

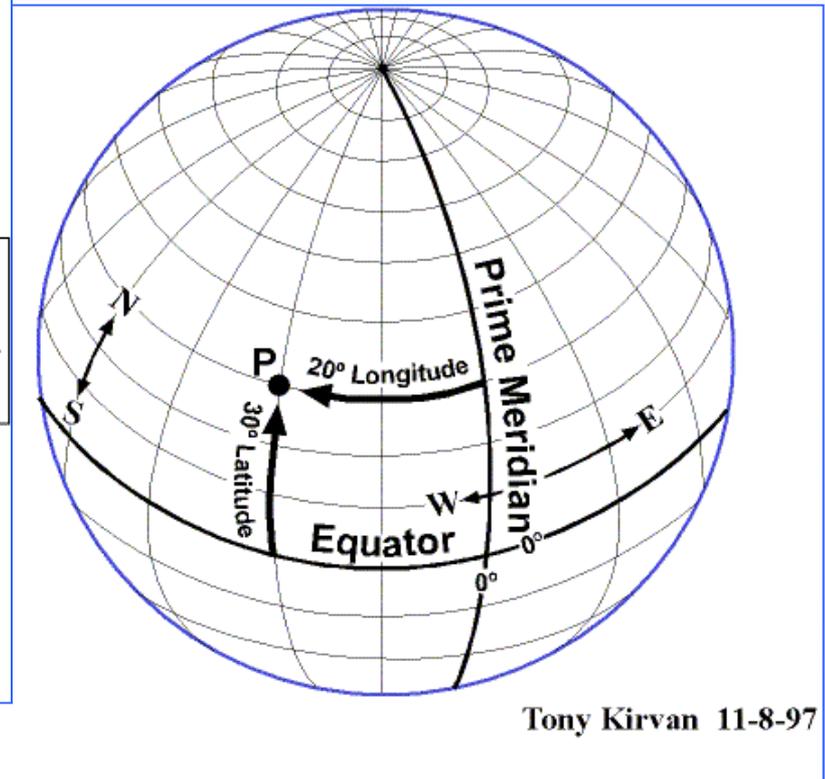
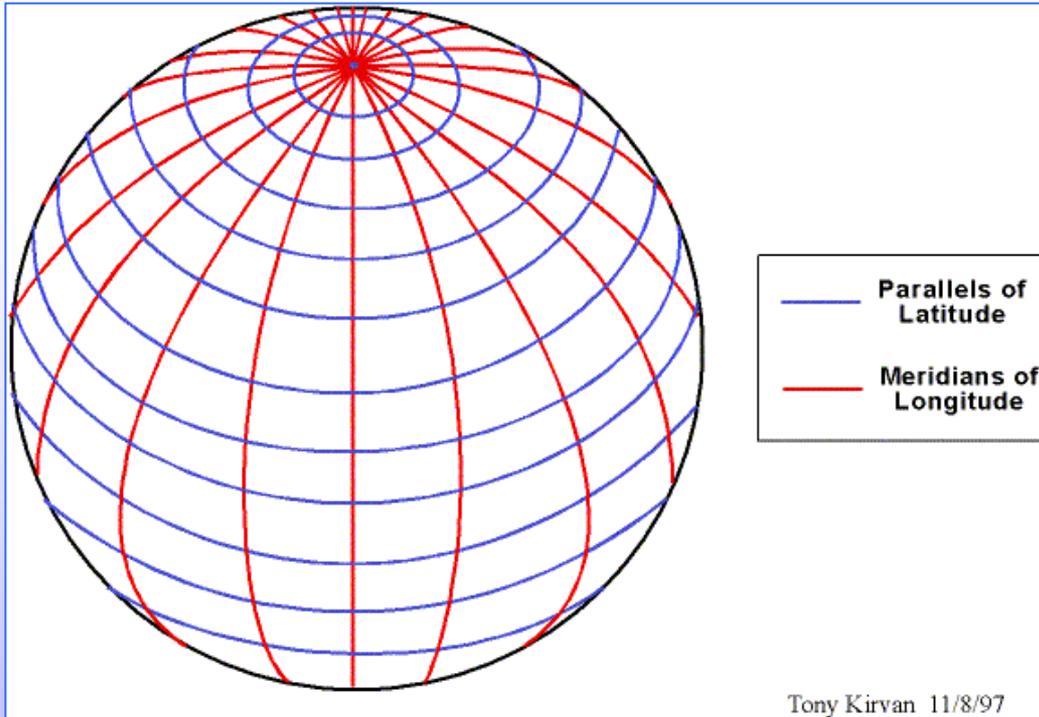
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# Geometria di un satellite

- **Sfera Celeste**
- **Sistemi di Coordinate**
- **Studio Eclissi**
- **Geometria Terra / Satellite**

SMAD Chapter 5  
p. 95

# Sfera Celeste 1/2



azimuth (longitudine  $\ell$ )

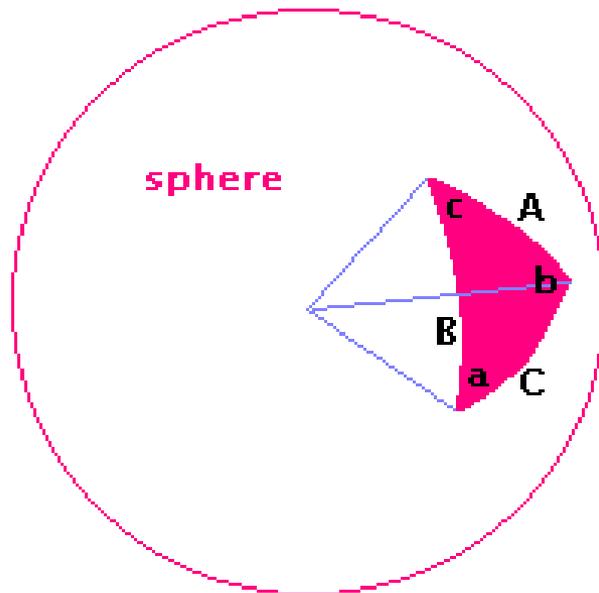
elevazione (latitudine  $\lambda$ )

$$x = \cos \ell \cos \lambda$$

$$y = \sin \ell \cos \lambda$$

$$z = \sin \lambda$$

# Sfera Celeste 2/2



A spherical triangle consists of Great Circle Arcs, extending from the sphere's center, forming Great Circle Angles. Relations among arcs and angles are:

$$\cos(A) = \cos(B) \cos(C) + \sin(B) \sin(C) \cos(a)$$

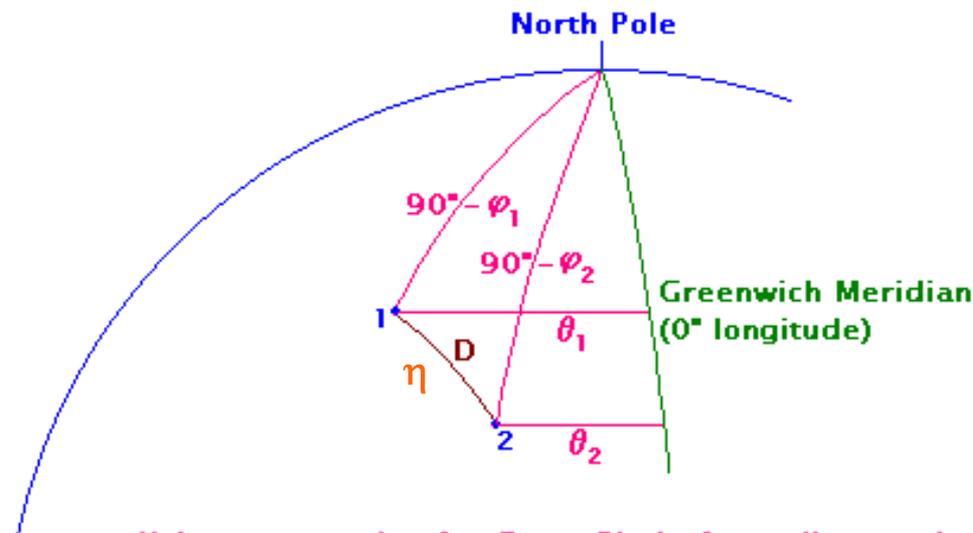
$$\cos(a) = -\cos(b) \cos(c) + \sin(b) \sin(c) \cos(A)$$

$$\sin(A)/\sin(a) = \sin(B)/\sin(b) = \sin(C)/\sin(c)$$

SMAD Appendix D  
Table D-3 p. 907

$\varphi_1, \varphi_2$  elevazione

$\theta_1, \theta_2$  azimuth

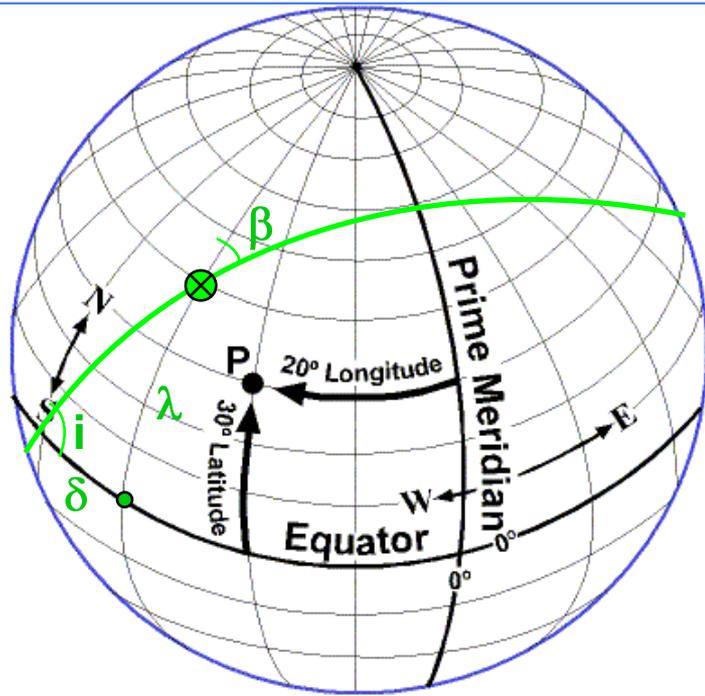


Using an equation for Great Circle Arcs, distance between 1 & 2 is estimated as:

$$\cos(\eta) = \cos(90^\circ - \varphi_1) \cos(90^\circ - \varphi_2) + \sin(90^\circ - \varphi_1) \sin(90^\circ - \varphi_2) \cos(\theta_1 - \theta_2)$$

$$D = 2\pi R_\oplus / (2\pi) \arccos(\sin(\varphi_1) \sin(\varphi_2) + \cos(\varphi_1) \cos(\varphi_2) \cos(\theta_1 - \theta_2))$$

# Finestre di Lancio



Tony Kirvan 11-8-97

$$\lambda > i ?$$

$$\lambda = i ?$$

$$\lambda < i ?$$

SMAD Appendix D  
Table D-1 p. 905 riga  
4 col. 3

SMAD Appendix D  
Table D-1 p. 905 riga  
5 col. 3

$$\sin \beta = \cos i / \cos \lambda$$

$$\cos \delta = \cos \beta / \sin i$$

$$LST = \Omega + \delta$$

$$LST = \Omega + 180^\circ - \delta$$

SMAD chapter 6.4  
p. 153-155

$$v_{sud} = -v_o \cos \gamma \cos \beta_L \quad \beta \text{ azimuth di lancio}$$

$$v_{est} = v_o \cos \gamma \sin \beta_L - v_\lambda \quad \gamma \text{ angolo traiettoria}$$

$$v_r = v_o \sin \gamma \quad (v_z)$$

$$v_\lambda = 464.5 \cos \lambda \text{ m/s}$$

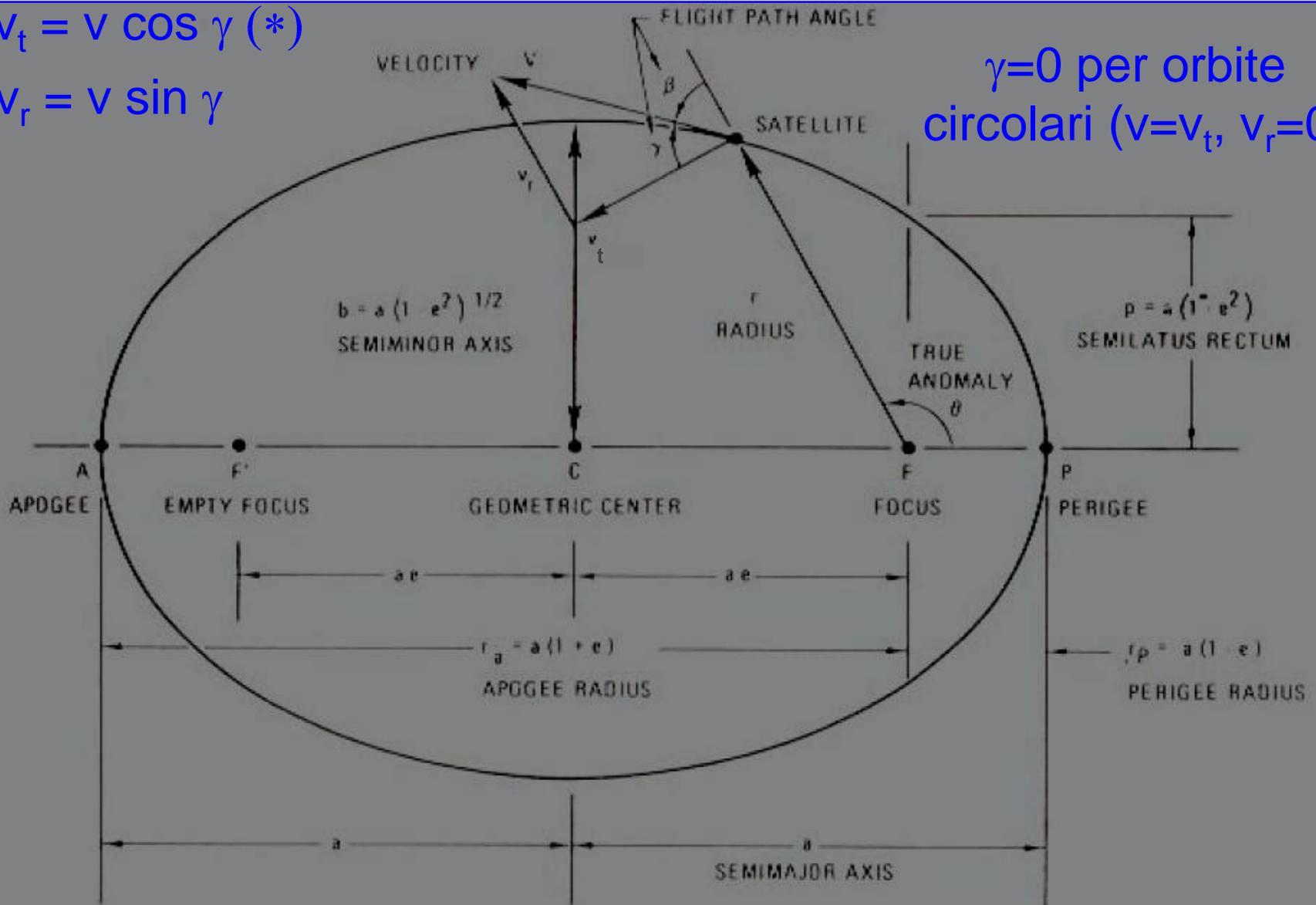
volo al *burn-out*  
(vedi ultima trasparenza  
su orbite)

# Parametri Ellisse

$$v_t = v \cos \gamma (*)$$

$$v_r = v \sin \gamma$$

$\gamma=0$  per orbite circolari ( $v=v_t$ ,  $v_r=0$ )



# Sistemi di Coordinate 1/3

## Sistema Geocentrico

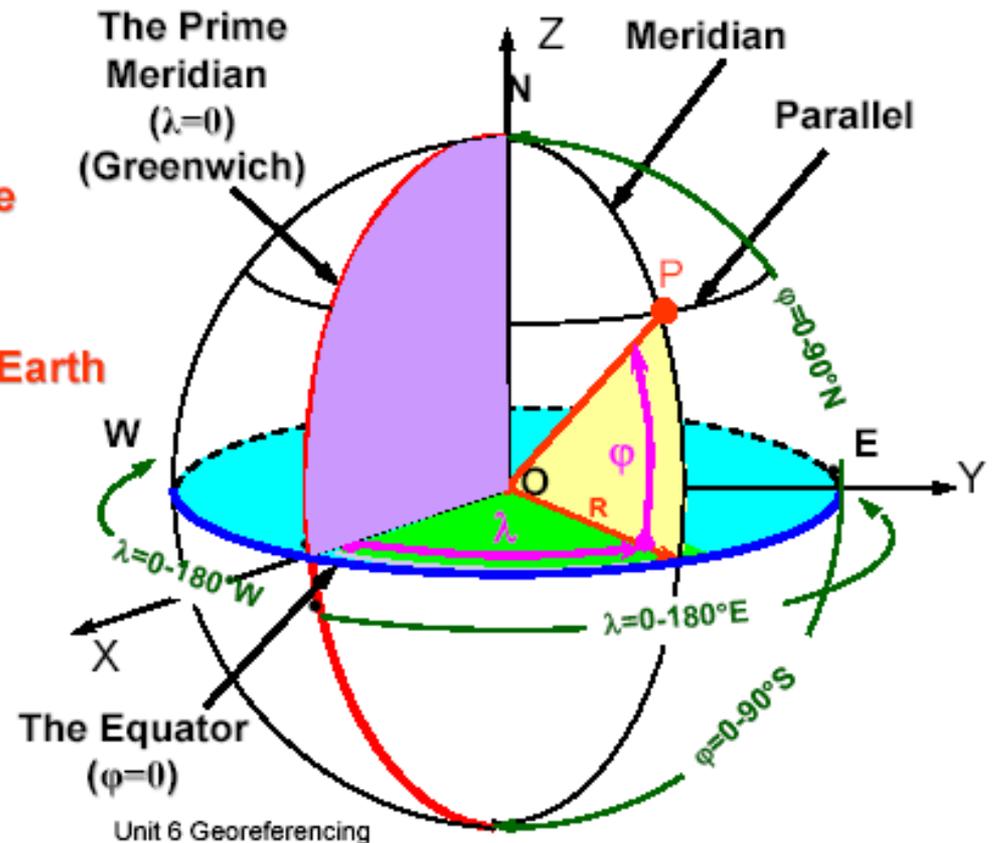
### “Geografico”

$\lambda$  - Geographic longitude

$\varphi$  - Geographic latitude

$R$  - Mean Radius of the Earth

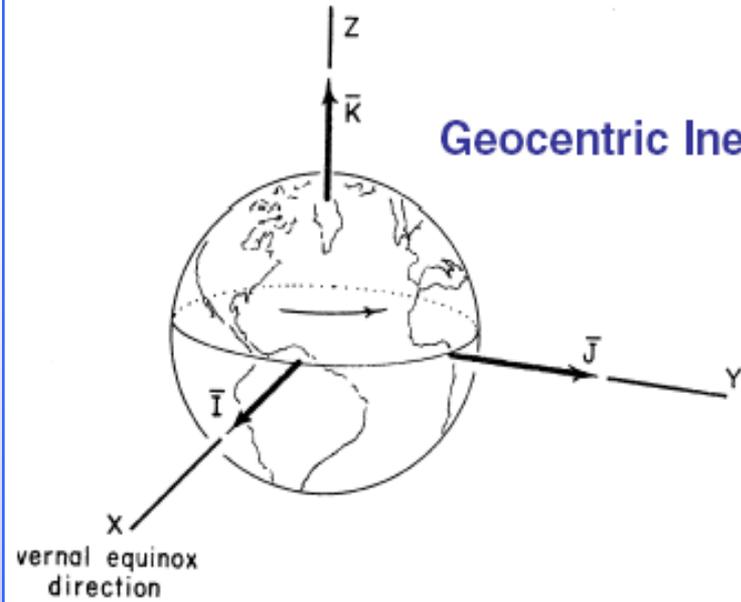
$O$  - The Geo-Center



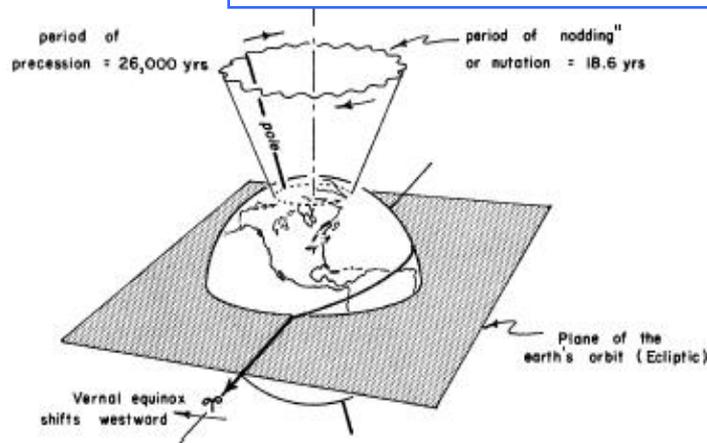
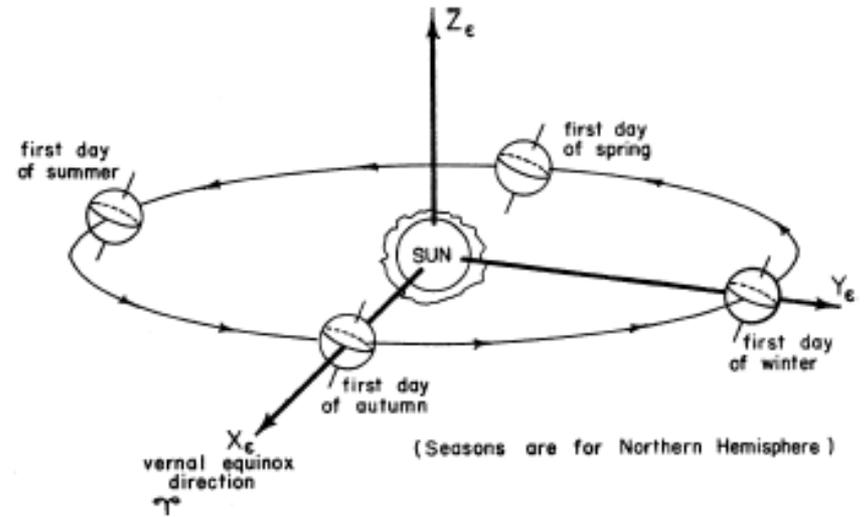
Attenzione all'indicazione lat/long !

# Sistemi di Coordinate 2/3

Geocentric Inertial System



Heliocentric Inertial System



50.2786" westerly drift of the Vernal Equinox per year

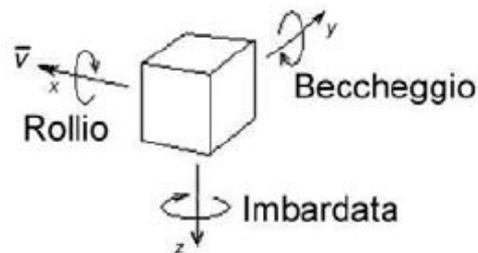
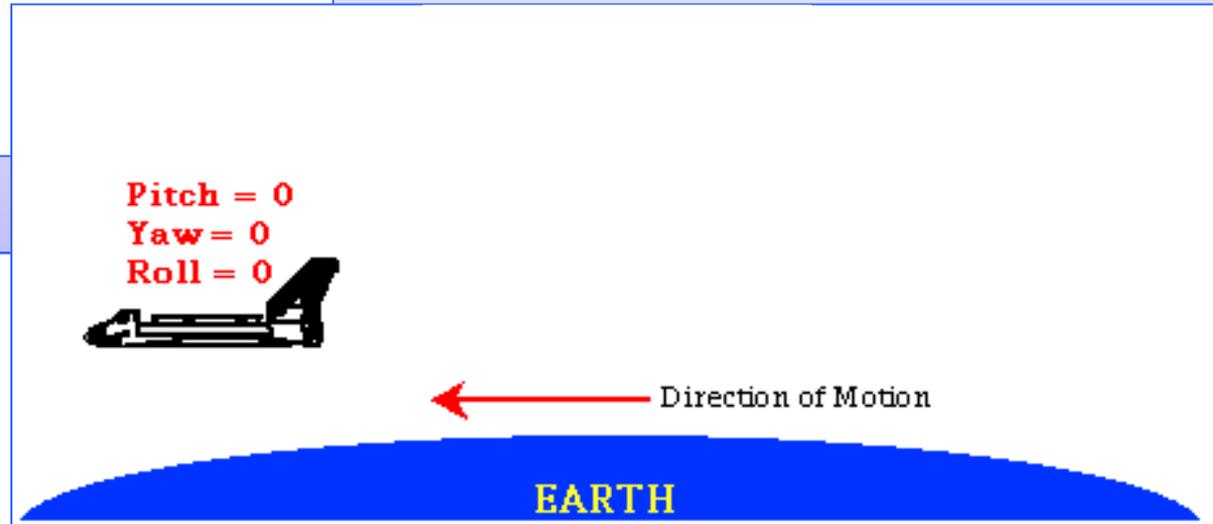
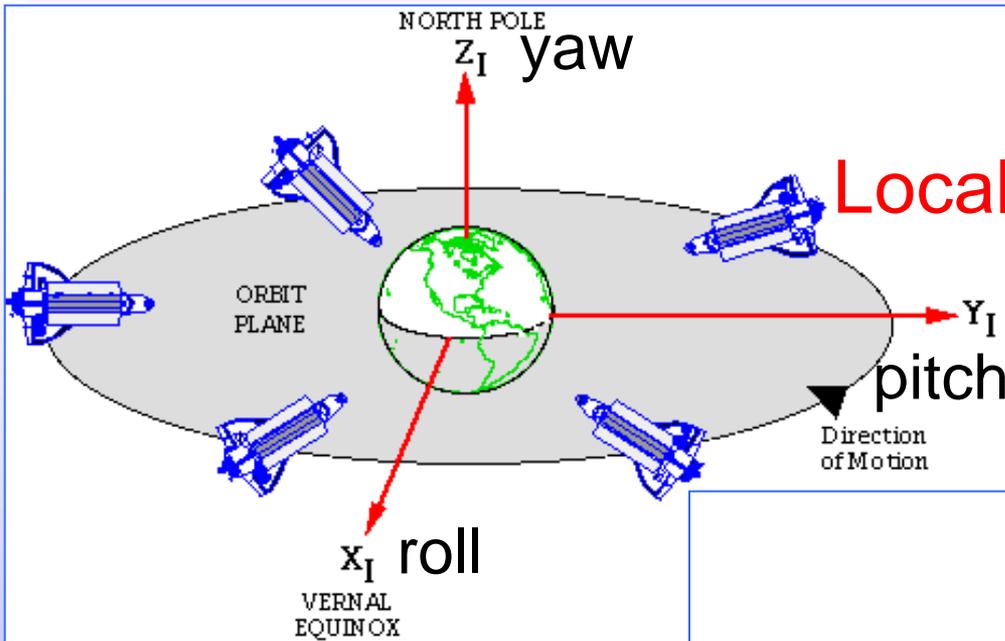
azimuth = ascensione retta  $\alpha$

elevazione = declinazione  $\delta$

# Sistemi di Coordinate 3/3

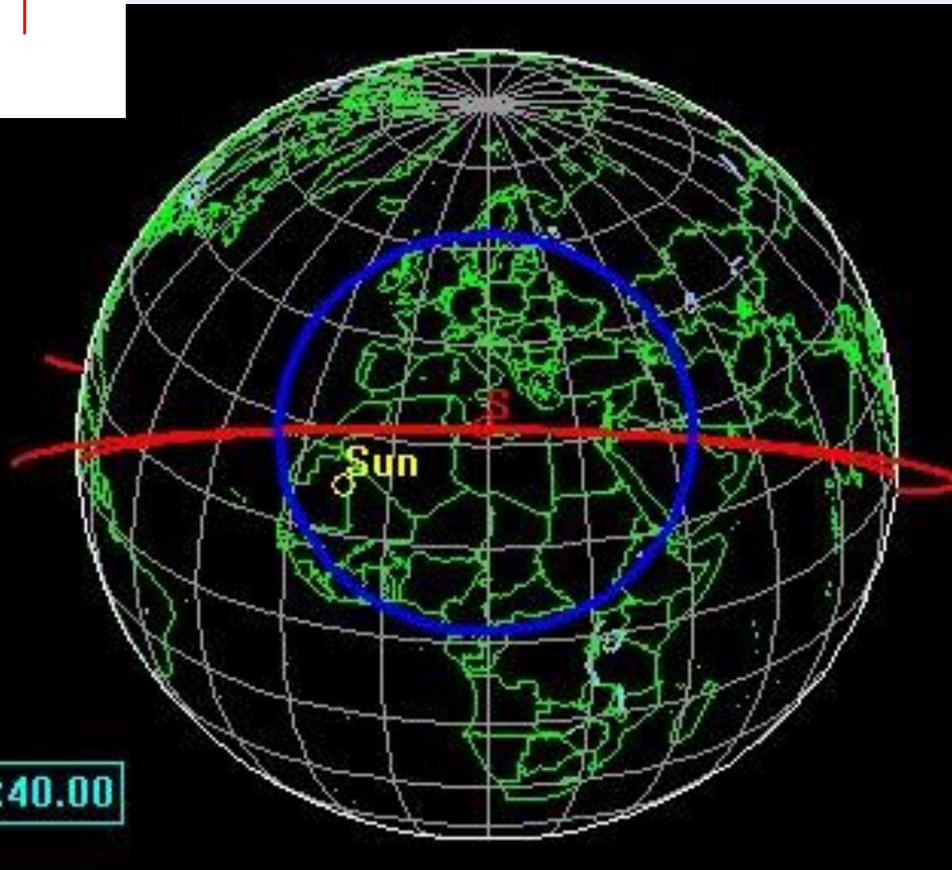
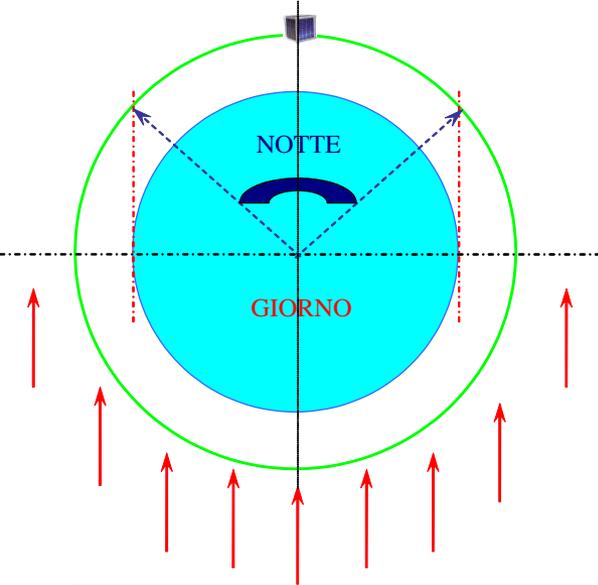
LVLH

Local Vertical – Local Horizontal



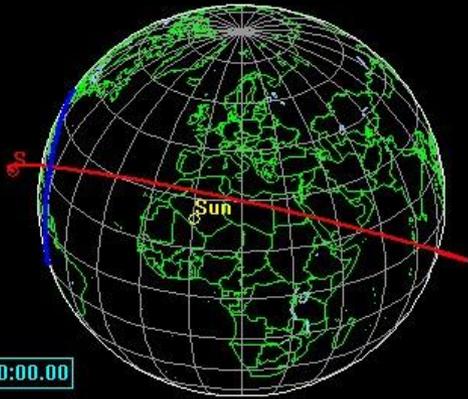
# Studio Eclissi 1/3

$h = 1000$  km,  $i = 32^\circ$

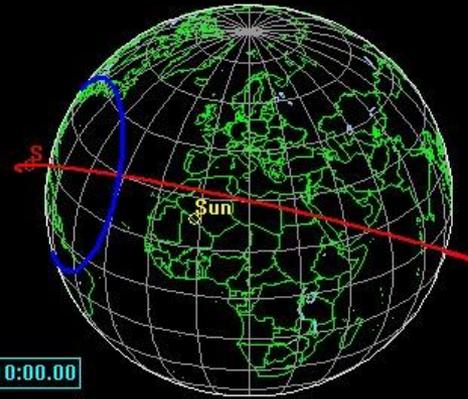


1 Jun 2004 12:26:40.00

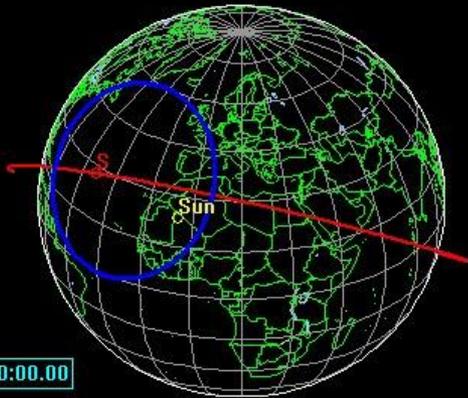
# Studio Eclissi 2/3



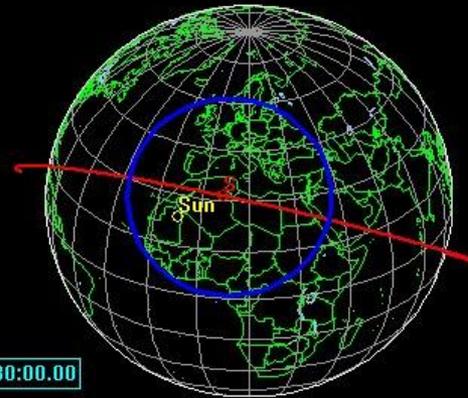
21 Jun 2004 12:00:00.00



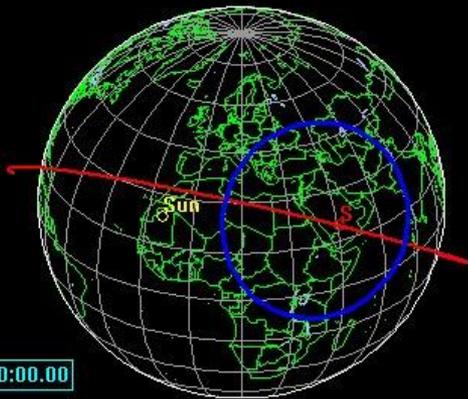
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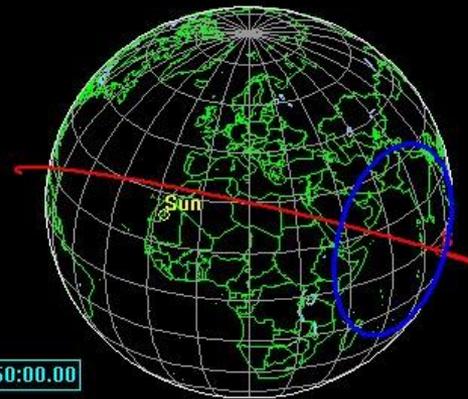
21 Jun 2004 12:20:00.00



21 Jun 2004 12:30:00.00

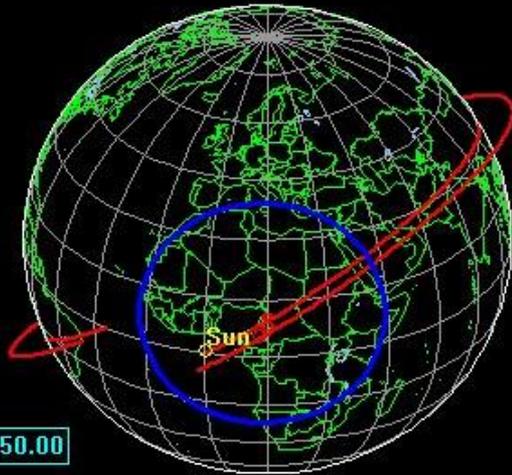


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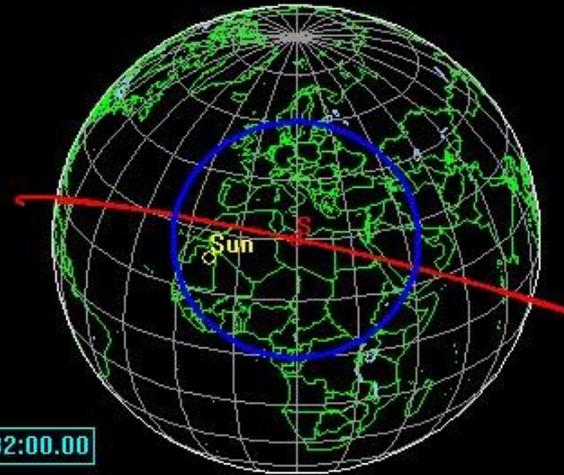


21 Jun 2004 12:50:00.00

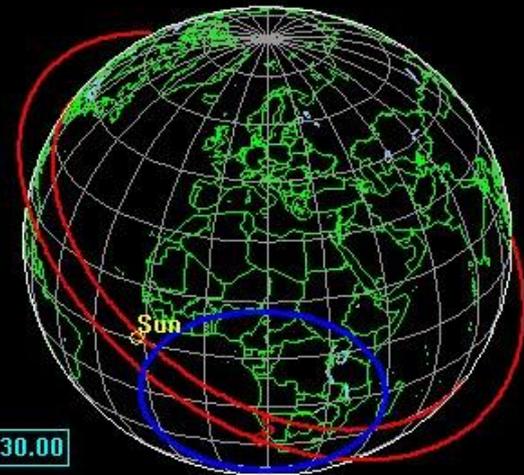
# Studio Eclissi 3/3



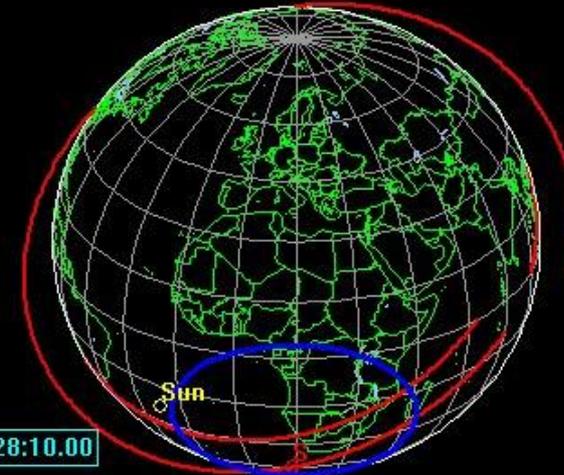
21 Mar 2004 12:04:50.00



21 Jun 2004 12:32:00.00



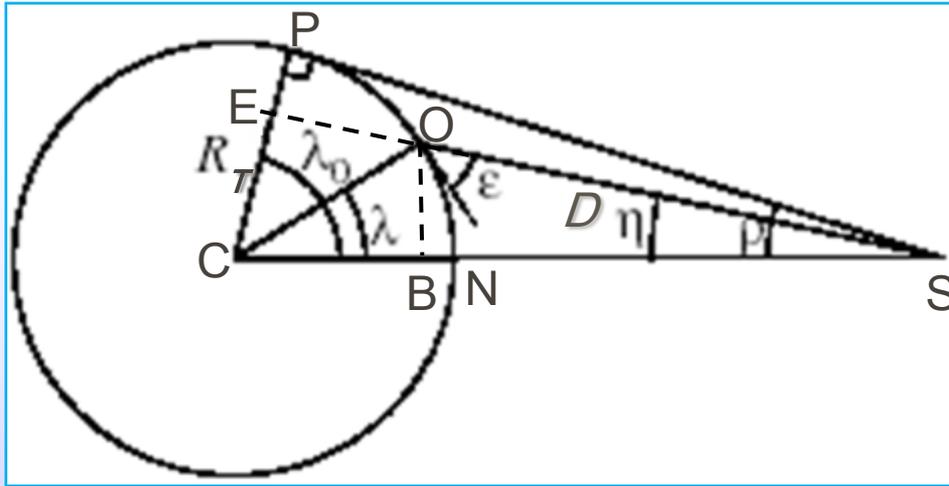
21 Sep 2004 13:02:30.00



21 Dec 2004 13:28:10.00

SMAD chapter 5.1  
Example 1, 2 e 3  
p. 105-110

# Geometria Terra / Satellite



$\rho$  raggio angolare Terra

$\eta$  angolo di nadir

$\varepsilon$  elevazione

$\lambda$  angolo centrale Terra

SMAD chapter 5.2  
fig 5-13 p. 110-113

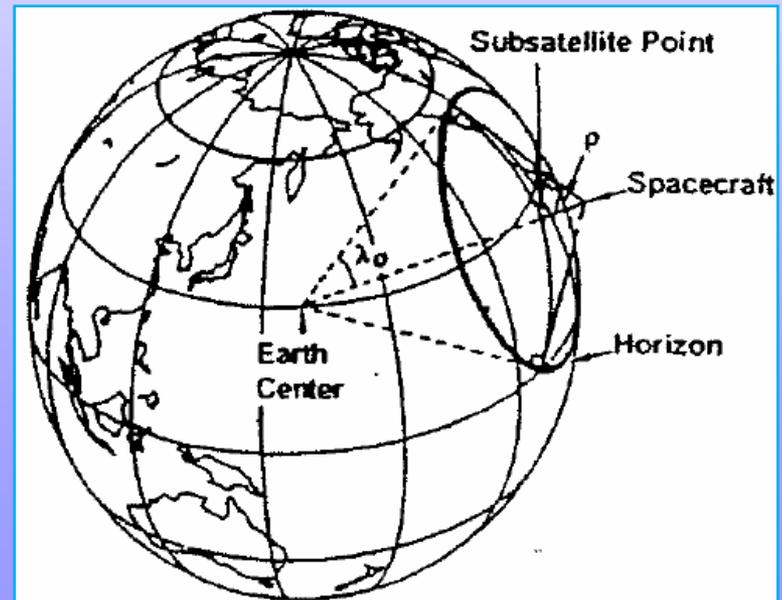
$$\sin \rho = \cos \lambda_0 = R_T / R = R_T / (R_T + h)$$

$$\sin \eta = \cos \varepsilon \sin \rho$$

$$\lambda = \pi/2 - \eta - \varepsilon$$

$$\text{tg } \eta = \sin \rho \sin \lambda / (1 - \sin \rho \cos \lambda)$$

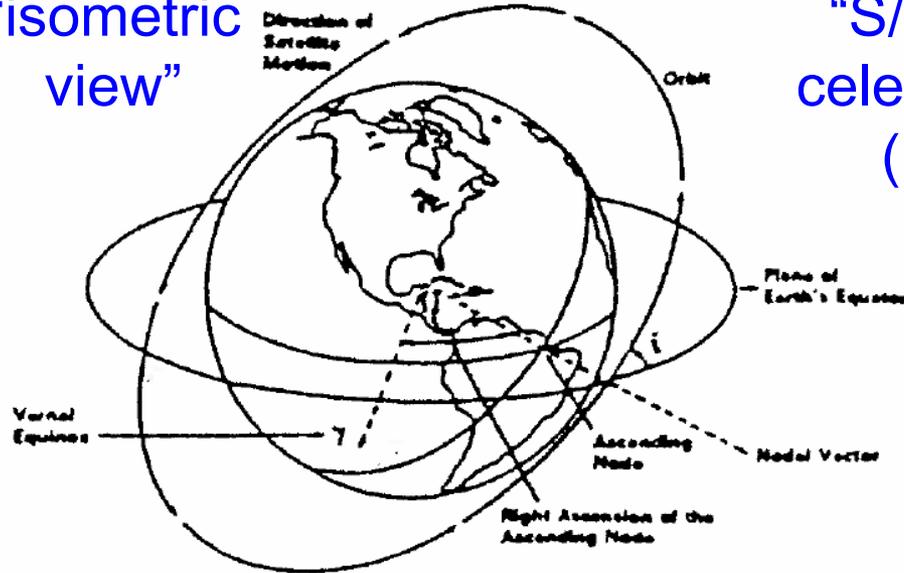
$$D = R_T \sin \lambda / \sin \eta$$



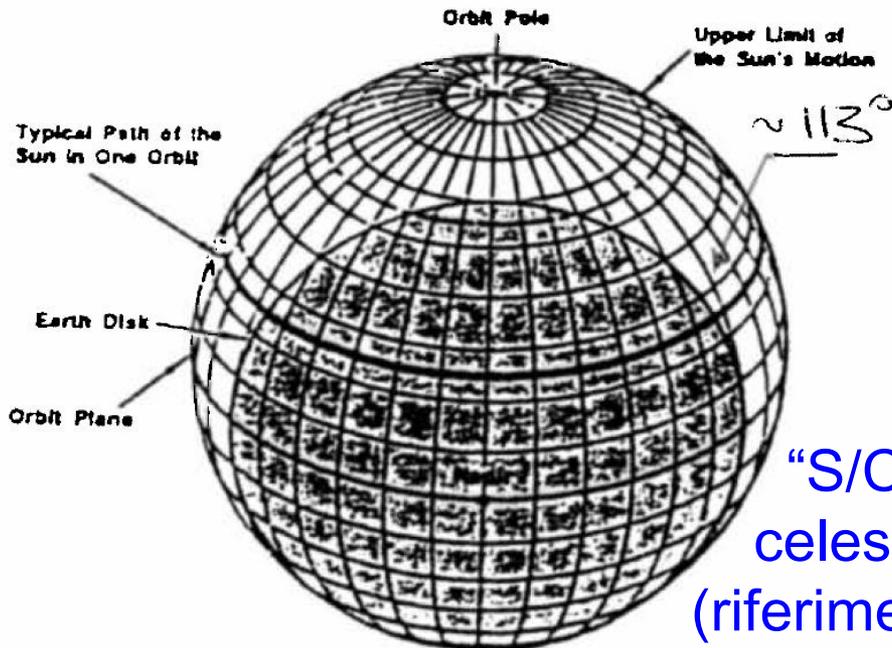
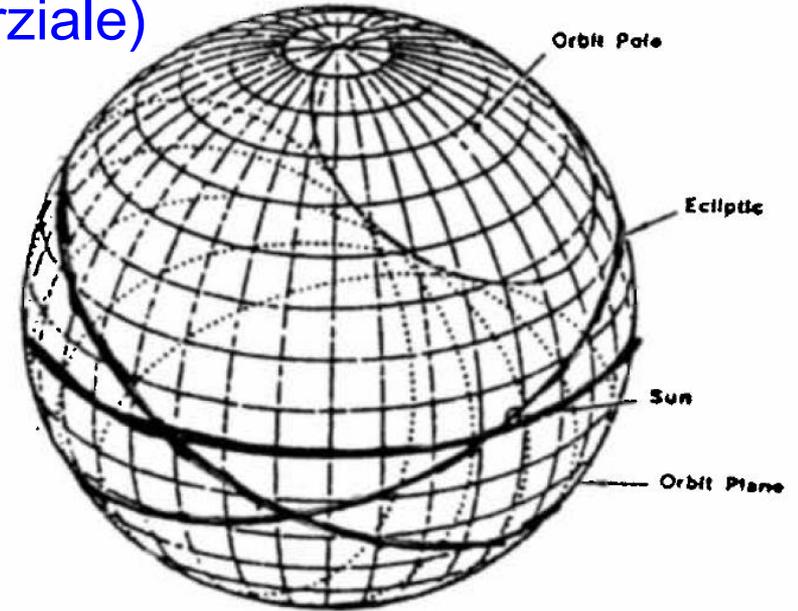
# Geometria di un satellite 1/3

“isometric view”

“S/C centered celestial sphere”  
(inerziale)



Celestial Pole



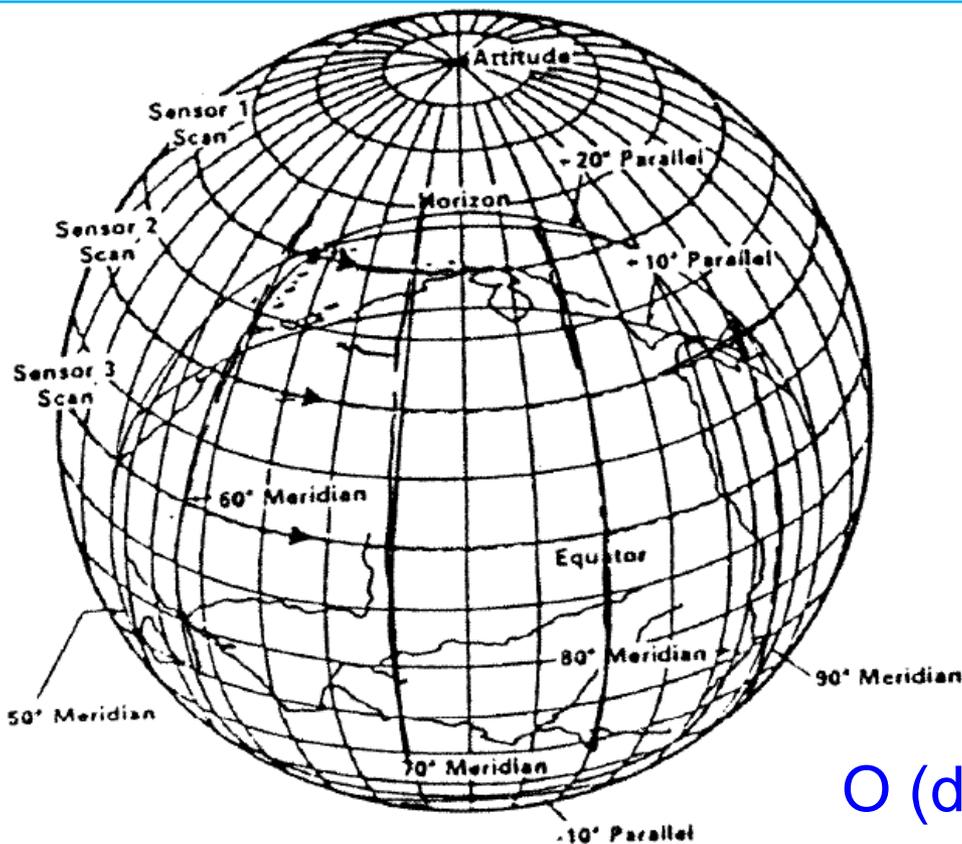
$$h = 1000 \text{ km}, i = 32^\circ \Rightarrow$$

$$\tau = 105 \text{ min}, \rho = 60^\circ$$

“S/C centered celestial sphere”  
(riferimento terrestre)



# Geometria di un satellite 3/3



O (da N)

$$l_o = 200^\circ, \lambda_o = 22^\circ$$

$$l_N = 185^\circ, \lambda_N = 10^\circ$$

⇒

$$\rho = 59.8^\circ, \lambda_o = 30.2^\circ,$$

$$D_{\max} = 3709 \text{ km}$$

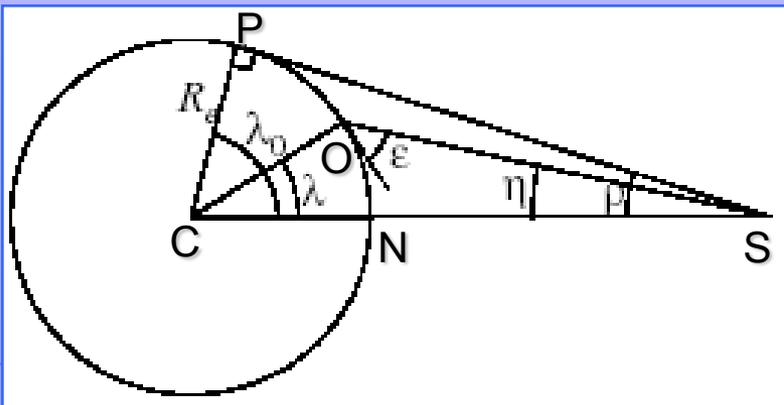
$$\lambda = 18.7^\circ \text{ (swath width)}$$

$$Az = 48.3^\circ$$

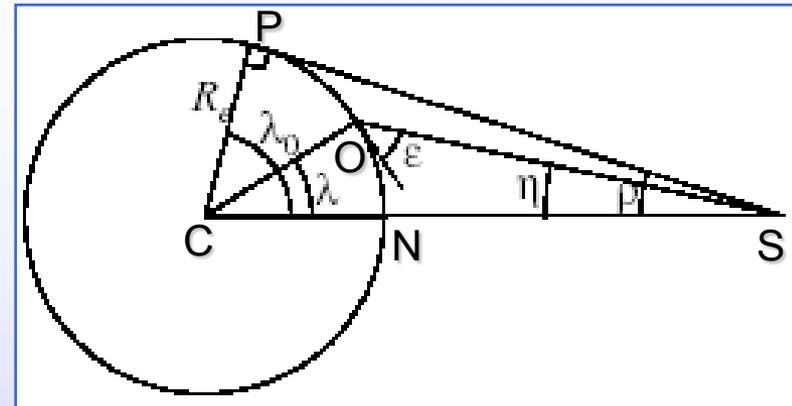
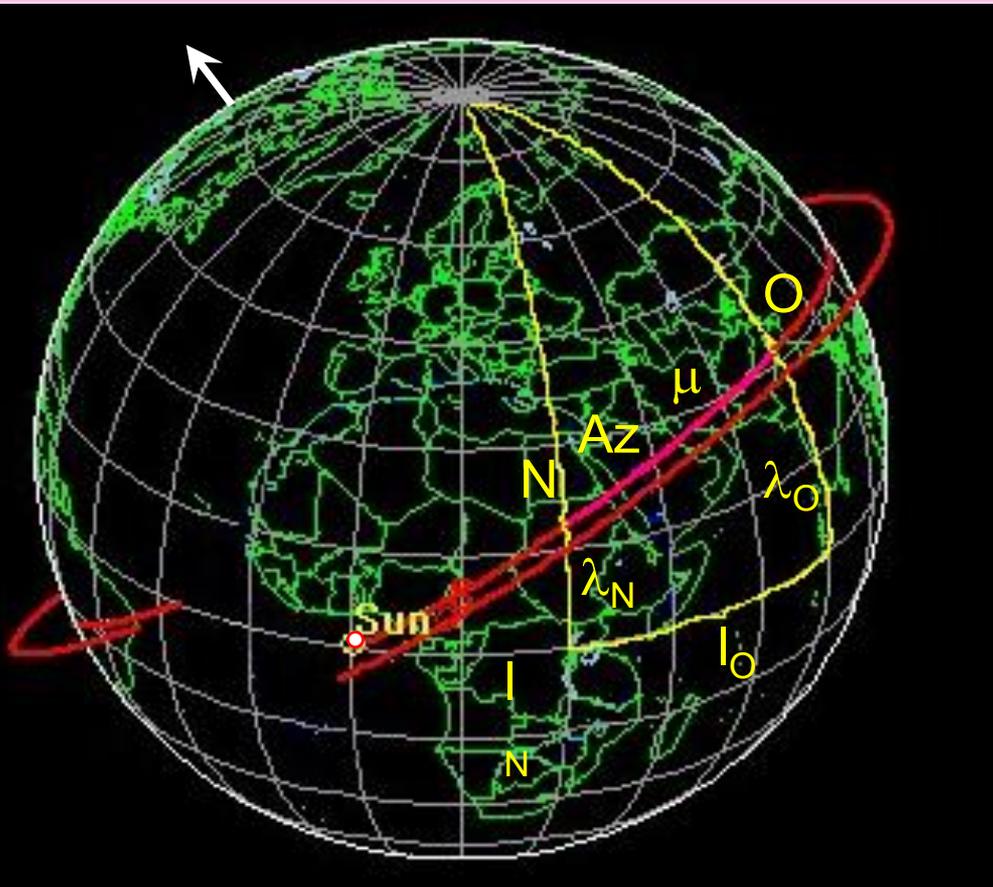
O (da satellite)

$$\eta = 56.8^\circ \text{ } (\epsilon = 14.5^\circ)$$

$$D = 2444 \text{ km}$$



# Passaggio sopra la stazione 1/2



Meglio: geometria vista dal satellite!  
Nota: quella segnata non è un'orbita del satellite ma un cerchio max che passa per O e N

N = Sub Satellite Point

O = Punto qls Terra!!!

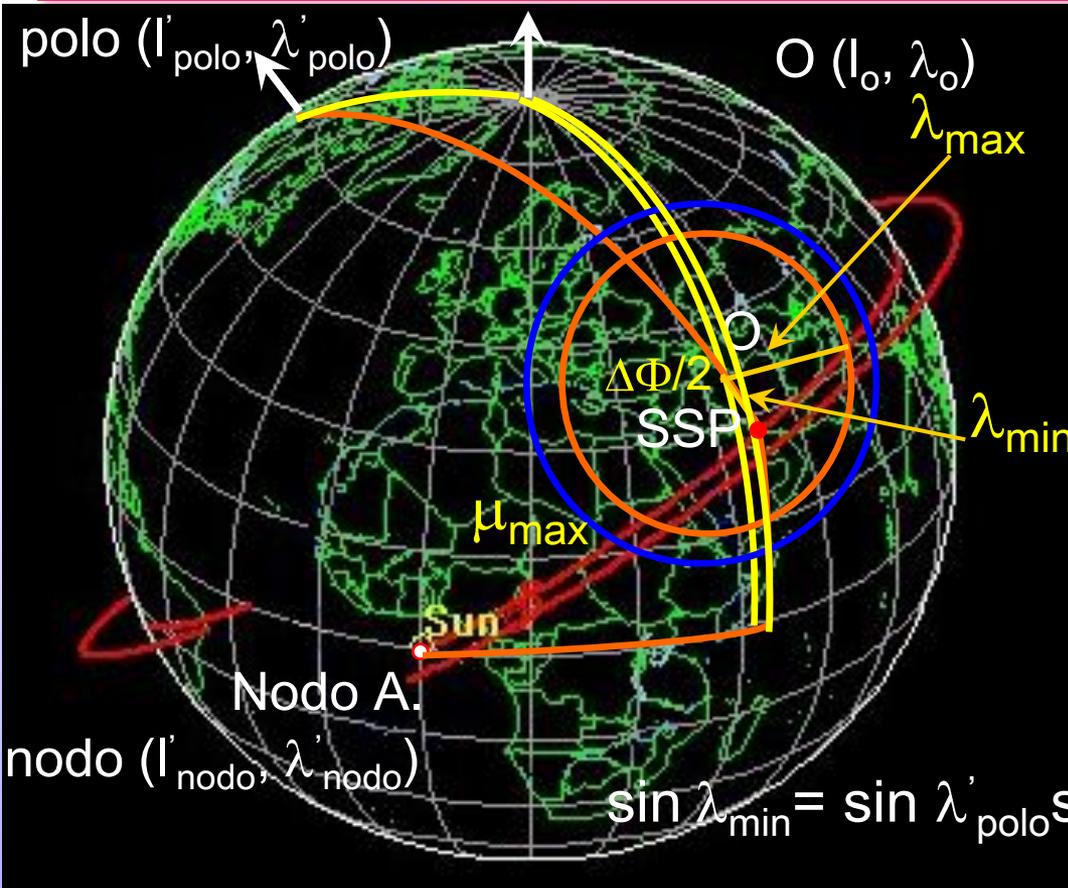
$$\cos \mu = \sin \lambda_N \sin \lambda_O + \cos \lambda_N \cos \lambda_O \cos(l_O - l_N) \Rightarrow \eta$$

$$\sin \lambda_O = \sin \lambda_N \cos \mu + \cos \lambda_N \sin \mu \cos Az \quad (\mu \equiv \lambda)$$

$$\cos Az = (\sin \lambda_O - \sin \lambda_N \cos \mu) / \cos \lambda_N \sin \mu$$

SMAD chapter 5.2  
 fig 5-12 p. 112

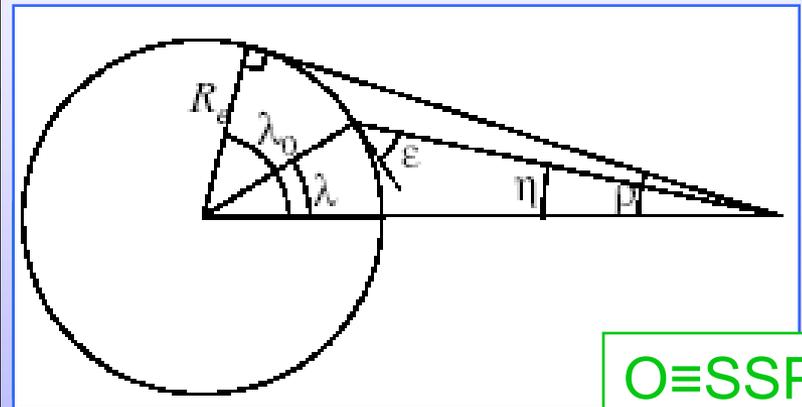
# Passaggio sopra la stazione 2/2



$$\sin \eta_{\max} = \cos \varepsilon_{\min} \sin \rho$$

$$\lambda_{\max} = \pi/2 - \eta_{\max} - \varepsilon_{\min}$$

$$D_{\max} = R_T \sin \lambda_{\max} / \sin \eta_{\max}$$



O=SSP

$$\sin \lambda_{\min} = \sin \lambda'_{\text{polo}} \sin \lambda_o + \cos \lambda'_{\text{polo}} \cos \lambda_o \cos(l_o - l'_{\text{polo}})$$

$$\tan \eta_{\min} = \sin \rho \sin \lambda_{\min} / (1 - \sin \rho \cos \lambda_{\min})$$

$$\varepsilon_{\max} = \pi/2 - \eta_{\min} - \lambda_{\min}$$

$$R_T \sin \lambda_{\min} = D_{\min} \sin \eta_{\min}$$

$$\omega_{\max} = \dot{\theta}_{\max} = v_{\text{sat}} / D_{\min}$$

$$\cos \Delta\Phi/2 = \tan \lambda_{\min} / \tan \lambda_{\max}$$

$$T = \tau/180^\circ \arccos(\cos \lambda_{\max} / \cos \lambda_{\min})$$

SMAD chapter 5.3.1  
fig 5-17 p. 118-121