Problem set constrained optimization

1)

a) Write the KT conditions of the following problem:

$$\max_{\{x,y\}} a \cdot (x \cdot y)^b$$
$$s.t. \quad 100 - \mathrm{m} \cdot x - n \cdot y \geq 0$$
$$x \leq 4$$
$$y \geq 0$$
$$\text{where } b < 0.5\, a < 0$$

b) Write the KT conditions of the modified Lagrangean

c) Check if these conditions are both necessary and sufficient

2)

a) Solve the following problem:

$$\max_{\{x,y\}} (100 - x) \cdot y$$
$$s.t. \quad x \cdot y \geq 10$$
$$x \leq 2$$
$$y \geq 0$$

b) Check if KT conditions are both necessary and sufficient

3) Solve the following problems:

a) $\max_{\{x,y)} \left(\mathrm{x} - 1\right)^2 + \left(\mathrm{y} - 1\right)^2$ s.t. $0 \leq x \leq 2$ and $0 \leq y \leq 2$

b) $\min_{\{x,y)} \left(\mathrm{x} - 1\right)^2 + \left(\mathrm{y} - 1\right)^2$ s.t. $0 \leq x \leq 2$ and $0 \leq y \leq 2$

4) For each possible value of the constant a, solve the problem

$$max_{\{x,y)}\ x + ay\ subject\ to\ \ x^2 + y^2 \leq 1\ and\ x + y \geq 0.$$

5) Consider the following problem.

$$max_{\{x\}} - x_1^2 - x_1 x_2 - x_2^2 \quad subject\ to\ x_1 - 2x_2 \leq -1\ and\ 2x_1 + x_2 \leq 2$$

a. Are the Kuhn-Tucker conditions necessary for a solution of this problem?
b. Are the Kuhn-Tucker conditions sufficient for a solution of this problem?
c. If possible, use the Kuhn-Tucker conditions to find the solution(s) of the problem.