



The Abdus Salam
International Centre
for Theoretical Physics

Postgraduate Diploma Programme
Earth System Physics

Wave physics
Seismic surface waves

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Surface Waves



Surface waves in an elastic half spaces: Rayleigh waves

- Potentials
- Free surface boundary conditions
- Solutions propagating along the surface, decaying with depth
- Lamb's problem

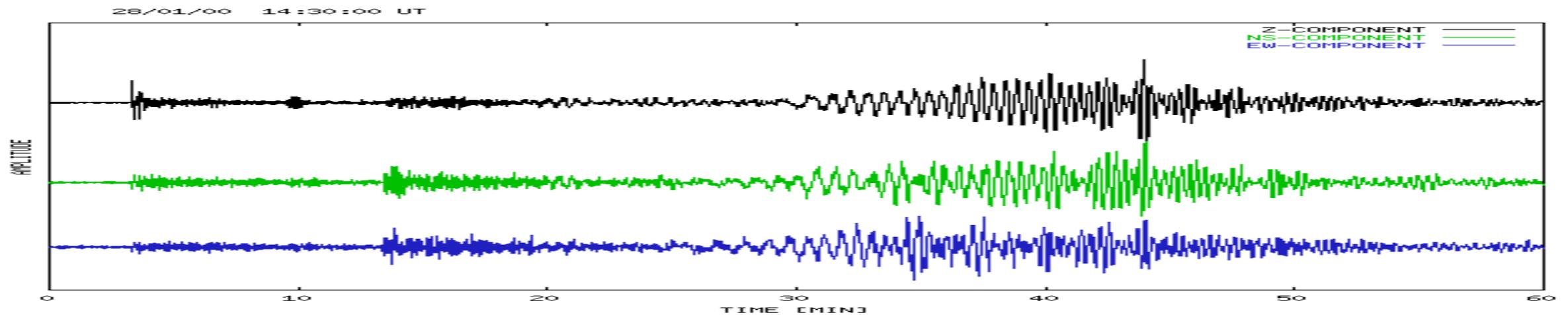
Surface waves in media with depth-dependent properties: Love waves

- Constructive interference in a low-velocity layer
- Dispersion curves
- Phase and Group velocity

Free Oscillations

- Spherical Harmonics
- Modes of the Earth
- Rotational Splitting

Data Example



Question:

We derived that Rayleigh waves are non-dispersive!

But in the observed seismograms we clearly see a highly dispersed surface wave train?

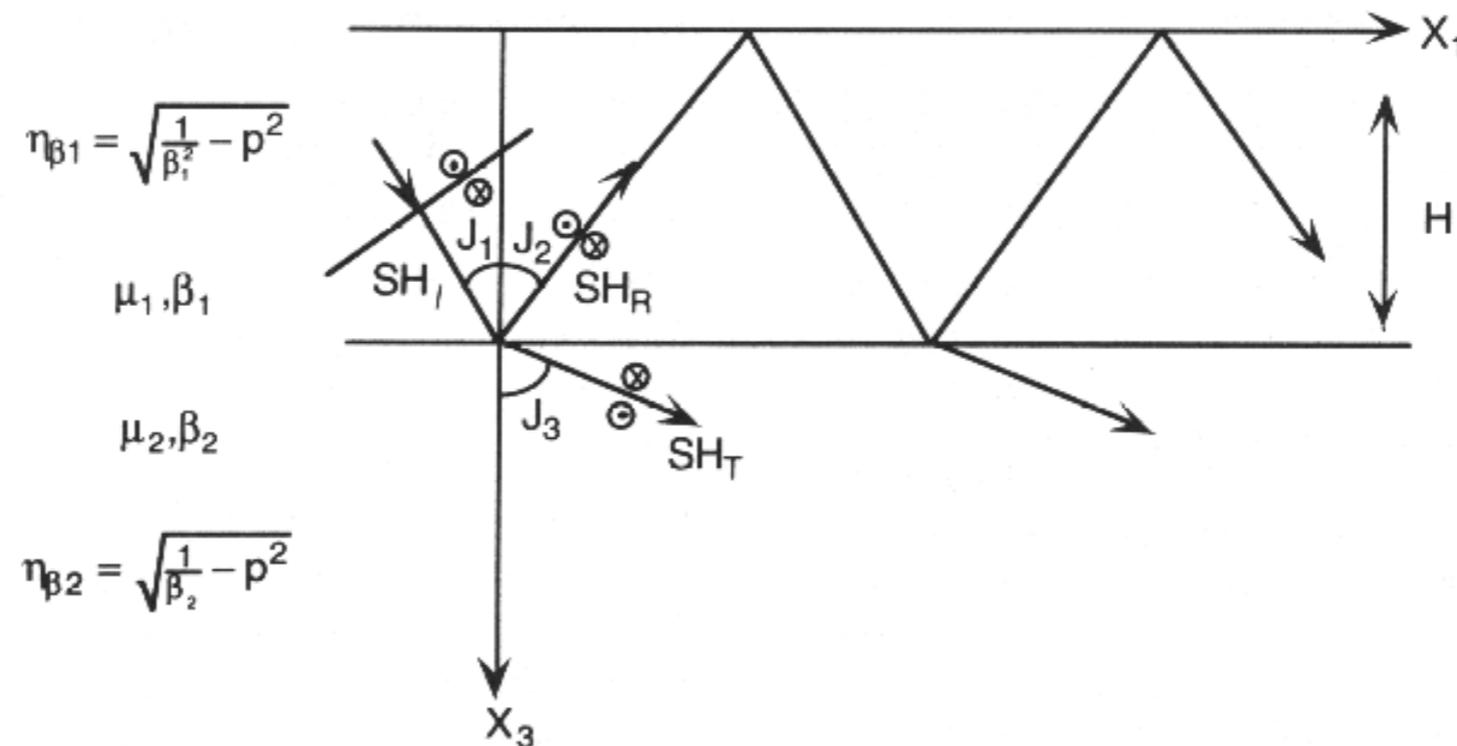
We also see dispersive wave motion on both horizontal components!

Do SH-type surface waves exist?

Why are the observed waves dispersive?

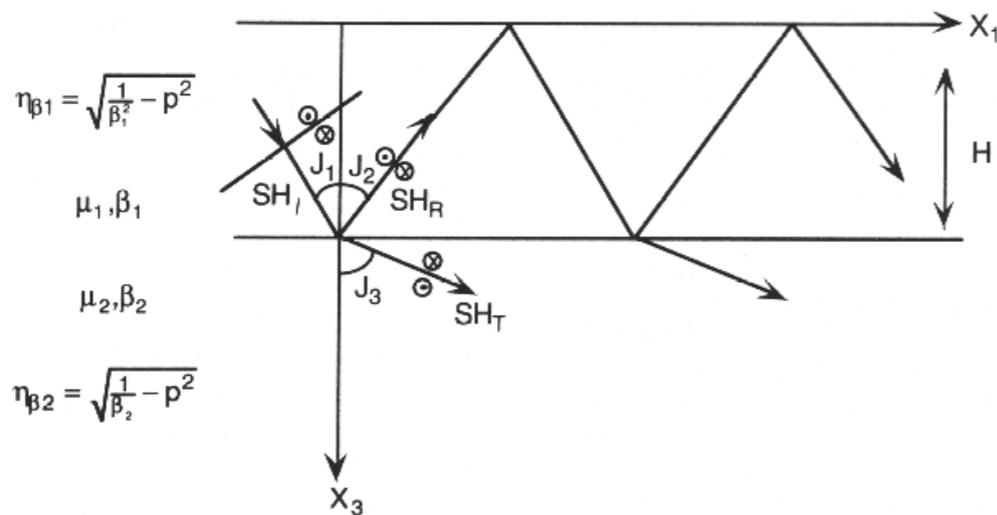
Love Waves: Geometry

In an elastic half-space no SH type surface waves exist. Why? Because there is total reflection and no interaction between an evanescent P wave and a phase shifted SV wave as in the case of Rayleigh waves. What happens if we have a layer over a half space (Love, 1911) ?



Repeated reflection in a layer over a half space.
 Interference between incident, reflected and transmitted SH waves.
 When the layer velocity is smaller than the halfspace velocity, then there is a critical angle beyond which SH reverberations will be totally trapped.

Love waves: trapping - 1



$$u_{y1} = A \exp[i(\omega t + \omega \eta_{\beta 1} z - kx)] + B \exp[i(\omega t - \omega \eta_{\beta 1} z - kx)]$$

$$u_{y2} = C \exp[i(\omega t - \omega \eta_{\beta 2} z - kx)]$$

$$k = k_x = \frac{\omega}{c}; \quad \omega \eta_{\beta} = k_z = \frac{\omega}{c} \sqrt{\frac{c^2}{\beta^2} - 1} = k r_{\beta}$$

$$u_{y1} = A \exp[i(\omega t + k r_{\beta 1} z - kx)] + B \exp[i(\omega t - k r_{\beta 1} z - kx)]$$

$$u_{y2} = C \exp[i(\omega t - k r_{\beta 2} z - kx)]$$

Love waves: trapping - 2

The formal derivation is very similar to the derivation of the Rayleigh waves. The conditions to be fulfilled are:

1. Free surface condition
2. Continuity of stress on the boundary
3. Continuity of displacement on the boundary
4. No radiation in the halfspace

$$1. \quad \sigma_{zy1}(0) = \mu_1 \left. \frac{\partial u_{y1}}{\partial z} \right|_0 = ikr_{\beta1} \left\{ A \exp[i(\omega t - kx)] - B \exp[i(\omega t - kx)] \right\} = 0$$

$$2. \quad \sigma_{zy1}(H) = \mu_1 \left. \frac{\partial u_{y1}}{\partial z} \right|_H = \sigma_{zy2}(H) = \mu_2 \left. \frac{\partial u_{y2}}{\partial z} \right|_H \quad 3. \quad u_{y1}(H) = u_{y2}(H)$$

$$4. \quad \lim_{\infty} u_{y2}(z) = 0 \quad \text{i.e. } c < \beta_2 \quad \text{i.e.} \quad r_{\beta2} = -i \sqrt{1 - \frac{c^2}{\beta_2^2}}$$

Love waves dispersion eqn.

We obtain a condition for which solutions exist. This time we obtain a frequency-dependent solution: a **dispersion relation**

$$\tan(H\omega \sqrt{1/\beta_1^2 - 1/c^2}) = \frac{\mu_2 \sqrt{1/c^2 - 1/\beta_2^2}}{\mu_1 \sqrt{1/\beta_1^2 - 1/c^2}}$$

... indicating that there are only solutions if ...

$$\beta_1 < c < \beta_2$$

Love Waves: Solutions

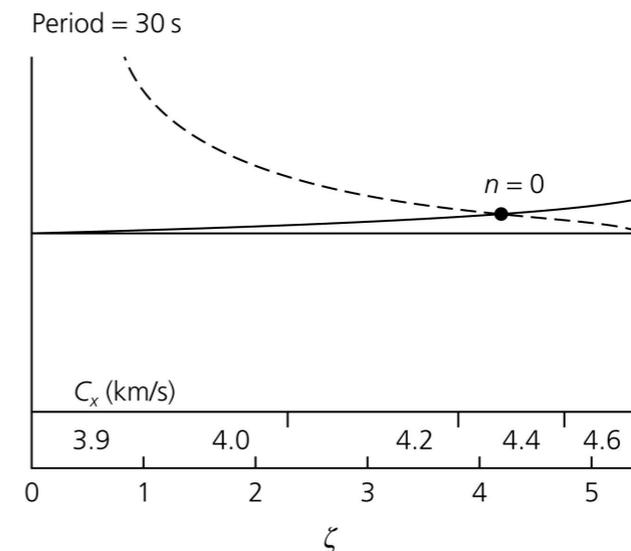
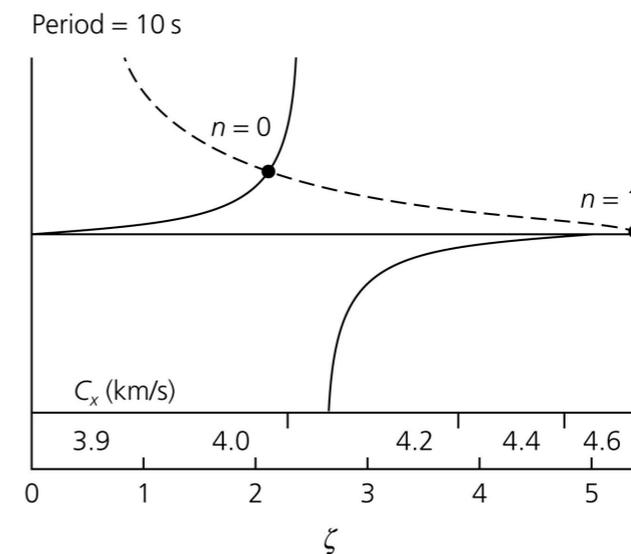
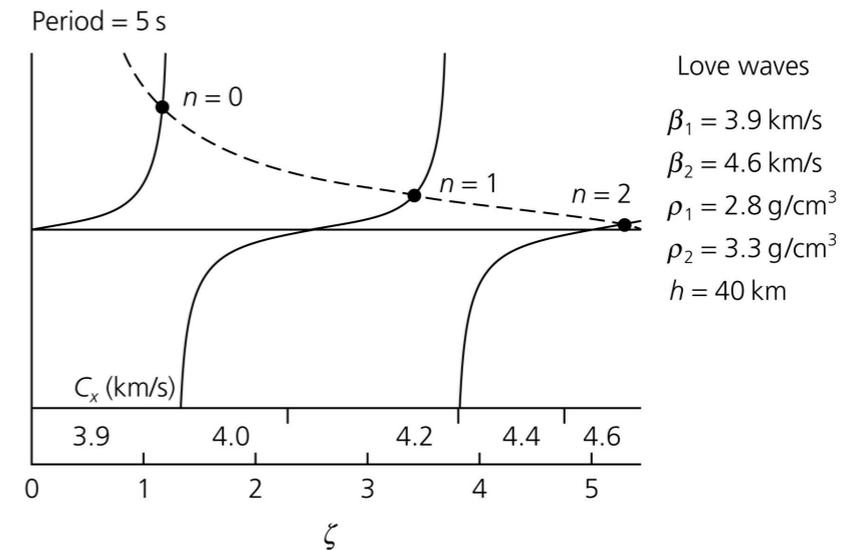
Graphical solution of the previous equation. Intersection of dashed and solid lines yield discrete modes.

$$\tan(H\omega\sqrt{1/\beta_1^2 - 1/c^2}) = \tan(\omega\zeta)$$

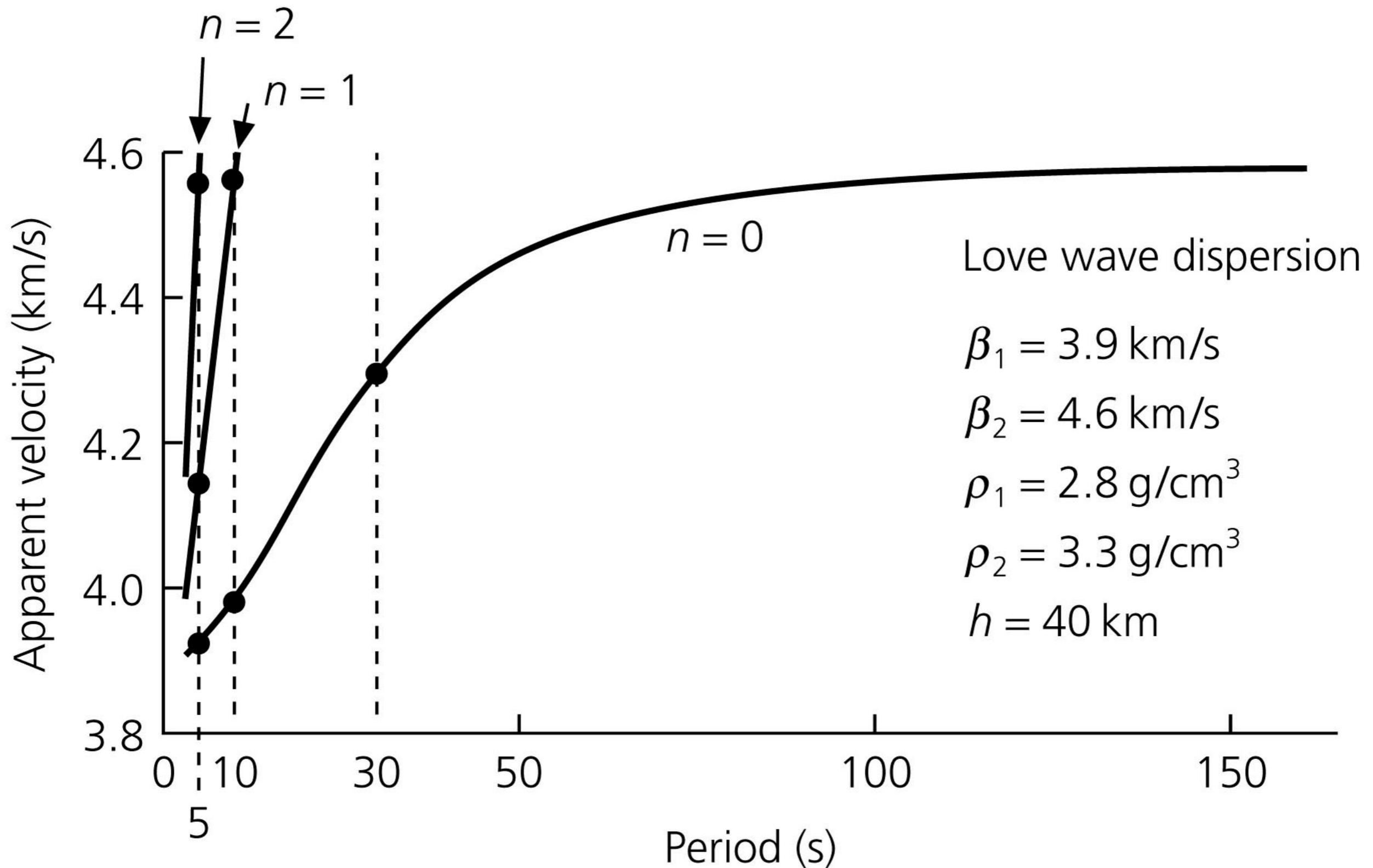
that vanishes when $\zeta = n \frac{\pi}{\omega}$

New modes appear at cut-off frequencies

$$\omega_n = \frac{n\pi}{H \left(\frac{1}{\beta_1^2} - \frac{1}{\beta_2^2} \right)^{1/2}}$$



Love Waves: Solutions



Love Waves: Solutions

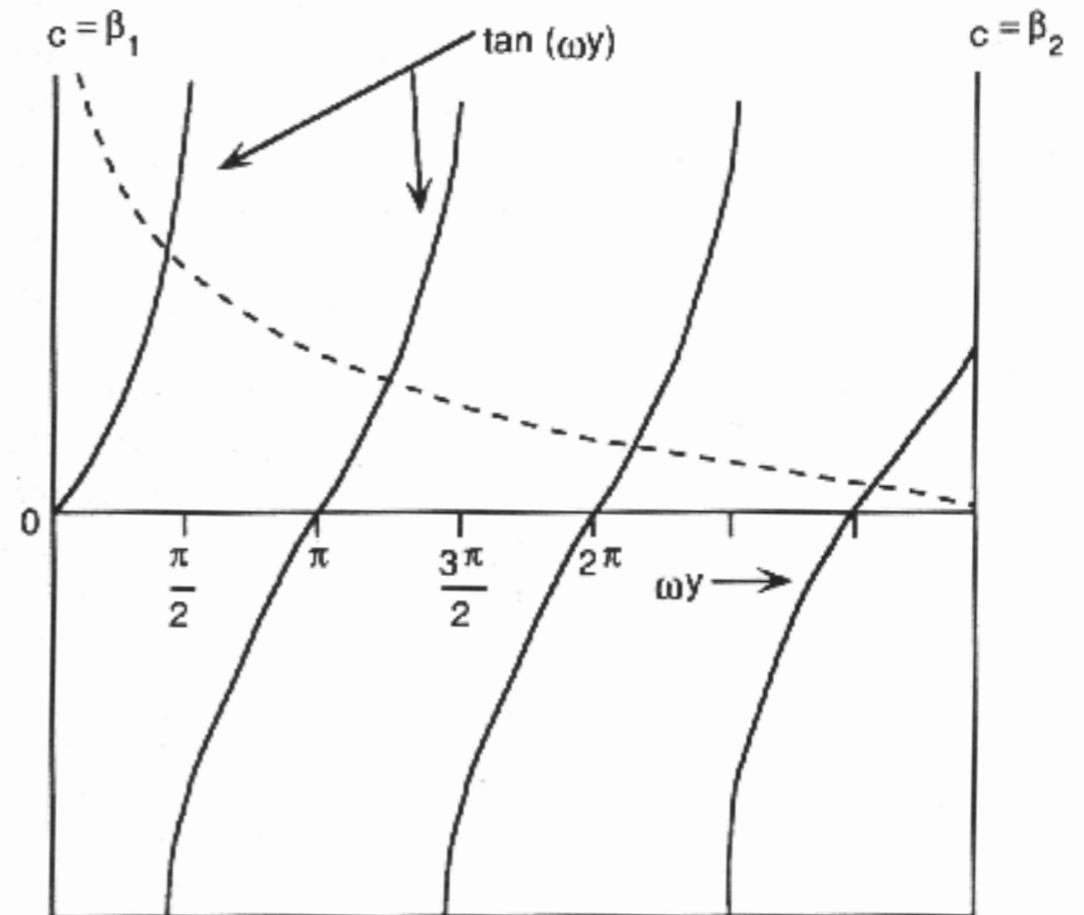
Graphical solution of the previous equation. Intersection of dashed and solid lines yield solutions while frequency is varying: discrete modes.

Every mode is characterized by a dispersion curve $c=c(\omega)$, showing the solution to the eigenvalue problem.

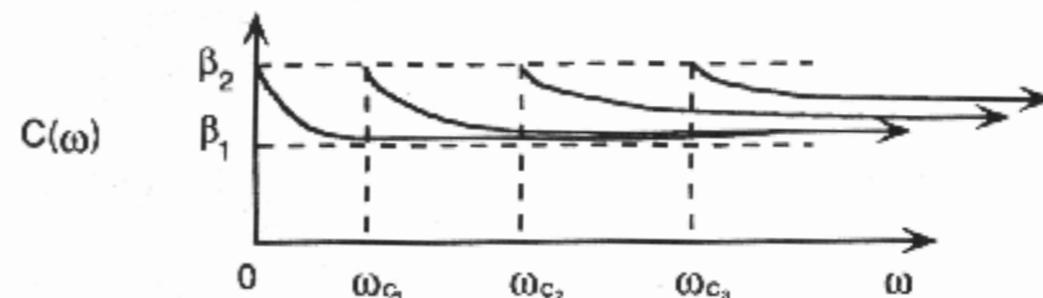
For every value of c one can calculate the eigenfunction, i.e. the displacement, u_y , versus depth.

a

$$\frac{\mu_2(1 - c^2/\beta_2^2)^{1/2}}{\mu_1(c^2/\beta_1^2 - 1)^{1/2}}$$

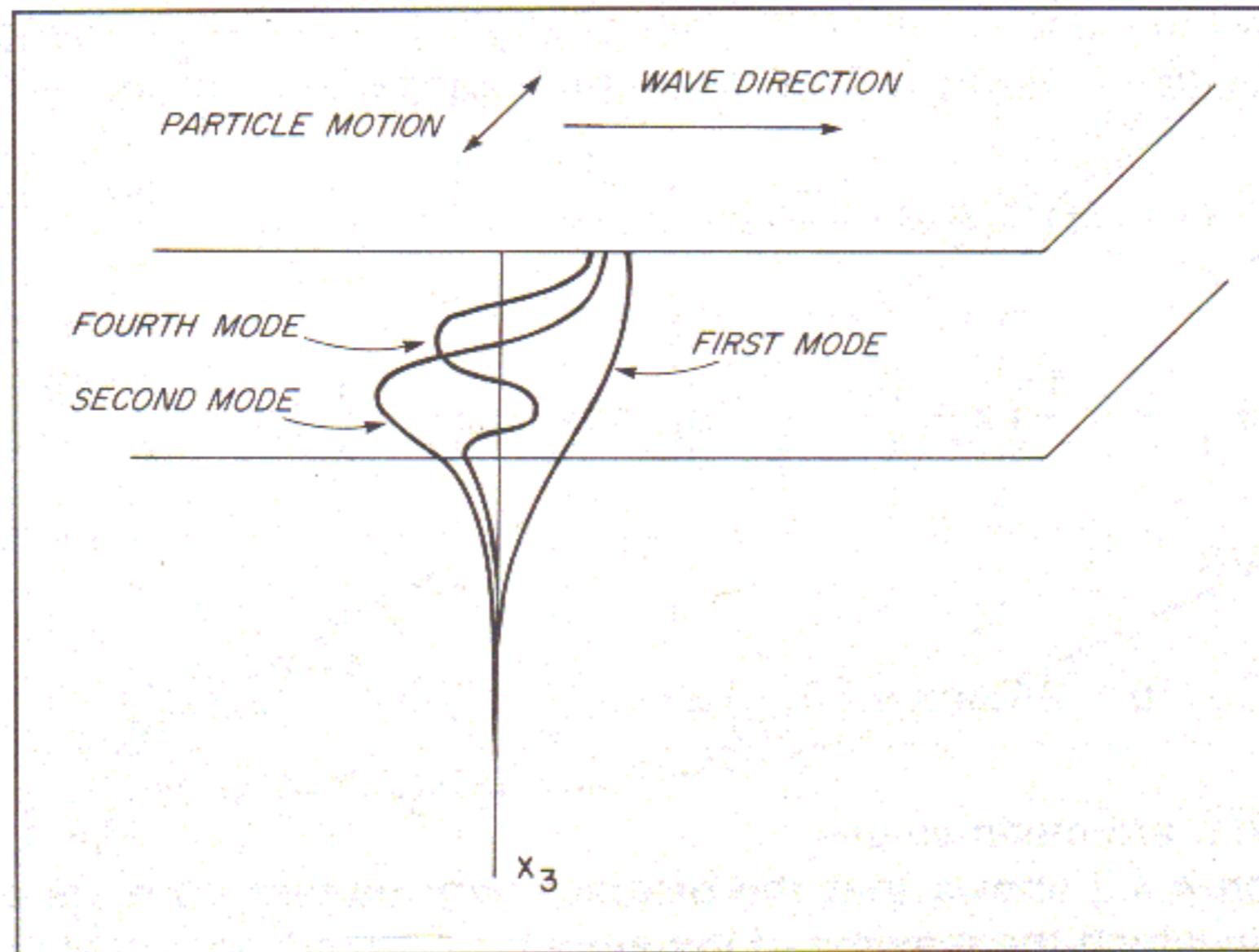


b



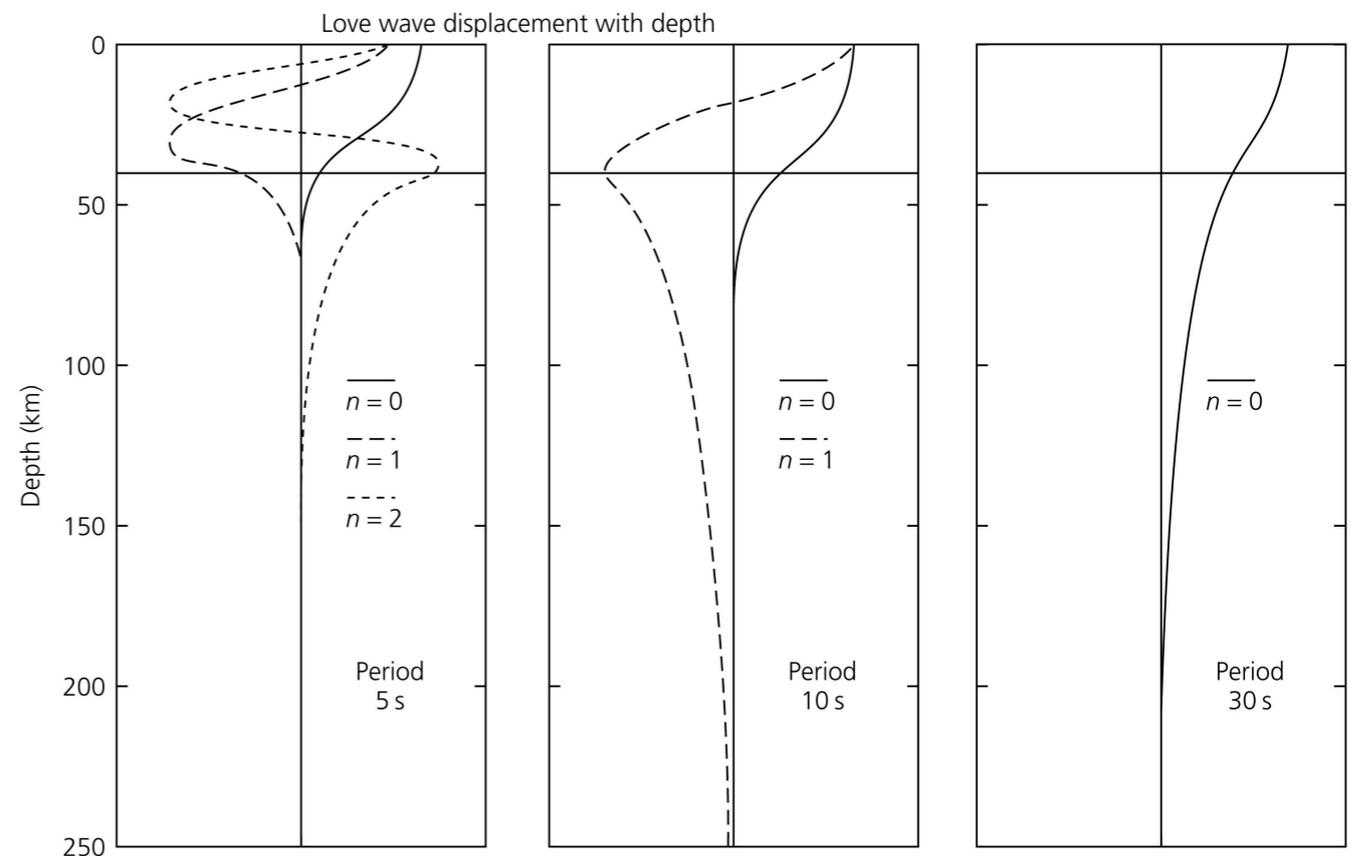
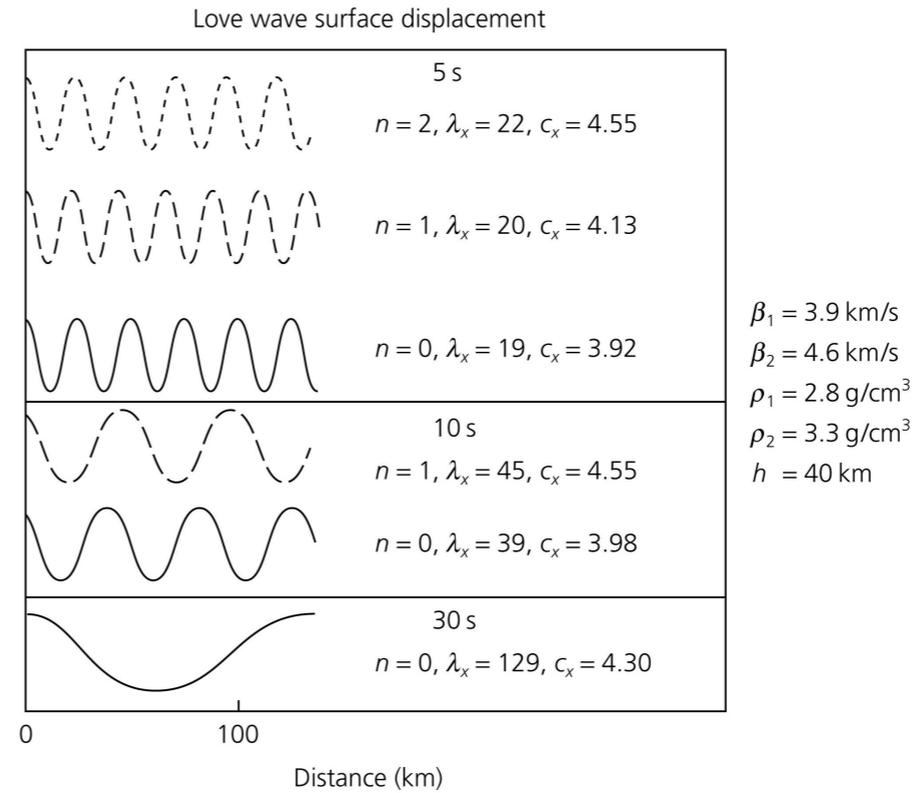
Love Waves: modes

Some modes for Love waves



Love Waves: modes

Some eigenvectors
(displacement) for
Love waves



Liquid layer over a half space

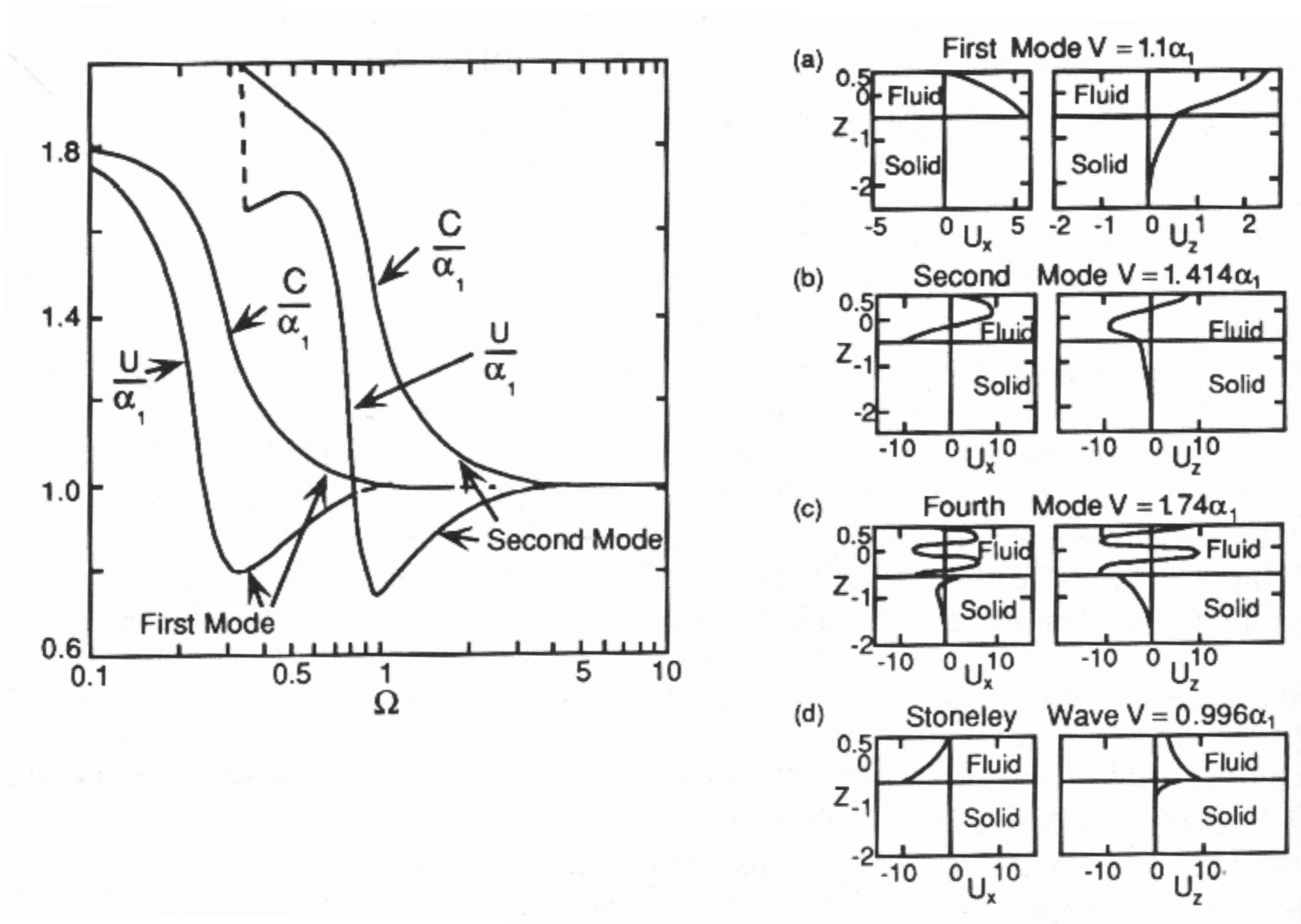
The conditions to be fulfilled are:

1. Free surface condition
2. No S-wave potential and shear stress in the liquid layer
3. Continuity of stress at the liquid-layer interface
4. Continuity of vertical component of displacement at the liquid layer interface (horizontal is free due to no viscosity in perfect liquid)

$$\tan(H\omega\sqrt{1/\alpha_w^2 - 1/c^2}) = \frac{\rho\beta^4\sqrt{c^2/\alpha_w^2 - 1}}{\rho_w c^4\sqrt{1 - c^2/\alpha^2}} \cdot \left[-(2 - c^2/\beta^2)^2 + 4(1 - c^2/\alpha^2)^{1/2}(1 - c^2/\beta^2)^{1/2} \right]$$

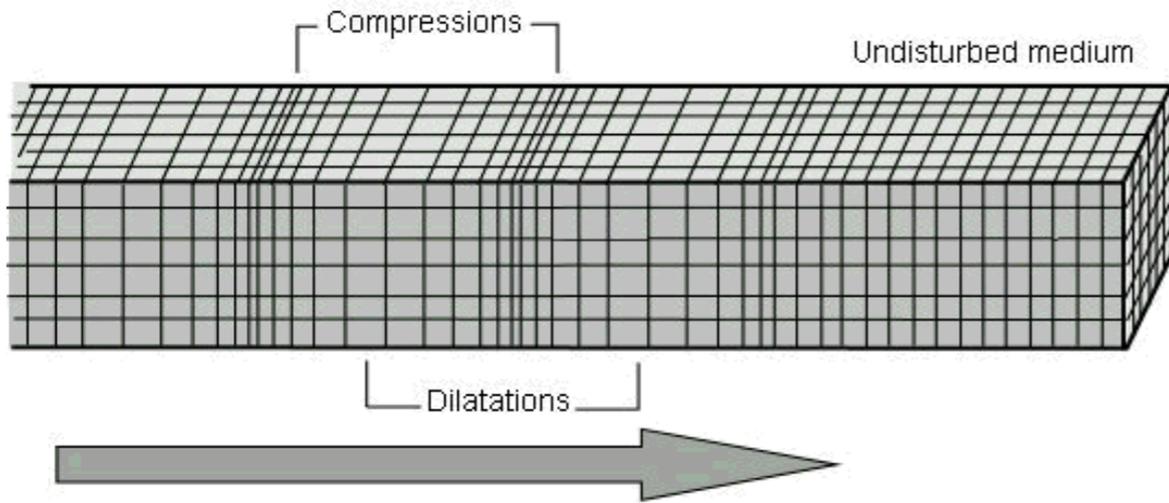
Liquid layer over a half space

Similar derivation for Rayleigh type motion leads to dispersive behavior

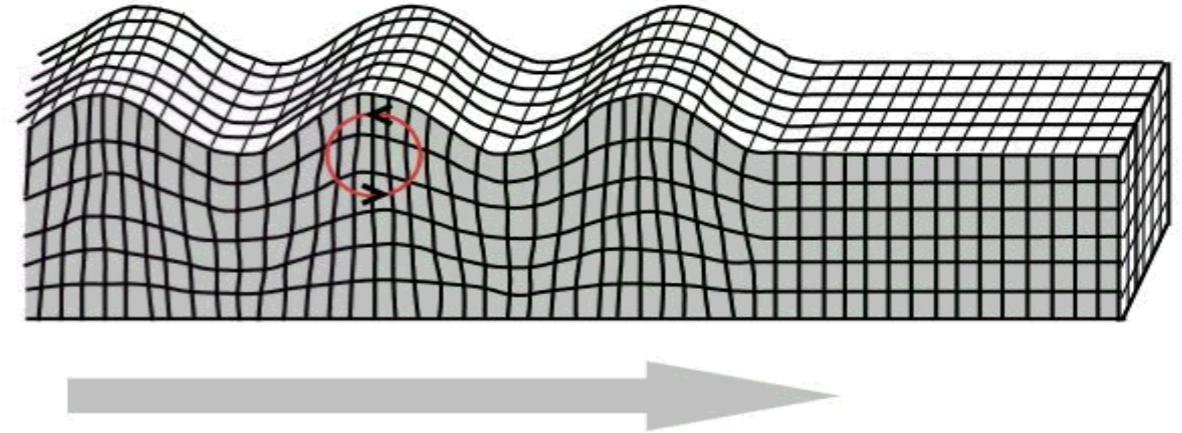


Wavefields visualization

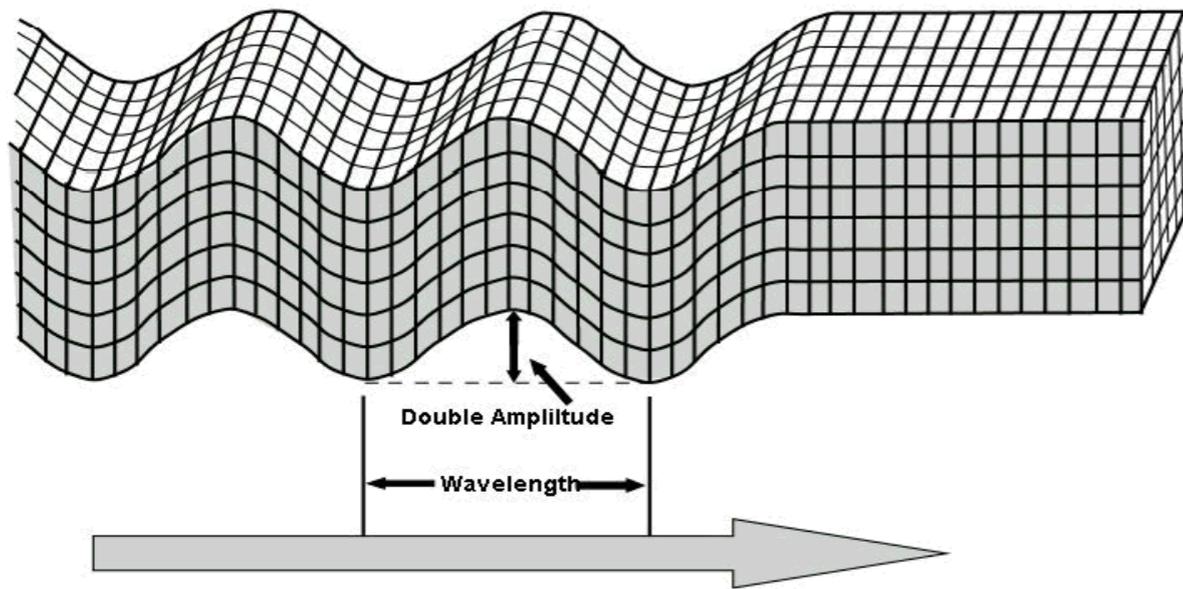
P Wave



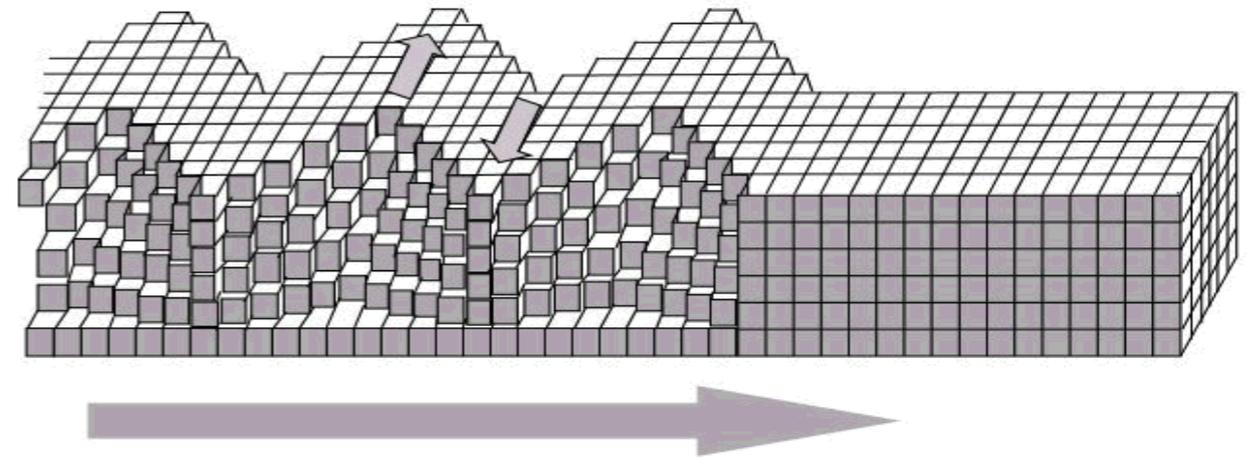
Rayleigh Wave



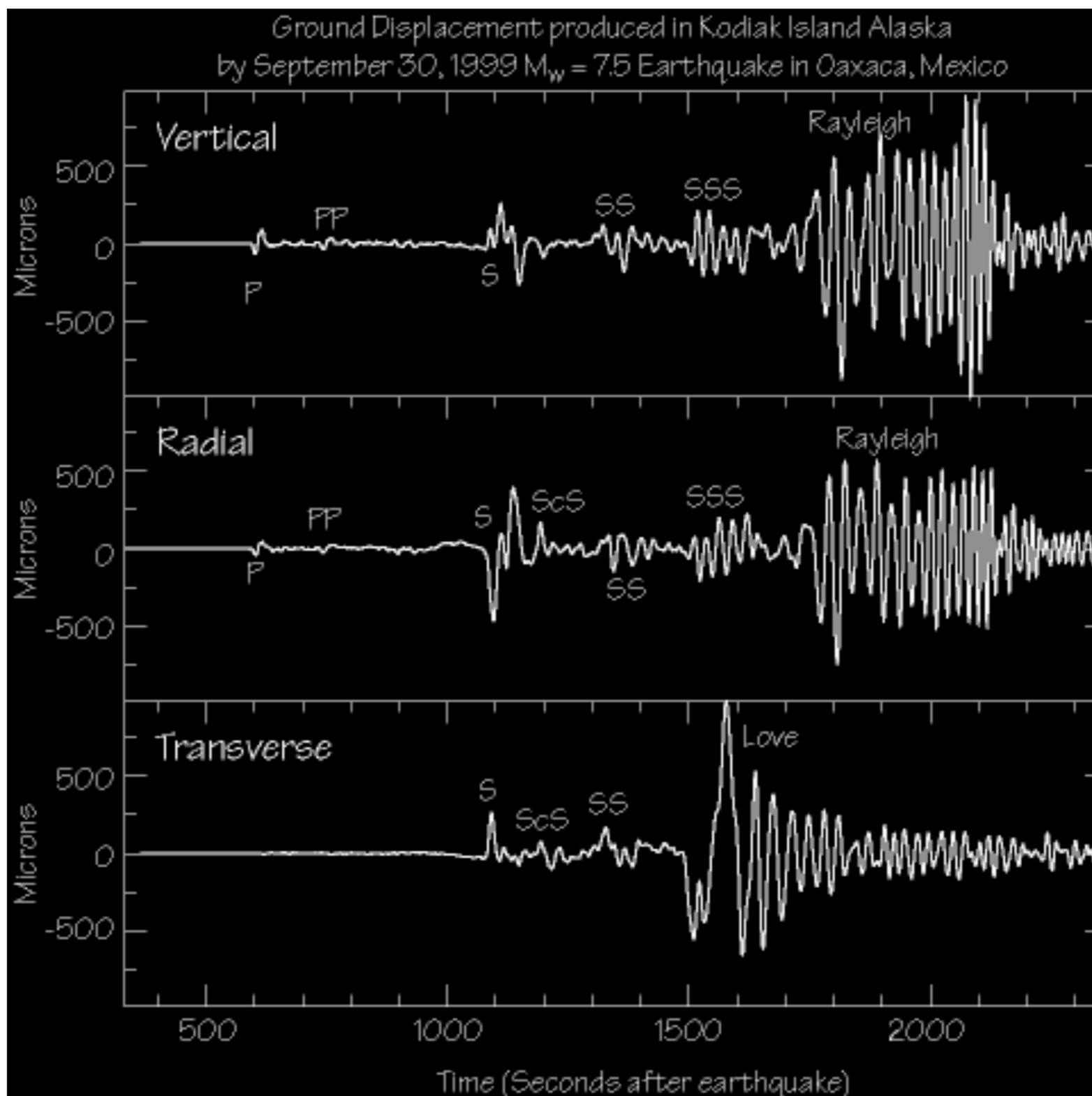
S Wave



Love Wave

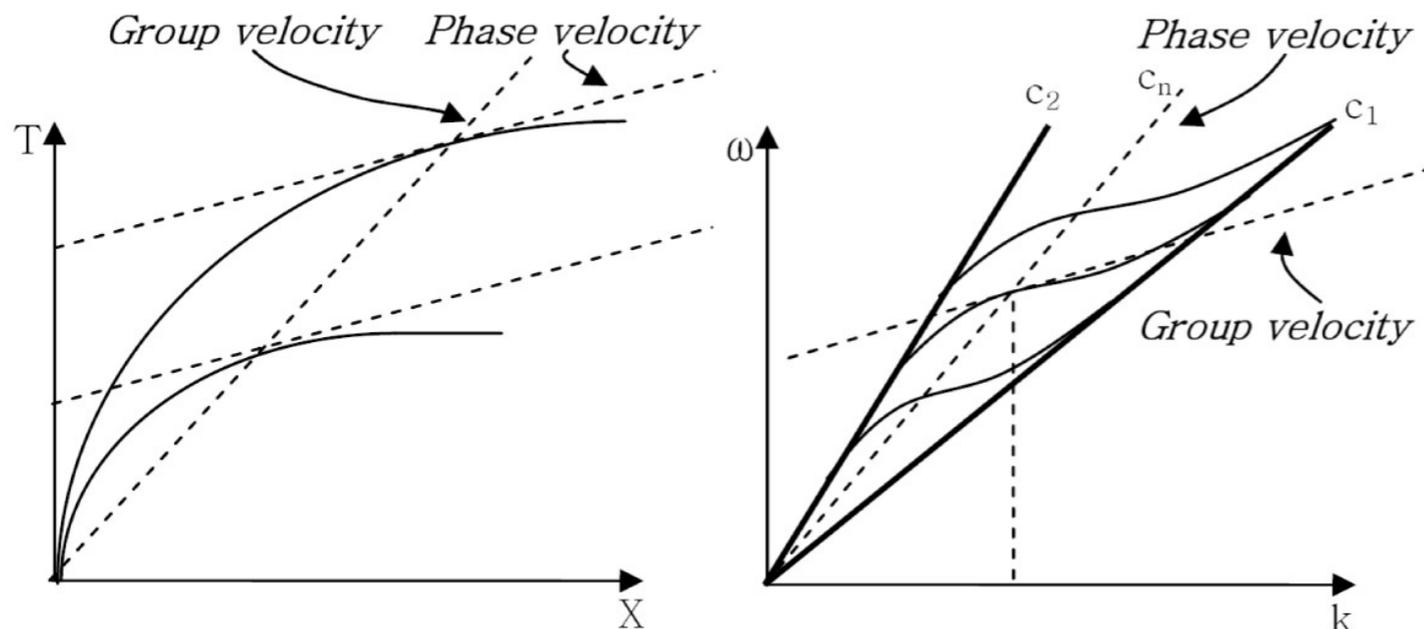


Data example - 2



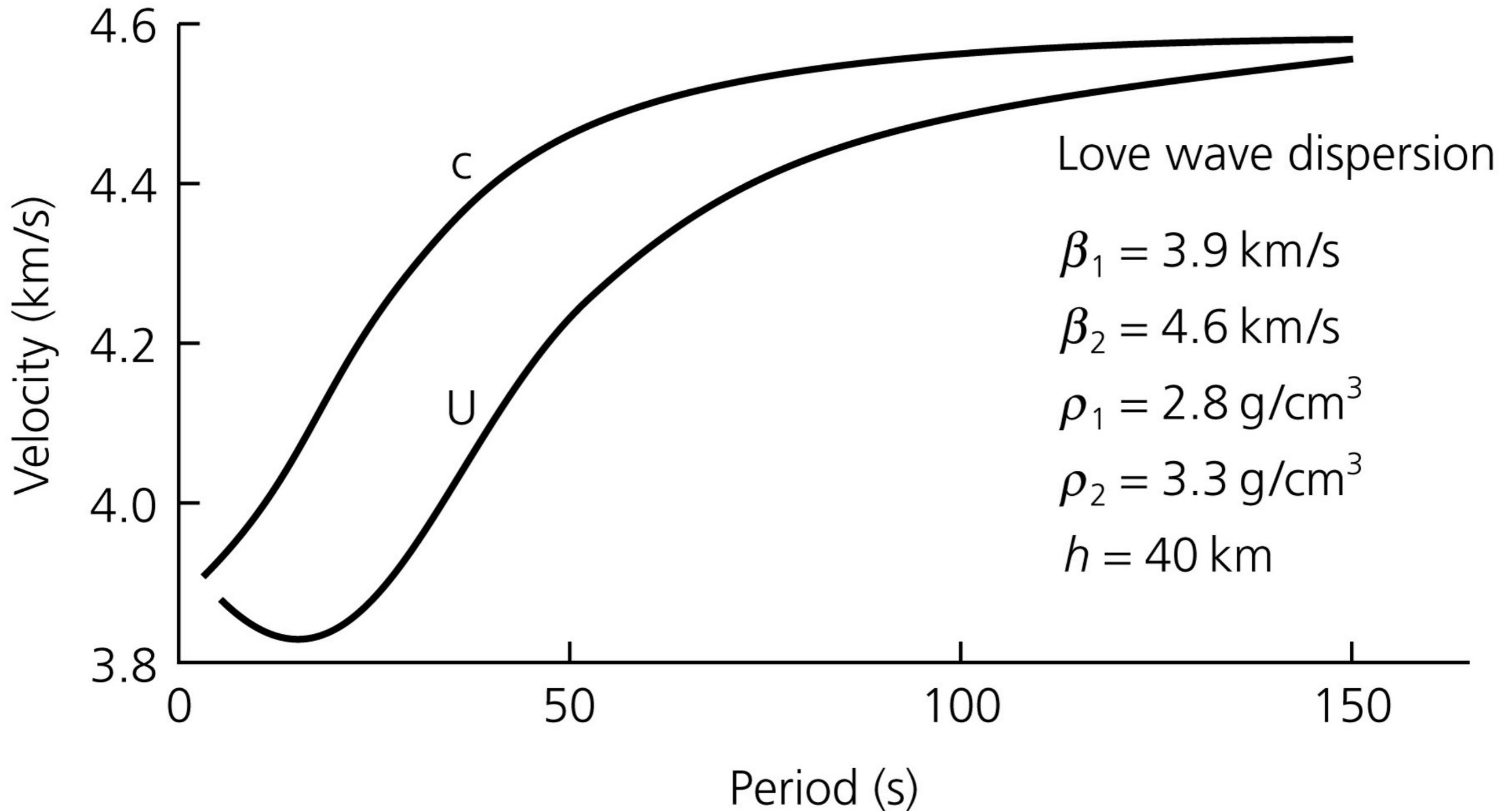
Dispersion...

$$D = \frac{dv_g}{d\omega}$$



Airy Phase -

wave that arises if the phase and the change in group velocity are stationary and gives the highest amplitude in terms of group velocity and are prominent on the seismogram.

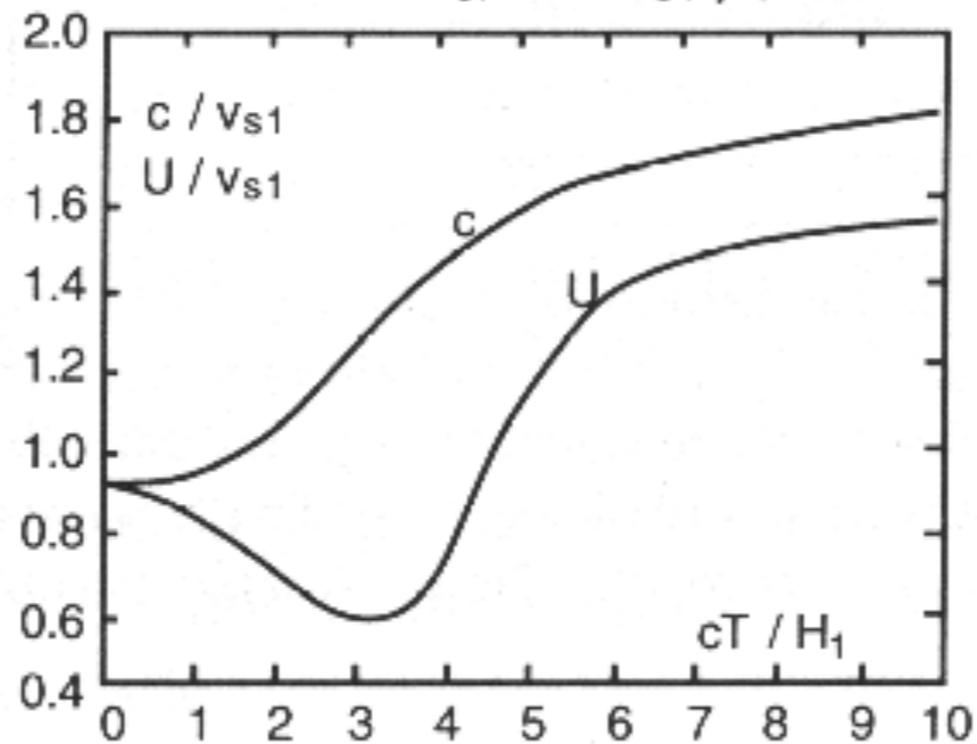


$$U = \frac{d\omega}{dk} = \frac{d(ck)}{dk} = c + k \frac{dc}{dk} = c - \lambda \frac{dc}{d\lambda}$$

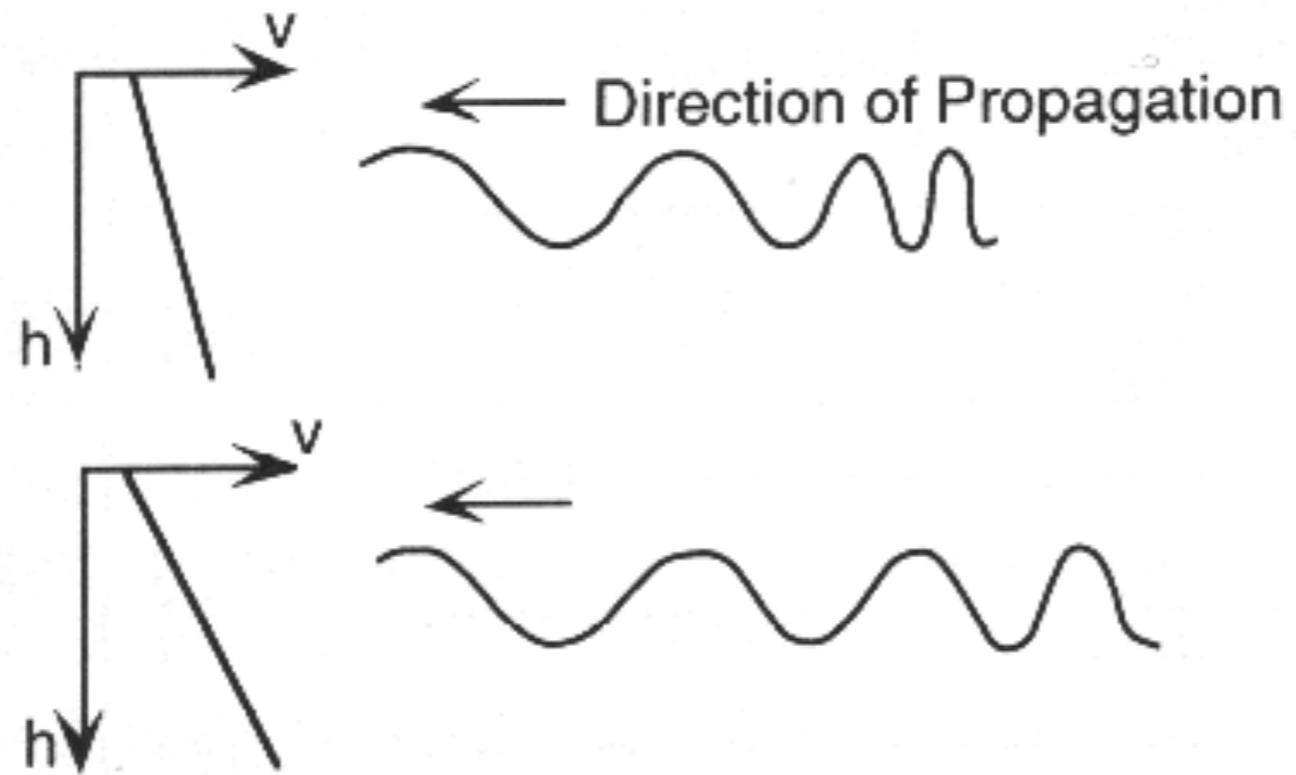
Dispersion

Fundamental Mode Rayleigh dispersion curve for a layer over a half space.

Layer	V_p	V_s	ρ	H
1	$1.732v_{s1}$	v_{s1}	ρ_1	H_1
2	$3.873v_{s1}$	$2.236v_{s1}$	ρ_1	∞

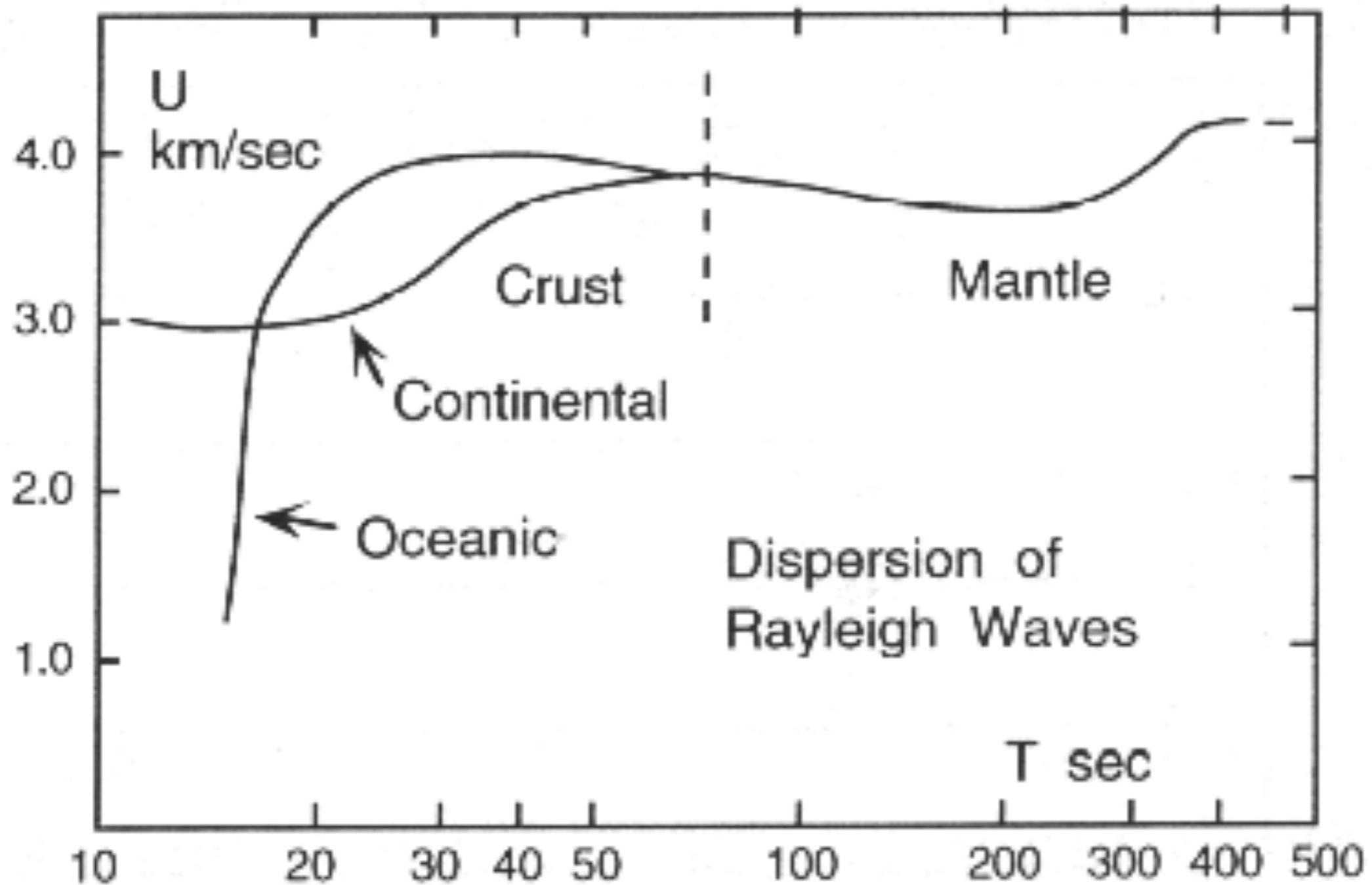


Dispersion

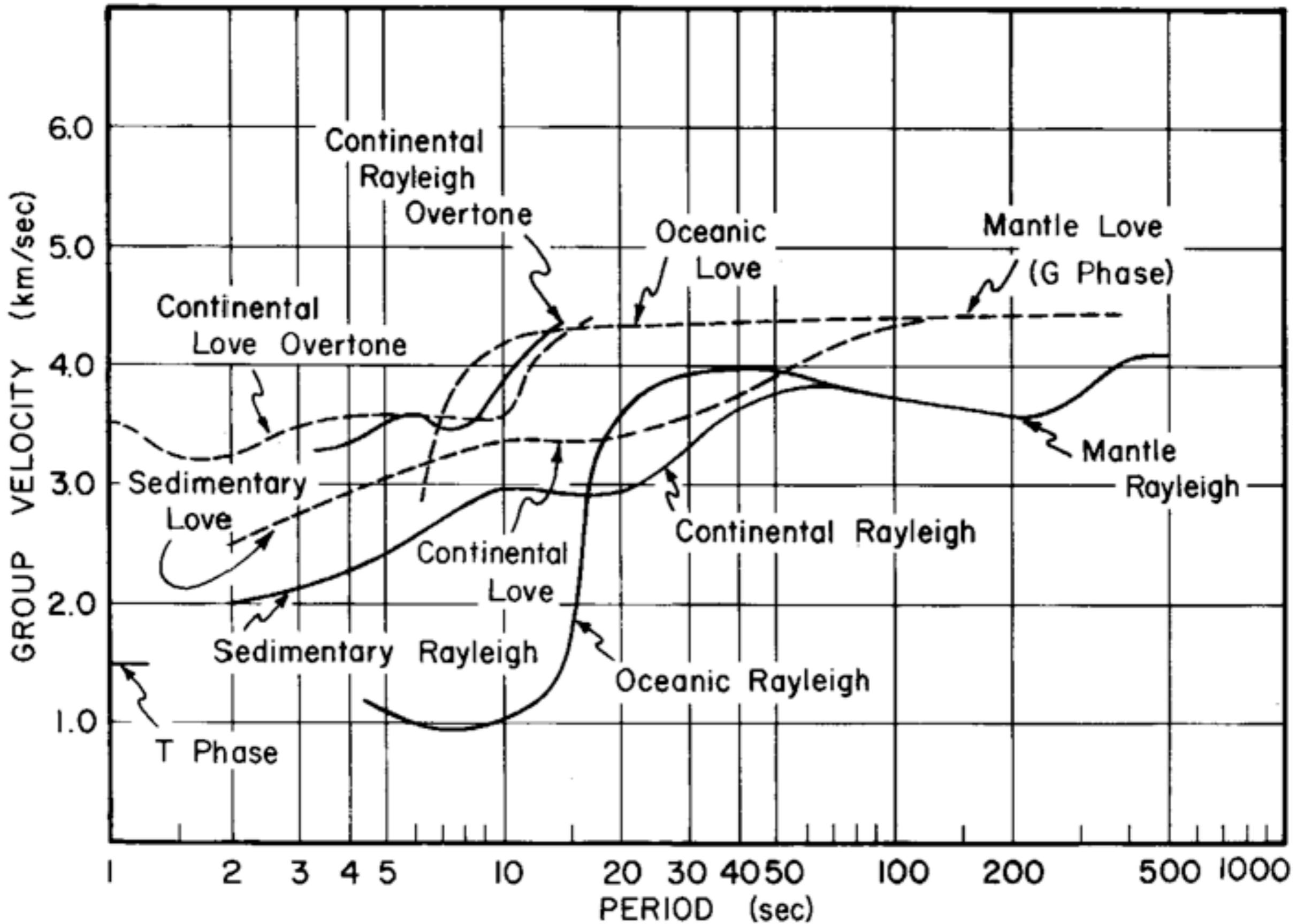


Stronger gradients cause greater dispersion

Observed Group Velocities ($T < 80s$)

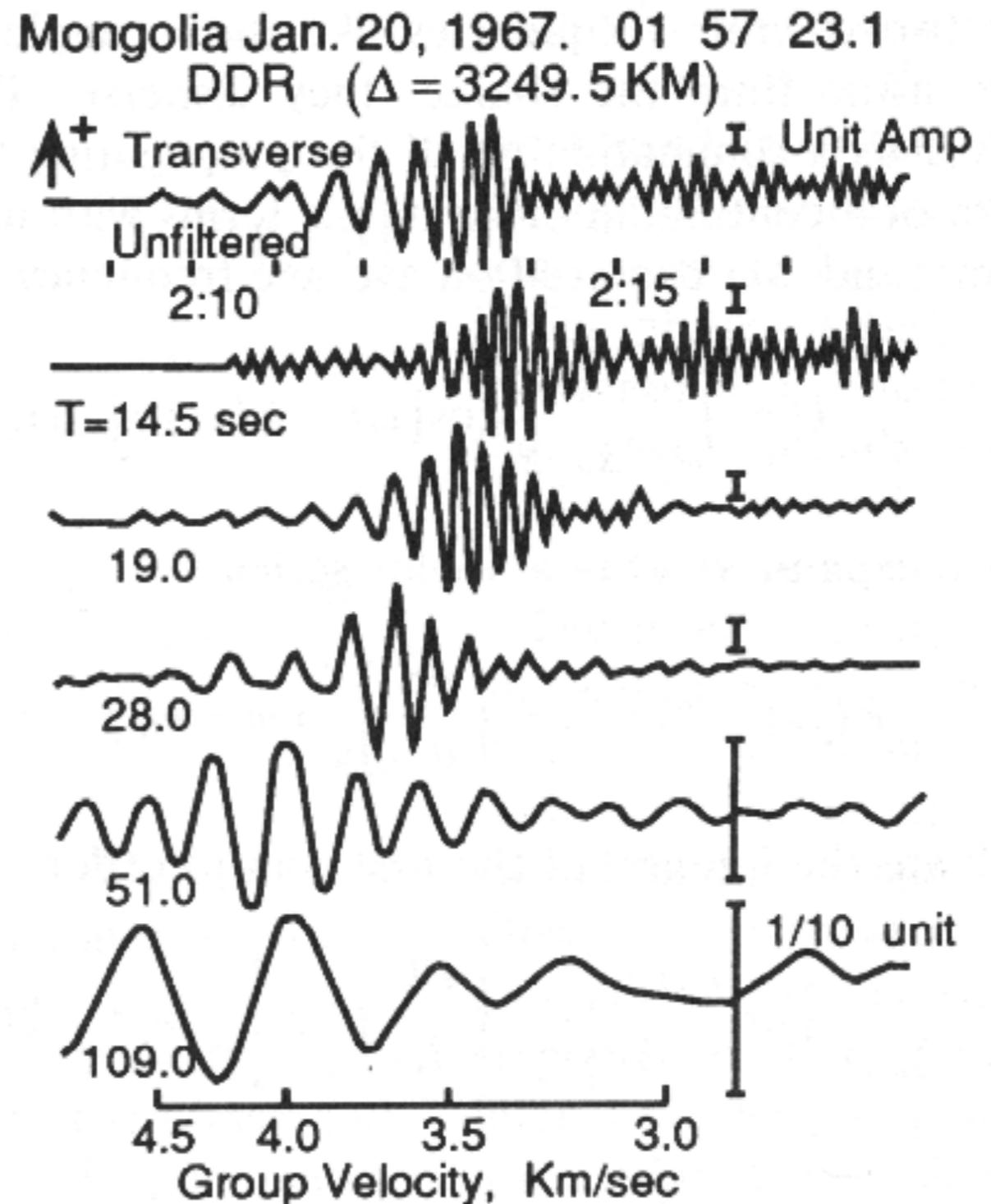


Group velocities



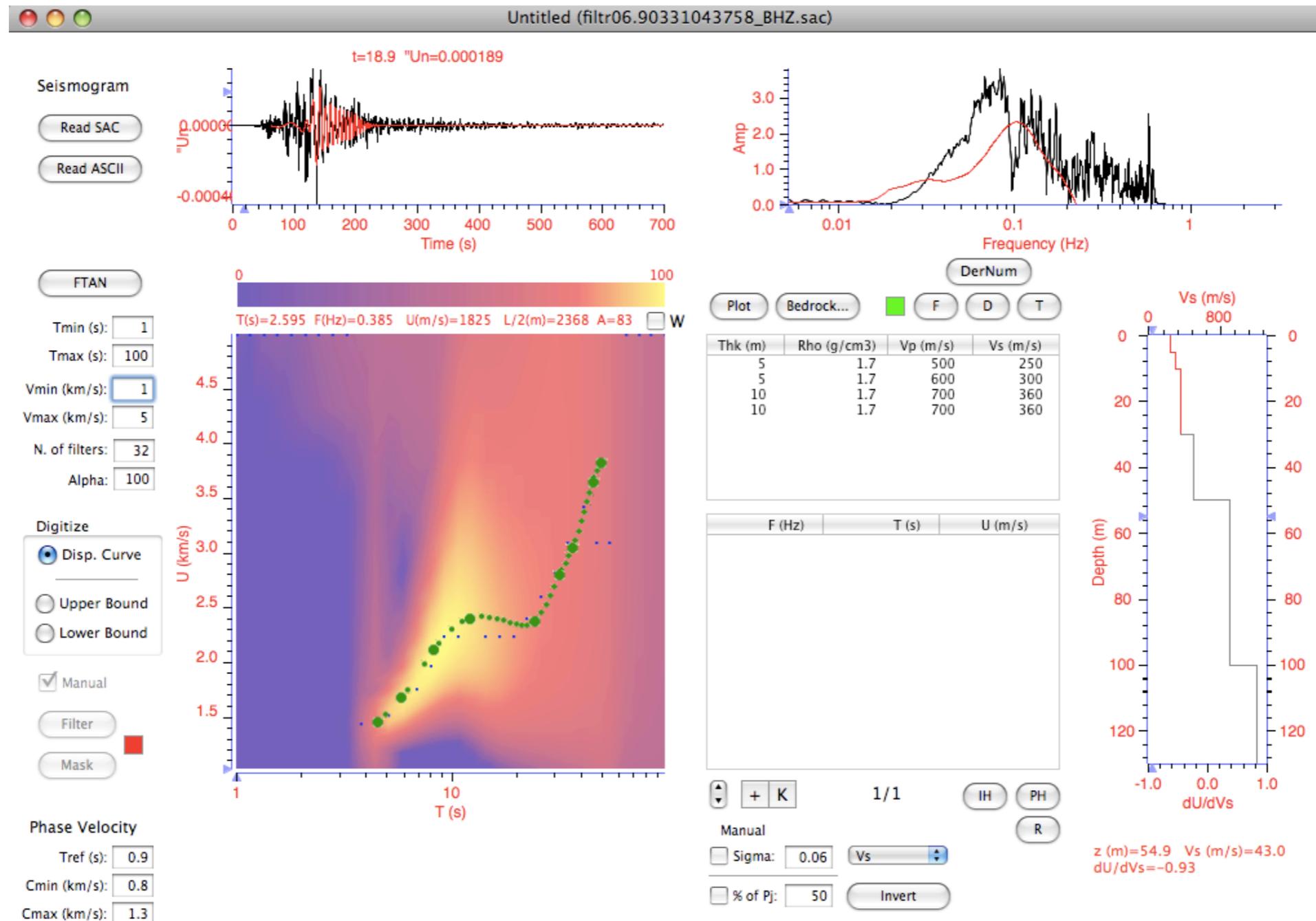
Wave Packets

Seismograms of a Love wave train filtered with different central periods. Each narrowband trace has the appearance of a wave packet arriving at different times.



- Frequency-Time representation:
- Gaussian filters; FTAN maps
- Floating filters: Phase equalization

e.g. Levshin et al., 1972



Measuring group velocity

One station method

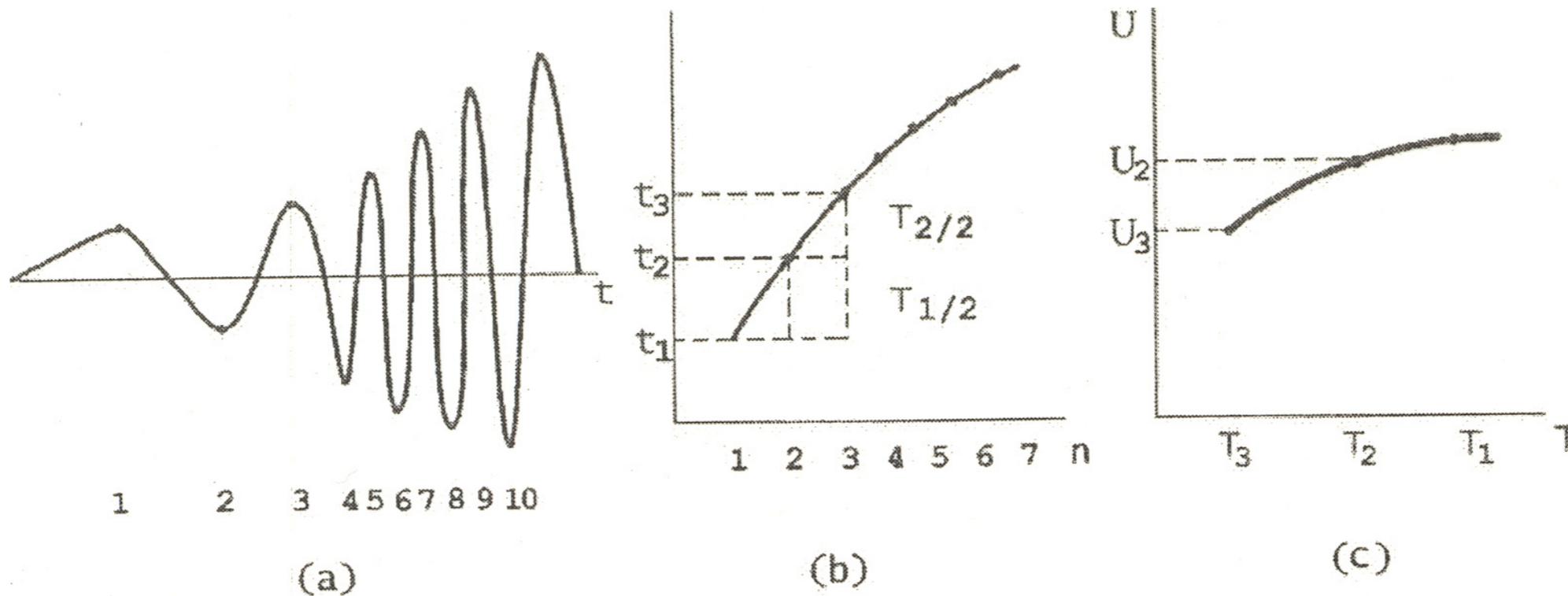
1. Directly measure the arrival time of the peaks and troughs on one seismogram
2. Narrow filtered the seismogram, measure the arrival of the peak of the wave packet (more accurate)

$$U(\omega) = \frac{x}{t}$$

Need know the origin time and the location of the earthquake source

Measuring group velocity

Determination of group velocities at one station



Measuring group velocity

Two stations method

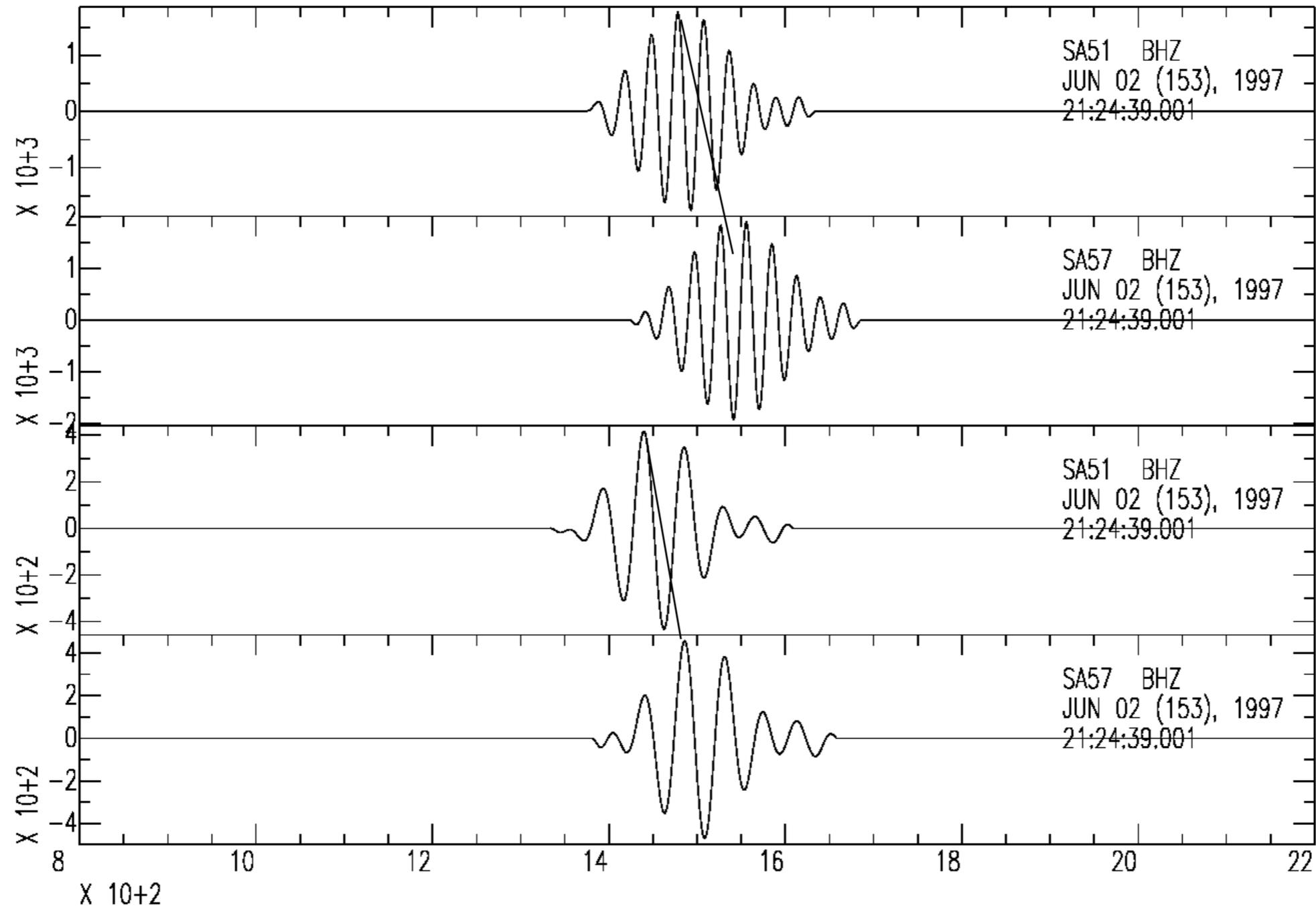
If two stations are located on the same great circle path, group velocity can be obtained by measuring the difference in arrival times of filtered wave packets.

$$U(\omega) = \frac{\Delta x}{\Delta t}$$

Distance between two stations

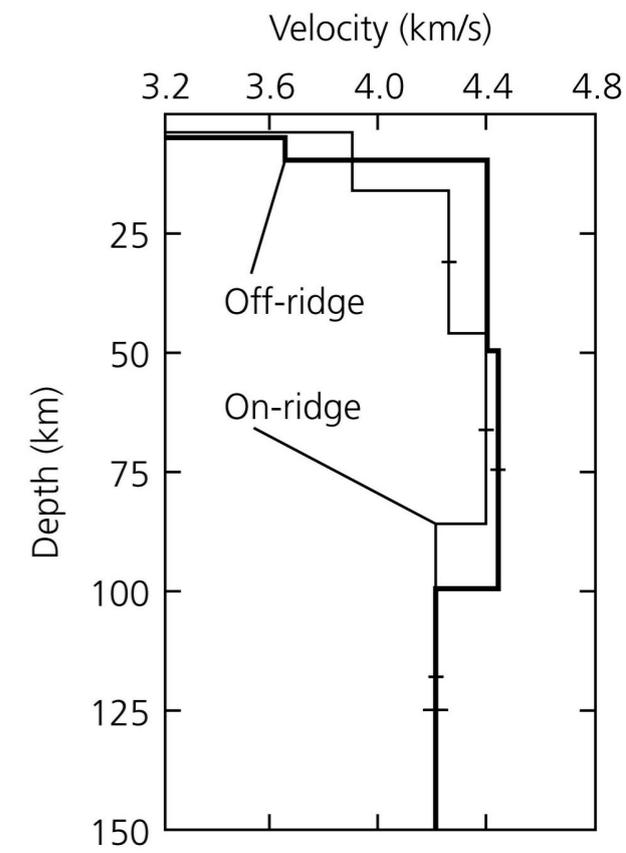
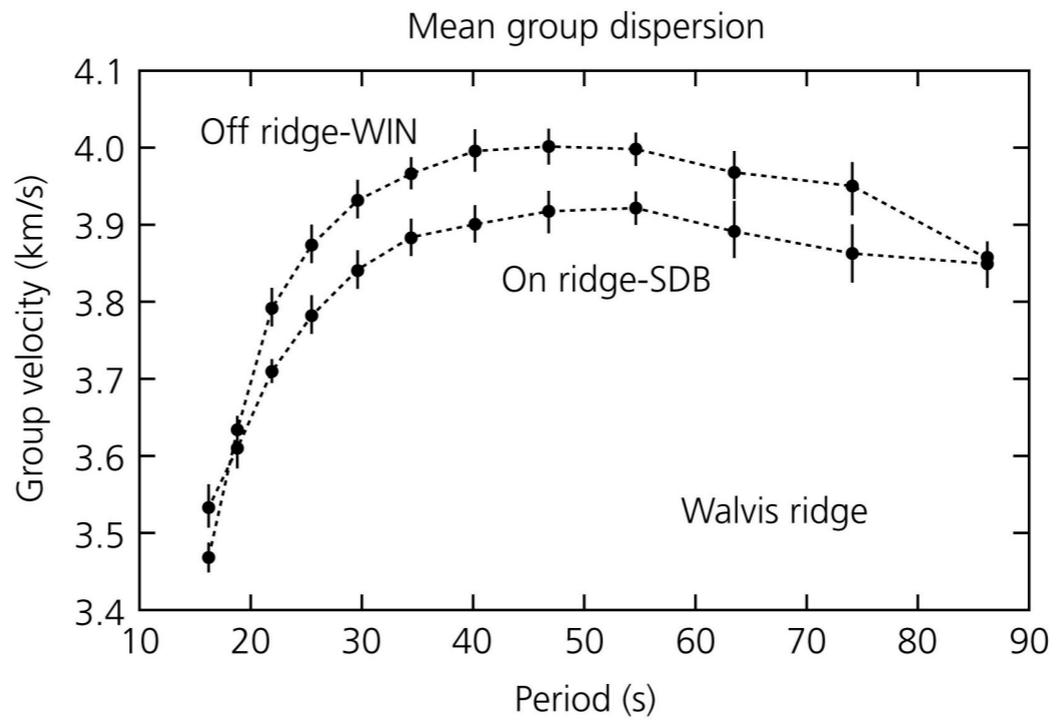
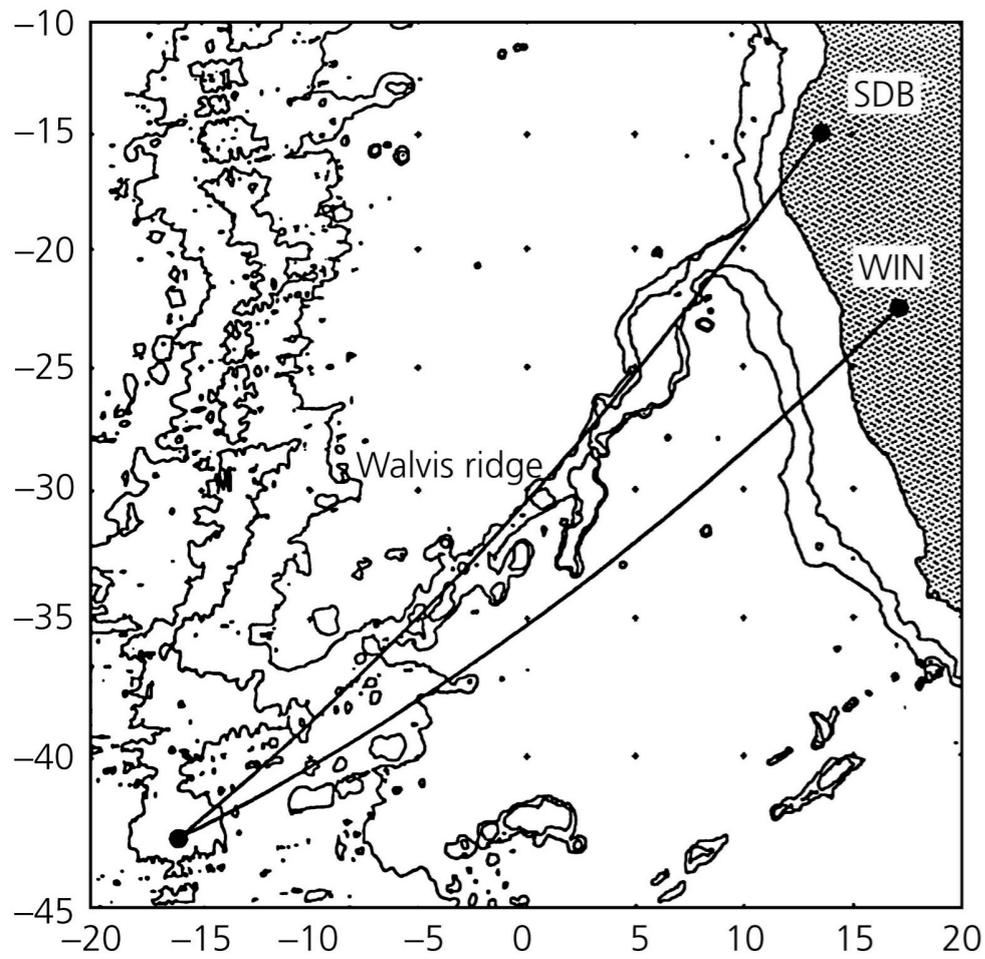
Different of arrival times at the two stations

Measuring group velocity



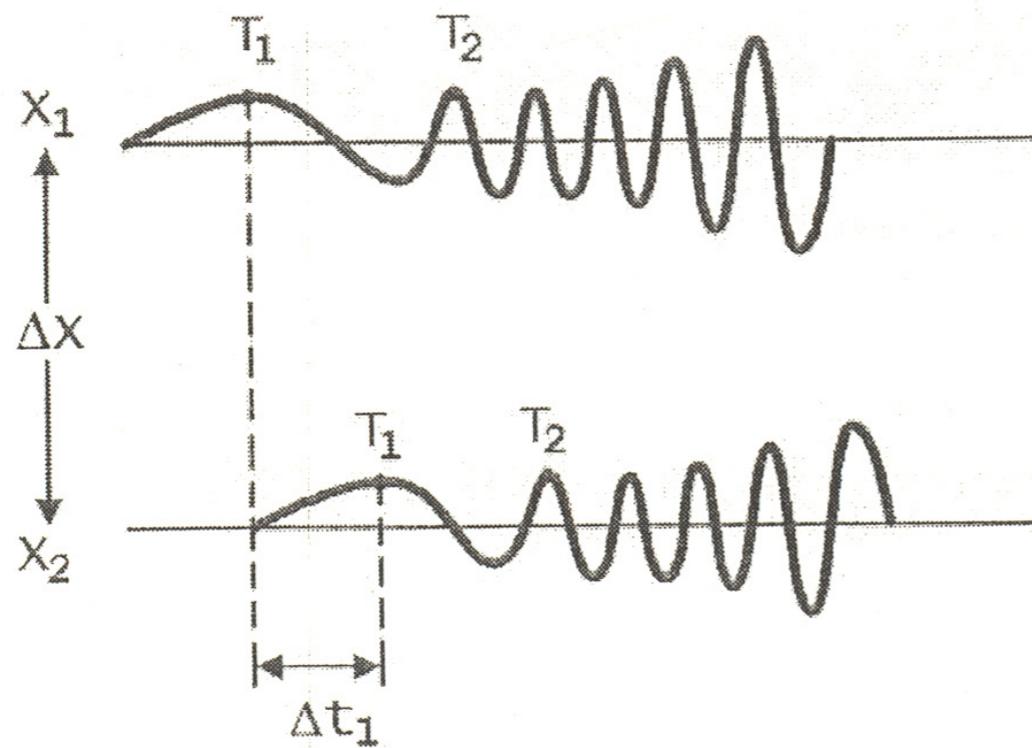
V_g measures

Figure 2.8-5: Rayleigh wave group velocity study of the Walvis ridge.

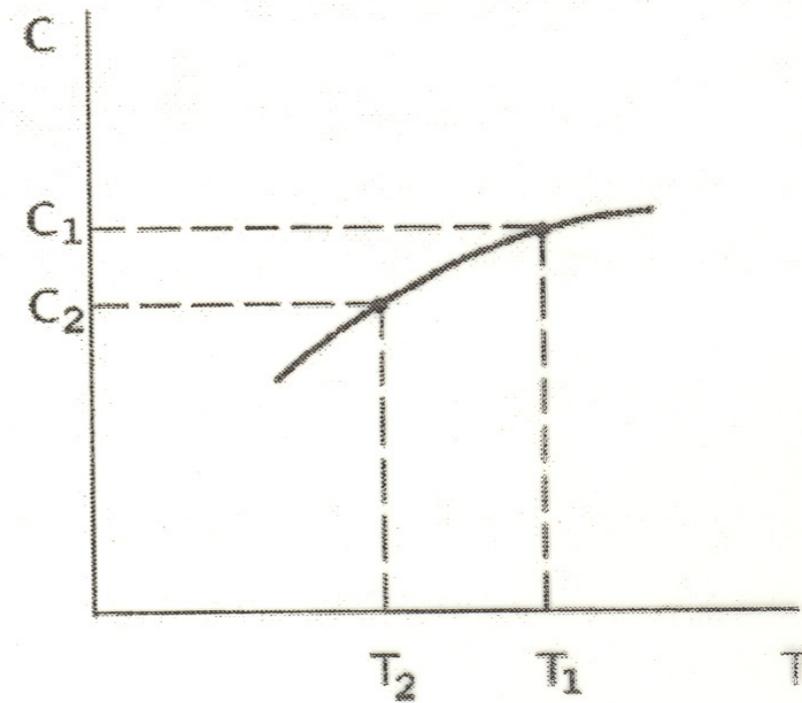


Measuring phase velocity

Directly measured at two stations



(a)



(b)

Measuring phase velocity

Measured by taking Fourier transform and obtaining phase spectrum

A surface wave can be represented:

$$u(x, t) = \frac{1}{\pi} \int_0^{\infty} A(\omega, x) \cos\left(\omega t - \frac{\omega}{c(\omega)} x + \phi_0(x)\right) d\omega$$

$$\phi(\omega) = \omega t - \frac{\omega}{c(\omega)} x + \phi_0(\omega) + 2\pi N$$

One-station method

$$\phi_1(\omega) = \omega t_1 - \frac{\omega}{c(\omega)} x_1 + \phi_0(\omega) + 2\pi N$$

Need know the initial phase ϕ_0

N can be determined by allowing $c(\omega)$ for the longest period converge to the global average

c measures

On a seismogram recorded at a distance x from the earthquake at time t after the earthquake, the phase has three terms:

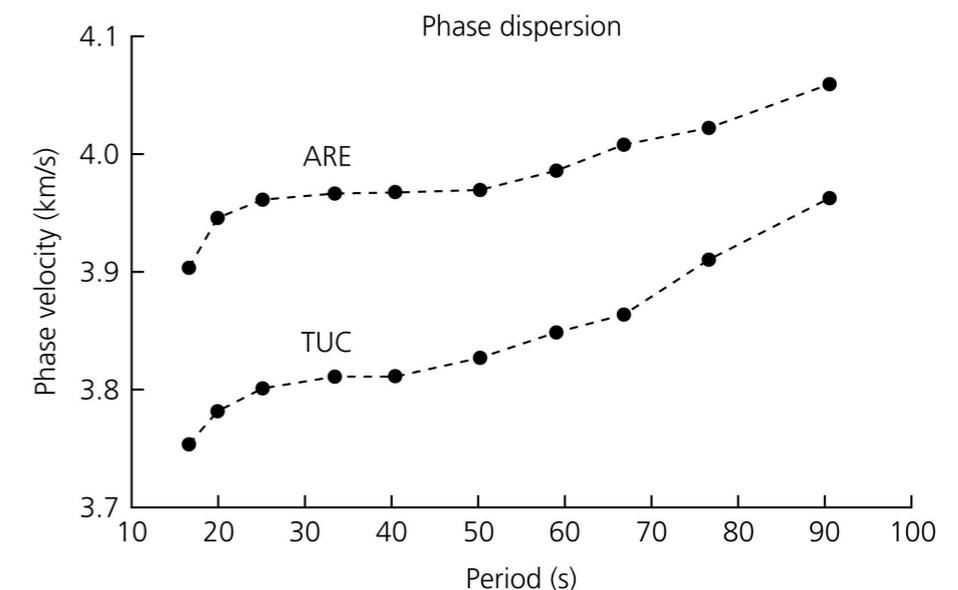
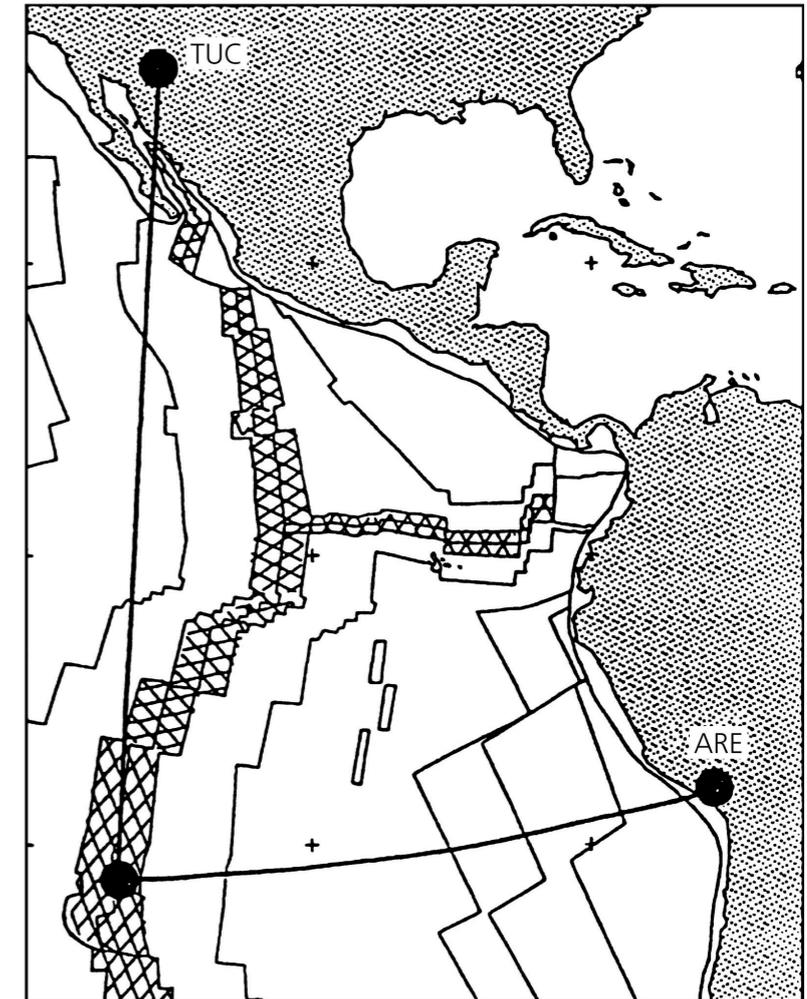
$$\begin{aligned}\Phi(\omega) &= [\omega t - k(\omega)x] + \phi_i(\omega) + 2n\pi \\ &= [\omega t - \omega x/c(\omega)] + \phi_i(\omega) + 2n\pi\end{aligned}$$

$\omega t - k(\omega)x$ is the phase due to the propagation of the wave in time and space.

$\phi_i(\omega)$ includes the initial phase at the earthquake and any phase shift introduced by the seismometer.

$2n\pi$ reflects the periodicity of the complex exponential, because adding an integral multiple of 2π to the argument yields the same value.

Figure 2.8-6: Example of Rayleigh wave phase velocities for ocean lithosphere.



c measures

Two station method:

$$\Phi_1(\omega) = \omega t_1 - \omega x_1/c(\omega) + \phi_i(\omega) + 2n\pi$$

$$\Phi_2(\omega) = \omega t_2 - \omega x_2/c(\omega) + \phi_i(\omega) + 2m\pi$$

Take the difference $\Phi_{21} = \Phi_2 - \Phi_1$, and solve for c:

$$c(\omega) = \omega(x_2 - x_1)/[\omega(t_2 - t_1) + 2(m - n)\pi - \Phi_{21}(\omega)].$$

The $2(m - n)\pi$ term is found empirically by ensuring that the phase velocity at long periods is reasonable.

Single station method:

Predict the phase at the earthquake from its focal mechanism.

If $\phi_i(\omega)$ is known, c is:

$$c(\omega) = \omega x/[\omega t + \phi_i(\omega) + 2n\pi - \Phi(\omega)]$$

Figure 2.8-6: Example of Rayleigh wave phase velocities for ocean lithosphere.

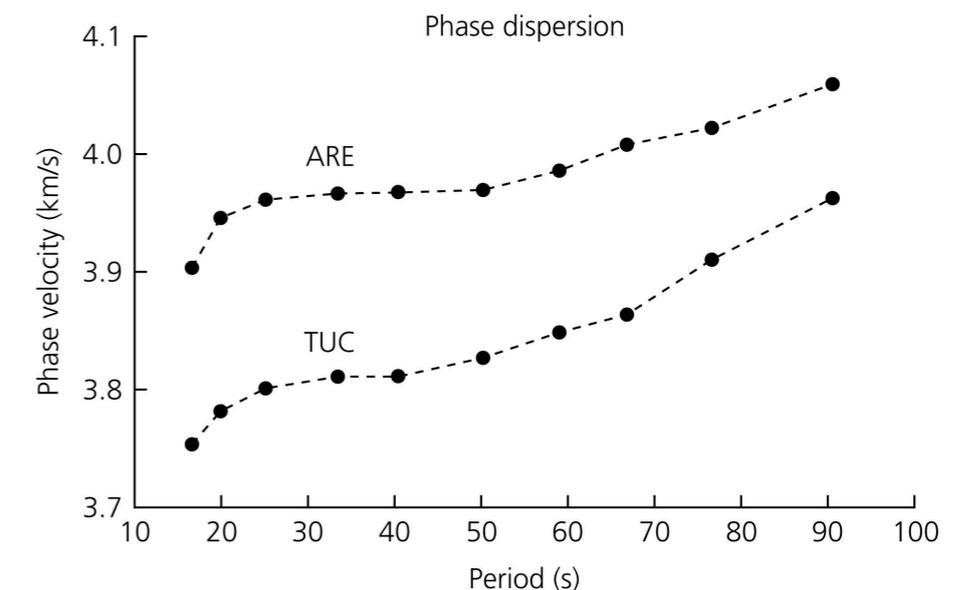
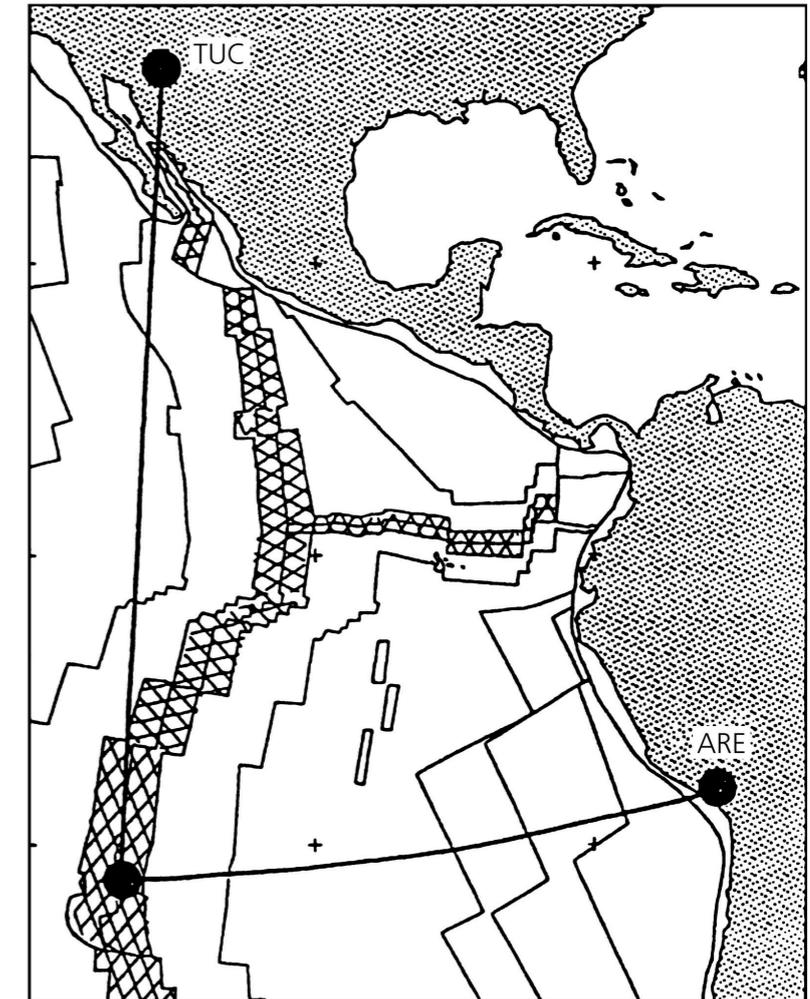




Figure 2.8-7: Rayleigh wave phase velocity dispersion as a function of oceanic plate age.

