Problem set 1

1. Read the section 3.4 Quasiconcavity and quasiconvexity in the math tutorial of Martin Osborne (http://www.economics.utoronto.ca/osborne/MathTutorial/QCCF.HTM) and check quasiconcavity for the following functions:
   a. $y + lnx \; for \; x, y > 0$
   b. $4\,xy + 4x \; for \; x, y < 0$

2. Check if the following matrix is positive/negative semi-definite
   a. $\begin{matrix} -4 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 2 & 1 \end{matrix}$

3. Check if the following functions are concave/convex
   a. $\sqrt{xy}$
   b. $\ln x + \ln y$

4. Find the all stationary points of the following functions. Then check the second order conditions and state if they are local/global maximizers/minimizers
   a. $x^3 - 12x + y^3 - 27y + z^3 - 3z$
   b. $-y^2 - 2x^2 + xy$
   c. $-y^2 - 2x^2 + 2xy$
   d. $e^x - xy$
   e. $\ln x + \ln y + \ln z - 4x - 5y - 6z$ for $z, x, y > 0$

5. Suppose that a firm that uses 2 inputs has the production function $f(x, y) = 12x^{\frac{1}{3}}y^{\frac{1}{2}}$ and faces the input prices (p_x, p_y) and the output price q.
   a. Show that f is concave, so that the firm's profit is concave.
   b. Find a global maximum of the firm's profit (and give the input combination that achieves this maximum).