Problem set 5

1. Find the general and the definite solution of the following differential equations. Check the validity of your answers.
   a. $\frac{dy}{dt} + 4y = 12; \quad y(0) = 2$
   b. $\frac{dy}{dt} - 2y = 0; \quad y(0) = 9$
   c. $\frac{dy}{dt} + 10y = 15; \quad y(0) = 0$
   d. $2\frac{dy}{dt} + 4y = 6; \quad y(0) = 1.5$

2. Solve the following first order linear differential equations
   a. $\frac{dy}{dt} + 2ty = t; y(0) = 1.5$
   b. $\frac{dy}{dt} + t^2y = 5t^2; \ y(0) = 6$
   c. $2\frac{dy}{dt} + 12y + 2e^t = 0; y(0) = \frac{6}{7}$
   d. $\frac{dy}{dt} + y = t$; y(0)=1

3. Verify that each of the following differential equation is exact, then solve it.
   a. $3y^2t\,dy + (y^3 + 2t)\,dt = 0$
   b. $t(1 + 2y)dy + y(1 + y)\,dt = 0$
   c. $\frac{dy}{dt} + \frac{2y^4t+3t^2}{4y^3t^2} = 0$

4. Are the following differential equations exact? If not try y, t, y^2 as possible integrating factors
   a. $2(t^3 + 1)\,dy + 3yt^2\,dt = 0$
   b. $4y^3t\,dy + (2y^4 + 3t)dt = 0$

5. Applying the method to solve an exact differential equation to the general exact differential equation $M\,dy + N\,dt = 0$ derive the following formula for the general solution of an exact differential equation $\int M\,dy + \int N\,dt - \int \frac{d\int M dy}{dt}\,dt = c$. Verify that this formula is the solution of exact differential equation $M\,dy + N\,dt = 0$

6. Exercises 1, 2,3, 4 in section 15.5 of the textbook