Anomalies in expected utility theory

Diminishing Marginal Utility of Wealth Cannot Explain Risk Aversion
M. Rabin (2000)

- Economists explain risk aversion by the realistic assumption of diminishing marginal utility of wealth.
- We dislike vast uncertainty in lifetime wealth because the marginal value of a dollar when we are poor is higher than when we are rich.
- Within the expected-utility framework, the concavity of the utility-of-wealth function is not only sufficient to explain risk aversion—it is also necessary
- Diminishing marginal utility of wealth is the *sole* explanation for risk aversion.

- it is an utterly implausible explanation for appreciable risk aversion, except when the stakes are very large.
- Any utility-of-wealth function that doesn't predict absurdly severe risk aversion over very large stakes predicts negligible risk aversion over modest stakes.
- Arrow (1971) shows that an expected-utility maximizer with a differentiable utility function will always want to take a sufficiently small stake in any positive-expected-value bet.

- i.e. Expected-utility maximizers are arbitrarily close to risk neutral when stakes are arbitrarily small.
- this approximate risk-neutrality prediction holds not just for very small stakes, but for quite sizable and economically important stakes.
- Diminishing marginal utility of wealth is not a plausible explanation of people's aversion to risk on the scale of \$10, \$100, \$1,000, or even more.

- Rabin, Calibration Theorem (Econometrica 2000):

  "If an expected-utility maximizer always turns down modest stakes Gamble X, she will always turn down large-stakes Gamble Y."

- If decision maker
  - turns down gambles where she loses $100 or gains $110, each with 50% probability, then she will turn down 50-50 bets of losing $1,000 or gaining *any* sum of money.
  - turns down 50-50 lose $1,000/gain $1,050 bets would always turn down 50-50 bets of losing $20,000 or gaining any sum.
  - turns down 50-50 lose $100/gain $101 bets would always turn down 50-50 bets losing $10,000 or gaining any sum.

Table I [Rabin(2000)]

If averse to 50-50 lose \$100 / gain \$$g$ bets for all wealth levels, will turn down 50-50 lose \$$L$ / gain \$$G$ bets; G's entered in table.

L	$g = \$101$	$g = \$105$	$g = \$110$	$g = \$125$
\$400	400	420	550	1,250
\$600	600	730	990	∞
\$800	800	1,050	2,090	∞
\$1,000	1,010	1,570	∞	∞
\$2,000	2,320	∞	∞	∞
\$4,000	5,750	∞	∞	∞
\$6,000	11,810	∞	∞	∞
\$8,000	34,930	∞	∞	∞
\$10,000	∞	∞	∞	∞
\$20,000	∞	∞	∞	∞

Ellsberg paradox

An urn contains 30 red balls and 60 other balls that are either **black** or yellow.

The number of **black** balls is unknown

Each individual ball is equally probable to be drawn (as any other.

Subjects have to choose between two gambles

Gamble A	Gamble B
You receive $100 if you draw a **red ball**	You receive $100 if you draw a **black ball**

Then subjects have to choose between other two gambles (about a different draw from the same urn):

Gamble C	Gamble D
You receive $100 if you draw a **red** or **yellow** ball	You receive $100 if you draw a **black** or **yellow** ball

A subject prefers A to B if and only if he believes that a **red** ball is more likely than a **black** ball.

A subject prefers C to D if and only if he believes that a **red** or **yellow** ball is more likely than drawing a **black** or **yellow** ball.

if drawing a **red** ball is more likely than drawing a **black** ball, then drawing a **red** or **yellow** ball is also more likely than drawing a **black** or **yellow** ball.

If a subject prefers A to B, it follows that he will also *prefer* C to D.

If a subject prefers B to A, it follows that he will also *prefer* D to C.

most people strictly prefer A to B and D to C.

Therefore, some assumptions of the expected utility theory are violated

Mathematically, your estimated probabilities of each colour ball can be represented as: r, y, and b. If $A \succ B$ must be:

$$r \cdot u(100) + (1 - r) \cdot u(0) > b \cdot u(100) + (1 - b) \cdot u(0)$$

$$r \cdot \big(u(100) - u(0)\big) > b \cdot \big(u(100) - u(0)\big)$$

$$r > b$$

If $D \succ C$ must be:

$$b \cdot u(100) + y \cdot u(100) + r \cdot u(0) > r \cdot u(100) + y \cdot u(100) + b \cdot u(0)$$

$$b \cdot u(100) + r \cdot u(0) > r \cdot u(100) + b \cdot u(0)$$

$$b \cdot \big(u(100) - u(0)\big) > r \cdot \big(u(100) - u(0)\big)$$

$$b > r$$

the exact chances of winning are known for Gambles A and D, and not known for Gambles B and C,

Then this experimental result can be taken as evidence for some sort of ambiguity aversion which cannot be accounted for in expected utility theory

Preference reversal phenomenon

Sarah Lichtenstein and Paul Slovic
(1971) and Harold Lindman (1971).

In the classic preference reversal experiment individuals carry out two distinct tasks

First task: subjects has to choose between two prospects:

one prospect (called the $-bet) offers a small chance of winning a "good" prize;

The other (the "P-bet") offers a larger chance of winning a smaller prize

Second task: subjects have to assign monetary values—usually minimum selling prices denoted M($) and M(P)—to the two prospects.

Repeated studies have revealed a tendency for individuals to chose the P-bet (i.e., reveal $P \succ \$$)

while placing a higher value on the \$-bet (i.e., $M(\$) > M(P)$).

It is a puzzle for economics because, viewed from the standard theoretical perspective, both tasks constitute ways of asking essentially the same question:

“which of these two prospects do you prefer?”

Allais paradox (1953):
the common consequence effect

	Option	0.1	0.01	0.89
Choice 1	A **	500	500	500
	B	2500	0	500
Choice 2	C	500	500	0
	D **	2500	0	0

Each row represents a prospect

Each column is an event (state of the world) with its associated probability

1) Subjects choose between prospect A and B

Note that the third state is irrelevant because the outcome is the same in the two prospects (independence axiom)

	Option	0.1	0.01	0.89
Choice 1	A **	500	500	500
	B	2500	0	500
Choice 2	C	500	500	0
	D **	2500	0	0

2) Subjects choose between prospect C and D

Note that the third state is irrelevant (as in previous choice). Then the choice between A and B is equivalent to the choice between C and D.

According to the independence axiom if $A \succcurlyeq B$ then $C \succcurlyeq D$ and viceversa.

Experimental evidence:

Many subjects reveal $A \succcurlyeq B$ but $D \succcurlyeq C$

→ Violation of independence axiom

Common ratio effect

	Option	
Choice 1	A	(6000, 0.45)
	B	(3000, 0.90)
Choice 2	C	(6000, 0.001)
	D	(3000, 0.002)

Choice 1 and Choice 2 imply the same payoffs and the same relative probability

According to the Independence axiom if $A \succcurlyeq B$ then $C \succcurlyeq D$ and viceversa.

Experimental evidence:

14% of subjects reveal $A \succcurlyeq B$ but 73% reveal $C \succcurlyeq D$

→ Violation of independence axiom

This phenomenon is observed in pair of choices of the following general form

$\boldsymbol{s}^{**} = (y,\ p;\ 0,\ 1 - p)$

$\boldsymbol{r}^{**} = (x,\ \lambda p;\ 0, 1 - \lambda p)$

where $x > y$

Assume that the ratio of "winning" probabilities (λ) is constant

Expected Utility Theory implies that preferences should not depend on the value of *p*

numerous studies reveal a tendency for individuals to switch their choice from $\boldsymbol{s}^{**}$ to $\boldsymbol{r}^{**}$ as *p* falls.

Alternative Theories

Classification by Starmer 2000:

Conventional and **nonconventional theories**

Conventional theories accept the first three axioms, completeness, transitivity and continuity and allow for violations of independence.

These theories maintain monotonicity or dominance

Nonconventional theories allows for more radical changes

Short review of **Conventional theories**

Weighted utility theory

Chew and MacCrimmon 1979,

Preference function

$$V(\boldsymbol{q}) = \frac{\sum p_i \cdot g(x_i) \cdot u(x_i)}{\sum p_i \cdot g(x_i)}$$

where $g(\cdot)$ and $u(\cdot)$ are two weighting functions and p_i is the probability of outcome i

Expected utility theory is a special case when weights $g(\cdot)$ are identical

It implies a weaker form of the independence axiom:

If $\boldsymbol{q} \succ \boldsymbol{r}$ then for each p_q there exists a corresponding p_r such that:

$$\left(\boldsymbol{q}, p_q; \boldsymbol{s}, 1 - p_q\right) \succ (\boldsymbol{r}, p_r; \boldsymbol{s}, 1 - p_r)\ \forall \boldsymbol{s}$$

people become more risk averse as the prospects they face improve

Advantage: they explain the violations of the independence axiom

Problem: no psychological foundation

Disappointment Theory

Bell 1985, Loomes and Sudgen 1986

Preference function

$$V(\boldsymbol{q}) = \sum p_i \cdot \left(u(x_i) + D\left[u(x_i) - \underline{U}\right]\right)$$

where $u(x_i)$ is the utility of $x_i$in isolation, $\underline{U}$ is the prior expectation of the utility of **q** and $D[\cdot]$ is the disappointment function

If the outcome of the prospect is worse than the expected → disappointment

If the outcome of the prospect is better than the expected → euphoria

$u(x_i) - \underline{U} < 0$ disappointment

$u(x_i) - \underline{U} > 0$ euphoria

When $\mathrm{D}\left[u(x_i) - \underline{U}\right] = 0$ the model reduces to EUT

Intuition: people are disappointment averse, i.e. the disappointment function $D[\cdot]$ is concave

Betweenneess models

Gull 1991, Neilson 1992

weakened form of independence

Betweenneess:

$$\text{If } \boldsymbol{q} \succ \boldsymbol{r} \text{ then } \boldsymbol{q} \succ (\boldsymbol{q}, p; \boldsymbol{r}, 1-p) \succ \boldsymbol{r}$$

For all $p < 1$

An individual is indifferent to all mixture among equally valued prospects

Nonbetweenneess models

Quadratic utility theory, Chew, Epstein and Segal 1991

No restriction as betweenneess

weakened form of betweenneess, mixture symmetry

$$\text{If } \boldsymbol{q} \sim \boldsymbol{r} \text{ then } (\boldsymbol{q}, p; \boldsymbol{r}, 1-p) \sim (\boldsymbol{q}, 1-p; \boldsymbol{r}, p)$$

Becker and Sarin (1987) no assumption regarding independence

Decision-weighting theories

- Subjective weights (utilities) to each outcomes
- Objective probabilities are combined with subjective weights
- Probability transformation functions convert objective probabilities into *subjective decision weights*
- Experimental evidence that individuals either underestimate or overestimate probabilities: probabilities of common events are underestimated and those of rare events are overestimated.

Edwards (1955, 1962): ***subjective expected value***

Subjective probabilities and objective outcomes, i.e. $u(x_i) = x_i$

Preference function

$$V(\boldsymbol{q}) = \sum w_i \cdot x_i$$

Handa (1977), probability weighting function $\pi(p_i)$ where $\pi(0) = 0$ and $\pi(1) = 1$

Variations are *simple decision weighted utility*

$$V(\boldsymbol{q}) = \sum \pi(p_i) \cdot u(x_i)$$

Monotonicity is not satisfied

Rank-dependent expected utility theory

Quigging 1982

Weights depend on objective probabilities as well as on the ranking of outcomes

Ranking of the outcomes (from the worse to the best):

$$x_1, x_2, \dots . x_n$$

Preference function is

$$V(\boldsymbol{q}) = \sum w_i \cdot u(x_i)$$

where

$$w_i = \pi(p_i + \cdots + p_n) - \pi(p_{i+1} + \cdots + p_n)$$

Transformation of cumulative probabilities

$\pi(p_i + \cdots + p_n)$ is the subjective probability to get an outcome good at least as x_i

$\pi(p_{i+1} + \cdots + p_n)$ is the subjective probability to get an outcome better than x_i

These model explain over and under estimation of objective probabilities

Preserve monotonicity

Shape of $\pi(\cdot)$.

If $\pi(\cdot)$ is convex, individual is pessimistic because attaches higher subjective probabilities to lower outcomes

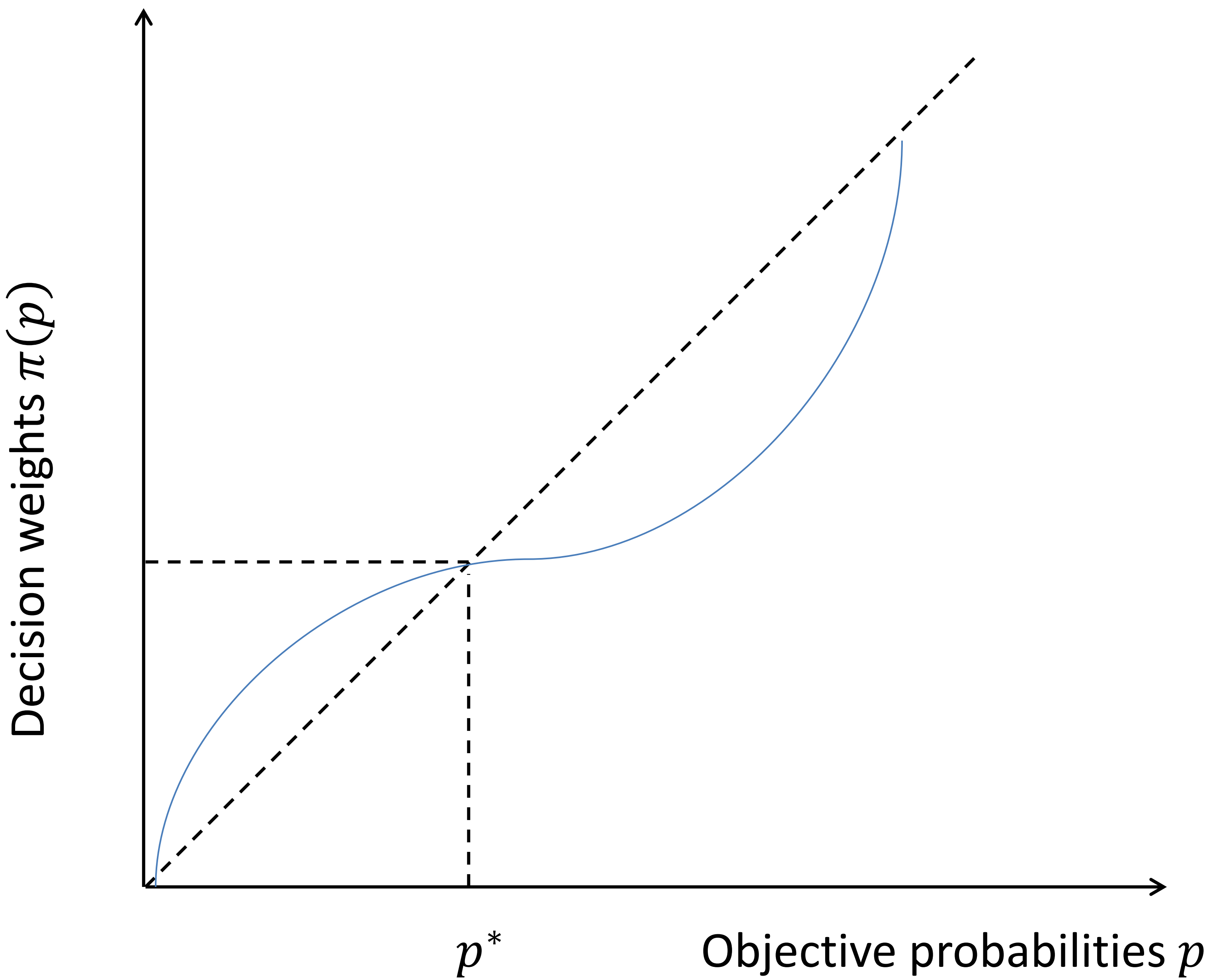
If $\pi(\cdot)$ is concave, individual is optimistic because attaches higher subjective probabilities to higher outcomes

Quigging 1982 proposes s-shaped function:

$\pi(p) = p$ if $p = p^*$

It is concave if if $p < p^*$

It is convex if if $p > p^*$



Decision weights $\pi(p)$
p^*
Objective probabilities p

Yaari's Dual Theory

Yaari, M. (1987) "The Dual Theory of Choice Under Risk."

Expected utility theory: linear in probabilities and non-linear in payoffs

Dual theory: non-linear in probabilities and linear in payoffs.

Aim: to explain behavioral traits that are at odds with expected utility theory

G_v *is the Decumulative distribution function (DDF)* of a random variable, $v \in V \equiv [0, 1]$:

$$G_v\,(t) = \Pr\{v > t\}, 0 \leq t \leq 1$$

G_v is nonincreasing

$G_v(1) = 0$

$\int_0^1 G_v(t)dt = Ev$ i.e. the expected value of v.

Neutrality: u & v are two lotteries, $G_u = G_v$ implies $u \sim v$.

Complete weak order: The preference relation is reflexive, transitive, and connected.

Continuity: For all DDFs G, G', H, H', such that $G > G'$, there exists $\varepsilon > 0$ such that $|G - H| < \varepsilon$ & $|G' - H'| < \varepsilon$ imply $H > H'$, where | m| is the integral of m(t)dt.

Monotonicity: With respect to first-order stochastic dominance, if $G_u(t) \geq G_v(t) \forall t$, then $u \succcurlyeq v$.

Rewiev of Allais paradox and common consequence effects