| | $x$ | $y$ | $\lambda_1$ | $\lambda_2$ | $(x-1)^2+(y-1)^2$ | |
|---|---|---|---|---|---|---|
| a) | 0 | 0 | 0 | 0 | 2 | ← |
| b) | 1 | 0 | 0 | 0 | 1 | |
| c) | 2 | 0 | 2 | 0 | 2 | ← |
| d) | 0 | 1 | 0 | 0 | 1 | |
| e) | 0 | 2 | 0 | 2 | 2 | ← |
| f) | 1 | 1 | 0 | 0 | 0 | |
| g) | 2 | 1 | 2 | 0 | 1 | |
| h) | 1 | 2 | 0 | 2 | 1 | |
| i) | 2 | 2 | 2 | 2 | 2 | ← |

SOLUTION OF THE PROBLEM ARE a), c), e), i)

3b) THE PROBLEM CAN BE WRITTEN AS

$$\max_{\{x,y\}} -(x-1)^2 - (y-1)^2$$
$$\text{s.t. } 0 \leq x \leq 2$$
$$0 \leq y \leq 2$$

THE HESSIAN OF THE OBJECTIVE FUNCTION IS

$$H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

IT IS NEGATIVE DEFINITE THEN THE FUNCTION IS CONCAVE

KT CONDITIONS ARE NECESSARY AND SUFFICIENT (NOTE CONSTRAINT ARE LINEAR)

MODIFIED LAGRANGEAN IS

$$M = -(x-1)^2 - (y-1)^2 - \lambda_1(x-2) - \lambda_2(y-2)$$

KT CONDITIONS ARE

$$-2(x-1) - \lambda_1 \leq 0 \qquad x \geq 0 \qquad x(-2(x-1) - \lambda_1) = 0$$
$$-2(y-1) - \lambda_2 \leq 0 \qquad y \geq 0 \qquad y(-2(y-1) - \lambda_2) = 0$$
$$\lambda_1 \geq 0 \qquad x-2 \leq 0 \qquad \lambda_1(x-2) = 0$$
$$\lambda_2 \geq 0 \qquad y-2 \leq 0 \qquad \lambda_2(y-2) = 0$$

SUPPOSE $x=0$ THEN $\lambda_1=0$ THE FIRST CONDITION BECOMES $2 \leq 0$, THAT IS NO TRUE.

SUPPOSE $y=0$ THEN $\lambda_2=0$ THE SECOND CONDITION BECOMES $2 \leq 0$ THAT IS NO TRUE.

THEN THE ONLY CANDIDATE TO A SOLUTION IS
$x>0 \quad y>0$
IN THIS CASE KT CONDITIONS ARE

$$-2(x-1)-\lambda_1=0$$
$$-2(y-1)-\lambda_2=0$$
$$\lambda_1 \geq 0 \quad x-2 \leq 0 \quad \lambda_1(x-2)=0$$
$$\lambda_2 \geq 0 \quad y-2 \leq 0 \quad \lambda_2(y-2)=0$$

SUPPOSE $\lambda_1>0$ THEN FROM THE THIRD CONDITION $x=2$
REPLACING IN THE FIRST CONDITION $\lambda_1=-2$
A CONTRADDICTION WITH THE ASSUMPTION $\lambda_1>0$
SUPPOSE $\lambda_2>0$ BY FOURTH CONDITION $y=2$ AND REPLACING IN THE SECOND $\lambda_2=-2$ A CONTRADDICTION
THE ONLY POSSIBLE SOLUTION IS FOR $\lambda_1=\lambda_2=0$
FROM THE FIRST AND SECOND CONDITION WE GET
$x=y=1$
CONDITION 3 AND 4 ARE SATISFIED

THEN

$$x=1 \quad y=1 \quad \lambda_1=0 \quad \lambda_2=0$$

IS THE SOLUTION OF THE PROBLEM

4) THE OBJECTIVE FUNCTION IS LINEAR

THEN CONCAVE

CONSTRAINTS ARE CONVEX

$x = y = 0.5$ SATISFIES STRICTLY BOTH CONSTRAINTS

THEN KT CONDITIONS ARE NECESSARY AND SUFFICIENT

$$L = x + ay - \lambda_1 (x^2 + y^2 - 1) - \lambda_2 (-x - y)$$

KT CONDITION ARE

$$1 - 2x\lambda_1 + \lambda_2 = 0$$

$$a - 2y\lambda_1 + \lambda_2 = 0$$

$$\lambda_1 \geq 0 \quad x^2 + y^2 \leq 1 \qquad \lambda_1 (x^2 + y^2 - 1) = 0$$

$$\lambda_2 \geq 0 \quad x + y \geq 0 \qquad \lambda_2 (x + y) = 0$$

WE HAVE TO CONSIDER THE FOLLOWING CASES

1) $\lambda_1 = 0 \quad \lambda_2 = 0$

2) $\lambda_1 > 0 \quad \lambda_2 = 0$

3) $\lambda_1 = 0 \quad \lambda_2 > 0$

4) $\lambda_1 > 0 \quad \lambda_2 > 0$

1) $\lambda_1 = 0 \quad \lambda_2 = 0$

THE FIRST CONDITION BECOMES $1 = 0$

NO TRUE. THIS CANNOT BE A SOLUTION

2) $\lambda_1 > 0 \quad \lambda_2 = 0$ KT CONDITIONS ARE

$$1 - 2x\lambda_1 = 0$$
$$a - 2y\lambda_1 = 0$$
$$x^2 + y^2 = 1$$
$$x + y \geq 0$$

FROM THE FIRST TWO CONDITION WE SEE THAT IN THE SOL

$x \neq 0 \quad y \neq 0$, THEN

$$\lambda_1 = \frac{1}{2x} \quad \lambda_1 = \frac{a}{2y} \rightarrow \frac{1}{2x} = \frac{a}{2y} \rightarrow y = ax$$

REPLACING IN THE FIRST CONSTRAINT

$$x^2 + a^2x^2 = 1 \qquad x^2 = \frac{1}{1+a^2} \qquad y^2 = \frac{a^2}{1+a^2}$$

$$x = \pm\sqrt{\frac{1}{1+a^2}} \qquad y = \pm\sqrt{\frac{a^2}{1+a^2}}$$

NEGATIVE SOLUTIONS PRODUCE A CONTRADDICTION INDEED IN SUCH CASE REPLACING A NEGATIVE $x$ ($y$) IN THE FIRST (SECOND) CONDITION WE GET $\lambda_1 < 0$

THEN $x = \sqrt{\frac{1}{1+a^2}} \quad y = \sqrt{\frac{a^2}{1+a^2}} \quad \lambda_1 = \sqrt{1+a^2} \quad \lambda_2 = 0$

IS A SOLUTION FOR ALL VALUES OF $a$

3) $\lambda_1 = 0 \quad \lambda_2 > 0$

FROM THE FIRST CONDITION WE GET

$\lambda_2 = -1$ A CONTRADDICTION

4) $\lambda_1 > 0 \quad \lambda_2 > 0$ KT CONDITIONS ARE

$1 - 2x\lambda_1 + \lambda_2 = 0$

$a - 2y\lambda_1 + \lambda_2 = 0$

$x^2 + y^2 = 1$

$x + y = 0 \longrightarrow y = -x$

$x^2 + (-x)^2 = 1 \qquad 2x^2 = 1 \qquad x^2 = \frac{1}{2} \qquad x = \pm \frac{1}{\sqrt{2}}$

a) $x = \frac{1}{\sqrt{2}}$ AND $y = -\frac{1}{\sqrt{2}}$

OR

b) $x = -\frac{1}{\sqrt{2}}$ AND $y = \frac{1}{\sqrt{2}}$

REPLACE a) IN THE FIRST TWO CONDITIONS

$1 - \sqrt{2}\lambda_1 + \lambda_2 = 0$

$a + \sqrt{2}\lambda_1 + \lambda_2 = 0$

$\longrightarrow \quad 1 - \lambda_1\sqrt{2} = a + \lambda_1\sqrt{2}$

$\lambda_1 = \frac{1-a}{2\sqrt{2}}$

$\lambda_2 = \sqrt{2}\lambda_1 - 1 = \frac{1-a}{2} - 1 = -\frac{a}{2} - 1$

$\lambda_1 > 0$ ONLY IF $a < 1$ $\qquad \lambda_2 > 0$ ONLY IF $a < -2$

THEN $x = \frac{1}{\sqrt{2}} \quad y = -\frac{1}{\sqrt{2}} \quad \lambda_1 = \frac{1-a}{\sqrt{2}} \quad \lambda_2 = -a$

IS A SOLUTION ONLY FOR $a <$

REPLACE b) IN THE FIRST TWO CONDITIONS

$$\begin{cases} 1 + \lambda_1 \sqrt{2} + \lambda_2 = 0 \\ a - \lambda_1 \sqrt{2} + \lambda_2 = 0 \end{cases}$$

$$1 + \lambda_1 \sqrt{2} = a - \lambda_1 \sqrt{2}$$

$$\lambda_1 = \frac{a-1}{2\sqrt{2}}$$

$$\lambda_2 = -1 - \lambda_1 \sqrt{2} = -1 - \frac{a-1}{2} = -\frac{a+1}{2}$$

$\lambda_1 > 0$ ONLY IF $a > 1$

$\lambda_2 > 0$ ONLY IF $a < -1$

THEN NO SOLUTION

(5)

a) CONSTRAINTS ARE LINEAR, THEN CONCAVE SO KT CONDITIONS ARE NECESSARY

b) THE HESSIAN OF THE OBJECTIVE FUNCTION IS

$$H = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

THE LEADING PRINCIPAL MINOR OF ORDER 1 IS EQUAL TO $-2$

THAT OF ORDER 2 IS

$$|H| = (-2)(-2) - (-1)(-1) = 4 - 1 = 3 > 0$$

FUNCTION IS CONCAVE

CONSTRAINTS ARE LINEAR

KT CONDITIONS ARE SUFFICIENT.

c) $$L = -x_1^2 - x_1 x_2 - x_2^2 - \lambda_1 (x_1 - 2x_2 + 1) - \lambda_2 (2x_1 + x_2 - 2)$$

$$-2x_1 - x_2 - \lambda_1 - 2\lambda_2 = 0$$

$$-2x_2 - x_1 + 2\lambda_1 - \lambda_2 = 0$$

$$\lambda_1 \geq 0 \quad x_1 - 2x_2 \leq -1 \qquad \lambda_1 (x_1 - 2x_2 + 1) = 0$$

$$\lambda_2 \geq 0 \quad 2x_1 + x_2 \leq 2 \qquad \lambda_2 (2x_1 + x_2 - 2) = 0$$

1) $\lambda_1 = 0 \quad \lambda_2 = 0$

$$\left.\begin{array}{l} -2x_1 - x_2 = 0 \\ -2x_2 - x_1 = 0 \\ x_1 - 2x_2 \leq -1 \\ 2x_1 + x_2 \leq 2 \end{array}\right| \longrightarrow \begin{array}{l} x_1 = x_2 = 0 \\ \lambda_1 = \lambda_2 = 0 \end{array}$$

IS NOT A SOLUTION
BECAUSE DOES NOT SATISFIES
THE THIRD CONDITION

2) $\lambda_1 > 0 \quad \lambda_2 = 0$

$$\begin{cases} -2x_1 - x_2 - \lambda_1 = 0 & \longrightarrow \lambda_1 = -2x_1 - x_2 \\ -2x_2 - x_1 + 2\lambda_1 = 0 & \longrightarrow -2x_2 - x_1 - 4x_1 - 2x_2 = 0 \\ & \qquad -5x_1 - 4x_2 = 0 \\ x_1 - 2x_2 = -1 \\ 2x_1 + x_2 \leq 2 \end{cases}$$

$$\begin{cases} -5x_1 - 4x_2 = 0 & \longrightarrow -10x_2 + 5 - 4x_2 = 0 \\ x_1 = 2x_2 - 1 & \qquad 14x_2 = 5 \qquad x_2 = \frac{5}{14} \\ 2x_1 + x_2 \leq 2 & \qquad x_1 = 2 \cdot \frac{5}{14} - 1 = -\frac{4}{14} \end{cases}$$

LAST CONDITION IS SATISFIED

$$\lambda_1 = \frac{8}{14} - \frac{5}{14} = \frac{3}{14}$$

$$x_1 = -\frac{4}{14} \quad x_2 = \frac{5}{14} \quad \lambda_1 = \frac{3}{14} \quad \lambda_2 = 0$$

IS A SOLUTION

3) $\lambda_1 = 0 \quad \lambda_2 > 0$

$$\begin{cases} -2x_1 - x_2 - 2\lambda_2 = 0 \\ -2x_2 - x_1 - \lambda_2 = 0 \\ x_1 - 2x_2 \leq -1 \\ 2x_1 + x_2 = 2 \end{cases}$$

$\lambda_2 = -2x_2 - x_1$

$-2x_1 - x_2 + 4x_2 + 2x_1 = 0 \longrightarrow x_2 = 0$

$x_1 = 1 \qquad \lambda_2 = -1$ INCONSISTENCY.

4) $\lambda_1 > 0 \quad \lambda_2 > 0$

$$\begin{cases} -2x_1 - x_2 - \lambda_1 - 2\lambda_2 = 0 \\ -2x_2 - x_1 + 2\lambda_1 - \lambda_2 = 0 \\ x_1 - 2x_2 = -1 \\ 2x_1 + x_2 = 2 \end{cases}$$

$x_1 = 2x_2 - 1$

$4x_2 - 2 + x_2 = 2 \qquad 5x_2 = 4 \qquad x_2 = \frac{4}{5}$

$x_1 = \frac{3}{5}$

$$\begin{cases} -\frac{6}{5} - \frac{4}{5} - \lambda_1 - 2\lambda_2 = 0 \\ -\frac{8}{5} - \frac{3}{5} + 2\lambda_1 - \lambda_2 = 0 \end{cases}$$

$$\begin{cases} \lambda_1 = 2\lambda_2 + 2 \\ 2\lambda_1 - \lambda_2 = 1 \end{cases}$$

$$4\lambda_2 + 4 - \lambda_2 = 1$$

$$5\lambda_2 = -3 \qquad \lambda_2 = -\frac{3}{5}$$

AN INCONSISTENCY ARISES