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## Fundamentals of vibrations and waves

### Introduction to the course

- Earth System

- PDEs

- What is a wave?

### 1DOF systems: Harmonic oscillator

- Differential equation

  - Harmonic solution

  - Initial conditions

- Natural frequency

- Mathematical & physical pendulum

### 1DOF systems: Damped and Forced oscillator

- Damping

  - Damping regimes; Q factor

- Forcing

  - Transients and stationary regime

  - Resonance

### 2&N DOF systems: Coupled oscillators

- Discrete propagating systems

  - “Normal” modes

**The wave equation**

Transverse waves on a string

The wave function

D'Alembert solution

Harmonic waves

Sound waves

Energy, Power & Intensity: Decibel

**Wave phenomena**

Superposition principle

Interference

Beats

Heterogeneous string

Reflection and transmission

Boundary conditions & modes

String with fixed and free ends

Air columns with fixed and free ends

**Tutorial: Fourier**

Complex sound waves

Fourier synthesis & analysis

Vibrating string

## Waves in solids (Seismic waves)

### Elasticity

- Theory of elasticity
  - Deformation
  - Strain tensor
  - Stress tensor
- Generalized Hooke's law
- Navier equations

### Elastic body waves

- Body waves (P and S)
  - Helmholtz's theorem
  - Harmonic and spherical body waves
- Body waves at interfaces
  - Free surface
  - Apparent velocity

### Tutorial: Wave propagation

- Reflection and refraction
- Snell's law

### Rays and body waves

- Traveltimes in layered media
  - Direct, reflected and head waves
  - Ray Parameter
- Traveltimes in layered spherical media
  - Ray Parameter

### Surface waves

- Surface waves
  - Rayleigh waves in a halfspace
  - Phase velocity

## Dispersion

Group and phase velocity  
SH waves in plates  
Modes  
Dispersion  
Surface waves in layered halfspaces  
Love & Rayleigh waves

## Free modes of the Earth

3D: wave equation in spherical coordinates  
Spherical harmonics  
Torsional & Spheroidal modes

## References

- 1) Scales, J., and Snieder, R., 1999. What is a wave?,  
Nature, 401, 739-740;
- 2) Main, I., 1993. **Vibrations and Waves in Physics,**  
**Wiley, 377 pp.**
- 3) Graff, K., 1975. Wave motion in elastic solids,  
Ohio State University Press, 649 pp.
- 4) Stein, S., and Wysession, M., 2003.  
An introduction to seismology, earthquakes, and earth  
structure,  
Blackwell Publishing, 498 pp.

## Useful applets and animation links:

<http://www.acs.psu.edu/drussell/demos.html>

<https://www.walter-fendt.de/html5/phen/>

<http://www.falstad.com/mathphysics.html>