Prospect theory

Introduction

- Kahneman and Tversky (1979)
- Kahneman and Tversky (1992) → cumulative prospect theory
- It is classified as nonconventional theory
- It is perhaps the most well-known of alternative theories
- PT models choice as a two-phase process:
  - Editing
  - Evaluation
- Two main characteristics:
  - The use of an editing phase
  - Outcomes as difference respect to a reference point

Editing phase

Aim: to get a simpler representation of the prospects for a easier evaluation.

Operations:

- Coding
- Combination
- Segregation
- Cancellation
- Simplification
- Detection of dominance

- **Coding**

outcomes are computed with respect to some reference point as gains or losses
Example: a prospect has 3 outcomes, 110, 150, 180. Using a reference point of 150 outcomes are -40, 0, 30.

- **Combination**

Probabilities associated with identical outcome are combined
Example: prospect $(100, 0.30; 150, 030; 100, 0.30)$ can be written as $(100, 0.60; 150, 030\ )$

- **Segregation**

A riskless component can be segregated from the risky component

Example:

$$(100, 0.10; 150, 0.20; 180, 0.70)$$

can be written as a sure gain of 100 plus a risky prospect of $(50, 0.20; 80, 0.70)$

- **Cancellation**

when comparing two prospects the identical components can be ignored

Example:

in the comparison between

$\boldsymbol{a} = (100, 0.30; 150, 0.50; 180, 0.20)$ and

$\boldsymbol{b} = (100, 0.30; 140, 0.60; 190, 0.10)$

The common component 100, 0.30 can be ignored

The comparison is between

$\boldsymbol{a} = (150, 0.50; 180, 0.20)$ and $\boldsymbol{b} = (140, 0.60; 190, 0.10)$

- **Simplification**

Prospects may be simplified by rounding outcomes and probabilities.
Sometime this operation is the first in the editing phase
Example: $(99, 0.51)$ can be simplified as $(100, 0.50)$

- **Detection of dominance**

Some prospect may dominate others
Example
$\boldsymbol{a} = (200, 0.30; 99, 0.51)$
$\boldsymbol{b} = (200, 0.40; 101, 0.49)$
b dominates ***a***

$$\boldsymbol{a} = (200, 0.29; 99, 0.71)$$
$$\boldsymbol{b} = (200, 0.41; 101, 0.59)$$

Simplification

$$\boldsymbol{a} = (200, 0.30; 100, 0.70)$$
$$\boldsymbol{b} = (200, 0.40; 100, 0.60)$$

Segregation

$$\boldsymbol{a} = 100 + (100, 0.30)$$
$$\boldsymbol{b} = 100 + (100, 0.40)$$

Dominance

$$(100, 0.40) \succ (100, 0.30)$$

Evaluation phase

After the editing phase, the decision maker is assumed to choose the prospect with the highest value

The value of a prospect $\boldsymbol{q}$, $V(\boldsymbol{q})$ depends on:

1) $v(\cdot)$ assigns to each outcome x_i a number, *the subjective value*

2) $\pi(\cdot)$ associates to each probability p_i a decision weight $\pi(p_i)$.

Important concepts:

Reference points

Loss aversion

Decreasing marginal sensitivity

Decision weighting

Framing

Some notation

Consider prospects of the form:
$\boldsymbol{r} = (x, p; y, q)$ where $p + q \leq 1$
with at most two nonzero outcomes

Prospect $\boldsymbol{r}$ is:
- strictly positive if $x, y > 0$ and $p + q = 1$
- strictly negative if $x, y < 0$ and $p + q = 1$
- regular otherwise

$(10, 0.2; 30, 0.8)$ is strictly positive
$(-10, 0.2; -30, 0.8)$ is strictly negative
$(10, 0.2; 30, 0.7)$ is regular
$(10, 0.2; -30, 0.8)$ is regular

Evaluation regular prospects

If $\boldsymbol{r} = (x, p; y, q)$ is regular

$$V(\boldsymbol{r}) = \pi(p) \cdot v(x) + \pi(q) \cdot v(y)$$

where $\pi(0) = 0, \pi(1) = 1, v(0) = 0$

Example:

$\boldsymbol{r} = (10, 0.2; 15, 0.6)$
$V(\boldsymbol{r}) = \pi(0.2) \cdot v(10) + \pi(0.6) \cdot v(15)$

$\boldsymbol{r} = (-10, 0.4; 15, 0.6)$
$V(\boldsymbol{r}) = \pi(0.4) \cdot v(-10) + \pi(0.6) \cdot v(15)$

Evaluation strictly positive/negative prospects

In the editing phase these prospects are segregated into two parts: the riskless component and the risky component.

Consider $\boldsymbol{r} = (x, p; y, q)$
if $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, then:

$$V(\boldsymbol{r}) = v(y) + \pi(p) \cdot \big(v(x) - v(y)\big)$$

Example: $\boldsymbol{r} = (10, 0.2; 30, 0.8)$

$$V(\boldsymbol{r}) = v(10) + \pi(0.8) \cdot \big(v(30) - v(10)\big)$$

In the following we discuss the following important concepts of the evaluation phase:

1. Reference points
2. Loss aversion
3. Shape of the utility function
4. Decision weighting

Reference points

Outcomes are defined with respect to a reference point, that becomes the zero in the value scale

Relevant reference point could be:

Current wealth

Expected wealth

Example

$$\boldsymbol{r} = (300, 0.6; 700, 0.4)$$

reference point is the expected wealth: 460

$$\boldsymbol{r} = (-160, 0.6; 240, 0.4)$$

The psychological concept of reference point is related to the following characteristics of the human body:

1. in the body, some system has an optimal set point. The body works to maintain this point and, after any deviation, try to restore it. Examples are the body temperature and the quantity of glucose in our blood (Homeostasis).
2. Our body sets the different systems at an optimal level, conditionally to the environment and the current activity. For example, the optimal heart rate and blood pressure (Allostasis)

Our body uses reference points in many situations:

- Temperature
- Noise
- Light
- ……..

Put for some time and simultaneously one hand in cold water and the other in hot water

After put both hands in the same container with tepid water

Different feelings

Experimental evidences: **Happiness treadmill**

US average income increased 40% in real term since 1972 but people report that they are not happier than previously

Similar evidences in other countries

Same results using different indicators (suicide rate)

Winners of lotteries report satisfaction levels not higher than that of the general population

This effect works in both directions, people that suffered a loss tend to quickly recover happiness

Sometime the current wealth is not a good reference point

Expectations are important in the definition of reference points. An individual will be disappointed when he expects a prize of 10 but then receives a prize of only 5. In this case the reference point is the expected wealth level, not the current one

Reference points are affected by the **status of the others**. An individual could be happy when receives a prize of 5, but he could be disappointed when discovers that his friends received a prize of 10 reference point is not the current wealth if a person has not yet adapted to the new wealth level.

Example:

A person experienced a loss of 2000

After he faces a choice between 1000 for sure and a prospect (2000, 0.50).

If he is not still adapted to the loss of 2000 his reference point is the previous wealth level. Then the choice is between:

-1000 for sure and (-2000, 0.50)

If he is adapted:

1000 for sure and (2000, 0.50)

Loss aversion

Losses loom larger that gains. The aggravation that one experiences in losing a sum of money appears to be greater than the pleasure associated with gaining the same amount.

Kahneman and Tversky 1979

For most of the people prospects of the type $(x, 0.50;\ -x, 0.50)$, where $x > 0$, are unattractive

Then

$$v(x) < -v(-x) \text{ where } x > 0$$

A justification is that in nature a gain can improve the chance of survival and reproduction but a loos can be fatal.

Suppose that your wealth is 1 million of pounds. To get an additional 1 million improves your condition, for example you can buy a bigger house.
But if you suffer a loss of 1 million you are not able to buy any house.

Empirical evidences

Asymmetric price elasticities of demand for consumer goods.
Demands are more elastic in response to price rises than in response to price reduction

Ratio of loss and gain disutilities → coefficient of loss aversion
But there is not agreement about the specifications of this ratio.

Disposition effect

It happens in the stock markets

Investors tend

i. To hold stocks that have lost value
ii. To sell stocks that have risen in price

The explanation involves loss aversion and reference points (purchase price as reference point).

Same effect is observed in the house market

People are unwilling to sell at a price lower that the price they paid

It is an anomaly for the EUT,

the decision to buy/sell has to depend on the expected price, not on the past.

End of the day effect

Observed in racetrack betting
At the end of the day bettors tend to bet away from favourites.

Explanation: reference point and loss aversion

At the last race, most of the bettors are suffering losses
Bettors use zero daily profits as reference point.
Betting away from favourites produce a prospect with high outcome (but low probability)

Example

An individual is suffering a loss of $2x$

He has to choose between the following two prospects:

$$\boldsymbol{r} = (x, 2p) \quad \boldsymbol{q} = (2x, p)$$

According to EUT if he is risk neutral $\boldsymbol{r} \sim \boldsymbol{q}$, if he is risk averse $\boldsymbol{r} \succ \boldsymbol{q}$

Using loss aversion and a zero reference point:

$\boldsymbol{r} = \left(-x, 2p;\ -2x, (1-2p)\right)$ $\boldsymbol{q} = (-2x, 1-p)$

$V(\boldsymbol{r}) = v(-x) + (1-2p) \cdot (v(-2x) - v(-x))$

$V(\boldsymbol{q}) = (1-p) \cdot v(-2x)$

$\boldsymbol{q} \succ \boldsymbol{r}$ if $V(\boldsymbol{q}) > V(\boldsymbol{r})$

$(1-p) \cdot v(-2x) > v(-x) + (1-2p) \cdot (v(-2x) - v(-x))$

$$v(-2x) > 2\, v(-x)$$

That is true if $v(\cdot)$ is convex for negative arguments.

Shape of the utility function

1) Standard economic model: the utility function is increasing and concave. This means that $u' > 0$ and $u'' < 0$.

2) Friedman – Savage utility function. It is concave for low and high values, it is convex for middle values. This modification still fails to explain various empirical observations.

3) Markovitz utility function. It is S-shaped in both regions of gain and loss. It is convex for small gains and concave for the large ones. It is concave for small losses and convex for the large ones.

   It entails reference points (all outcomes are coded as losses or gains) and loss aversion.

4) Prospect theory utility function. It entails decreasing marginal sensitivity. As the Markovitz utility function it entails reference points (all outcomes are coded as losses or gains) and loss aversion.

It is an increasing function, concave above the reference point and convex below:

$v''(x) < 0 \; for \; x > 0$ and $v''(x) > 0 \; for \; x < 0$

It implies risk aversion in the region of gain and risk seeking in the region of losses

Reflection effect

A justification is given by the observation that many sensory dimension share the property of decreasing marginal sensitivity.

“It is easier to discriminate between a change of 3° and a change of 6° in the room temperature than to discriminate between a change of 13° and a change of 16°”

Problem: a big loss can be fatal and individual could be risk averse when they face prospects that imply large losses

Empirical evidences

Experiment with 95 subjects
Two choice tasks

1) Choice between (3000) and (4000, 0.8)
2) Choice between (-3000) and (-4000, 0.8)

In the first task 80% of subjects choose (3000) (risk averse)

In the second task 92% of subjects choose (-4000, 0.8) (risk seeking)

In according EUT if a subject choose (3000) is risk averse (lower expected value but zero variance).

Then this individual has to choose (-3000) in the second task, because higher expected value and zero variance.

Attitudes toward insurance

By EUT, buying an insurance implies risk aversion (utility function is concave)

But many people prefer insurance policies with limited coverage over policies with maximal coverage. By EUT this implies risk seeking.

A contradiction

Probabilistic insurance

Individual pays a x% of the full insurance premium

If an accident occurs, it cover with probability x%

Decision weighting

Decision weights are not probabilities because they do not obey the probability axioms

Two aspect of decision weighting:

1) Estimation of probabilities: it happens when probabilities are unknown

2) Weighting of probabilities: probabilities are known but don’t reflect preferences according EUT

Estimation of probabilities

People are often very bad probability estimators

Very often they overestimate small probabilities as to win a lottery or being hit by a meteorite.

Another situation is when people are estimating conditional probabilities:

- Tossing a coin. After several consecutive heads many people believe that tail is more probable.
- Medical test for a disease

- Medical test for a disease

consider a disease that affects 1 individual out 1000 and a test that fail 5% of the times

The probability to have this disease given that test is positive is given by Bayes rule:

$$\Pr(Dis|Pos) =$$

$$= \frac{\Pr(Pos|Dis)\Pr(Dis)}{\Pr(Pos|Dis)\Pr(Dis) + \Pr(Pos|noDis)\Pr(noDis)} =$$

$$= \frac{0.95 \cdot 0.001}{0.95 \cdot 0.001 + 0.05 \cdot 0.999} = 0.019$$

Even the majority of Harvard Medical School doctors failed to get the right answer.

Weighting of probabilities

Decision weights are stated as function of objective probabilities

$$\pi(p) = f(p)$$

The decision weights measure the impact of the event on the desirability of the prospect

Suppose a prospect (100, 0.50), i.e. tossing a coin, if tail the prize is 100 otherwise is 0.

It is empirically observed that $\pi(0.50) < 0.50$ meaning that there is risk aversion.

Properties of decision weights:

- $\pi(0) = 0$
- $\pi(1) = 1$
- $\pi(p)$ is increasing in p
- Subadditivity
- Subcertainty
- Subproportionality

Subadditivity

Kahneman and Tversky (1979) show that73% of people prefer the prospect $(6000, 0.001)$ to $(3000, 0.002)$.

It contradicts the risk aversion for gains (diminishing marginal sensitivity), but subadditivity can explain it.

$$\pi(0.001)v(6000) > \pi(0.002)v(3000)$$

By diminishing marginal sensitivity we know that

$$v(3000) > 0.5v(6000)$$

Then we have that

$$\pi(0.001)v(6000) > \pi(0.002)v(3000)$$
$$> \pi(0.002)0.5 \cdot v(6000)$$
$$\pi(0.001) > 0.5 \cdot \pi(0.002)$$
$$\pi(0.5 \cdot 0.002) > 0.5 \cdot \pi(0.002)$$

In general terms the subadditivity principle is:

$$\pi(r \cdot p) > r \cdot \pi(p) \ \forall p \in (0, 1)$$

Similar result for losses

70% of people prefer the prospect $(-3000, 0.002)$ to $(-6000, 0.001)$

Moreover they didn't find such evidence for large probabilities, for example when $p = 0.90$

- <u>Subcertainty</u>

People tend to overweight sure outcomes with respect to probabilistic outcomes.

Consider the two prospects

$\boldsymbol{r} = (2400)$ and $\boldsymbol{q} = (2500, 0.33; 2400, 0.66)$

83% of the respondents prefer $\boldsymbol{r}$

Consider the two prospects

$\boldsymbol{r}' = (2400, 0.34)$ and $\boldsymbol{q}' = (2500, 0.33)$

83% of the respondents prefer $\boldsymbol{q}'$

A violation of the independence axiom of EUT

$\boldsymbol{r} = (2400)$ and $\boldsymbol{q} = (2500, 0.33; 2400, 0.66)$

$\boldsymbol{r}' = (2400, 0.34)$ and $\boldsymbol{q}' = (2500, 0.33)$

$\boldsymbol{r} \succ \boldsymbol{q}$ if $v(2400) > \pi(0.33)v(2500) + \pi(0.66)v(2400)$

$\boldsymbol{q}' \succ \boldsymbol{r}'$ if $\pi(0.33)v(2500) > \pi(0.34)v(2400)$

Replacing the second inequality in the first

$$v(2400) > \pi(0.34)v(2400) + \pi(0.66)v(2400)$$

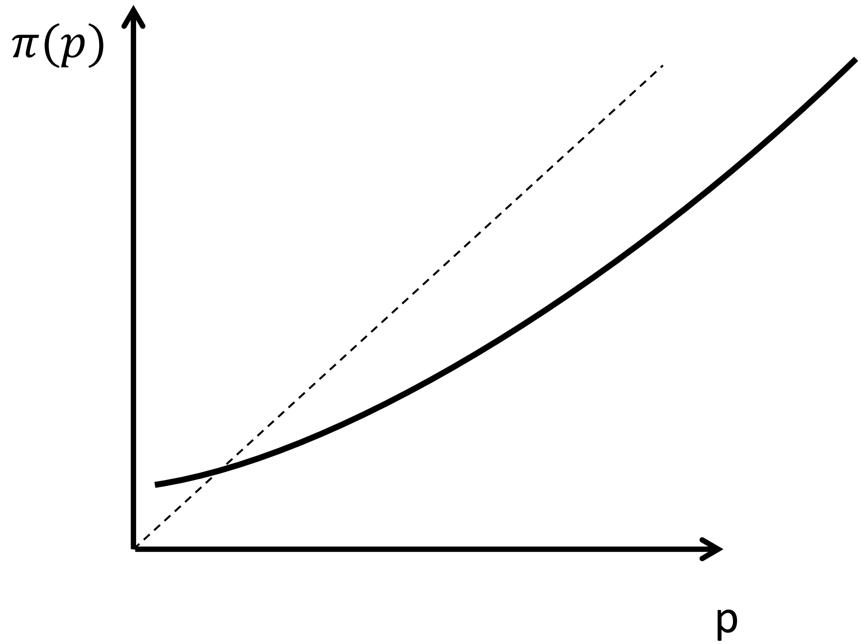
Dividing by $v(2400)$ we get:

$$1 > \pi(0.34) + \pi(0.66)$$

In general terms the subcertainty principle is:

$$\pi(p) + \pi(1-p) < 1 \; \forall p \in (0,1)$$

Main implication: preferences are less responsive to change in probabilities



- $\pi(p)$ is less steep that 45° line

Note there are two discontinuities, one near $\mathrm{p} = 0$ and another near $p = 1$

- <u>Subproportionality</u>

Another violation of independence axiom

Consider the two prospects

$\boldsymbol{r} = (3000)$ and $\boldsymbol{q} = (4000, 0.8)$

80% of the respondents prefer $\boldsymbol{r}$

Consider the two prospects obtained dividing by 4 the probabilities

$\boldsymbol{r}' = (3000, 0.25)$ and $\boldsymbol{q}' = (4000, 0.20)$

65% of the respondents prefer $\boldsymbol{q}'$

By independence axiom, the EUT predicts that if $\boldsymbol{r} \succ \boldsymbol{q}$ then $(\boldsymbol{r}, a) \succ (\boldsymbol{q}, a)\ \forall a \in (0,1)$

In the example $a = 0.25$

$\boldsymbol{r} = (3000)$ and $\boldsymbol{q} = (4000, 0.8)$

$\boldsymbol{r}' = (3000, 0.25)$ and $\boldsymbol{q}' = (4000, 0.20)$

$\boldsymbol{r} \succ \boldsymbol{q}$ if $v(3000) > \pi(0.80)v(4000)$

$\boldsymbol{q}' \succ \boldsymbol{r}'$ if $\pi(0.20)v(4000) > \pi(0.25)v(3000)$

$\boldsymbol{r} \succ \boldsymbol{q}$ if $\frac{v(3000)}{v(4000)} > \frac{\pi(0.80)}{\pi(1)}$ and $\boldsymbol{q}' \succ \boldsymbol{r}'$ if $\frac{\pi(0.20)}{\pi(0.25)} > \frac{v(3000)}{v(4000)}$

$$\frac{\pi(0.20)}{\pi(0.25)} > \frac{\pi(0.80)}{\pi(1)}$$

In general terms the subproportionality principle is:

$$\frac{\pi(p \cdot q \cdot r)}{\pi(p \cdot r)} \geq \frac{\pi(p \cdot q)}{\pi(p)} \quad 0 \leq p, q, r \leq 1$$

$$\frac{\pi(p \cdot q \cdot r)}{\pi(p \cdot r)} \geq \frac{\pi(p \cdot q)}{\pi(p)} \quad 0 \leq p, q, r \leq 1$$

For a fixed ratio of probabilities, the ratio of the corresponding decision weights is higher when the probabilities are low than when the probabilities are high.

It means that people judge probabilities more similar when are small

In the example 0.25 is judged more similar to 0.20 than 1 respect to 0.80

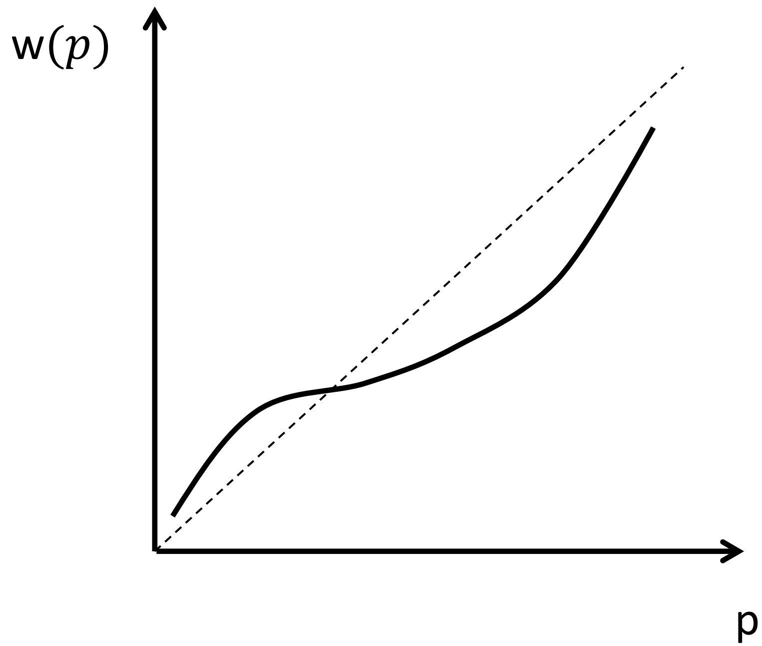
An evolution of this prospect theory was the **Cumulative Prospect Theory** (Kahneman and Tversky (1992))

Essential difference: the principle of decreasing marginal sensitivity is applied to decision weights

… the impact of a change in probabilty decreases with its distance from the boundaries.

Example: an increase of 0.1 the probability to win a prize has more impact when the probability of winning changes from 0 to 0.1 or from 0.9 to 1 than when it changes from 0.4 to 0.5

Then the weighting function is concave near 0 and convex near 1



The general shape is described by

$$w(p) = \frac{p^{\gamma}}{[p^{\gamma} + (1-p)^{\gamma}]^{1/\gamma}}$$

Parameter γ determines the curvature of the function

Prospect theory:

Risk aversion for gains, risk seeking for losses

Cumulative Prospect theory:

Risk aversion for gains, risk seeking for losses of high probabilities

Risk seeking for gains, risk aversion for losses of low probabilities

Empirical evidence

Prospect	Description	EV	Median CE	Attitude to risk
(100, 0.95)	Gain, high probability	95	78	Averse
(-100, 0.95)	Loss, high probability	-95	-84	Seeking
(100, 0.50)	Gain, medium probability	50	36	Averse
(-100, 0.50)	Loss, medium probability	-50	-42	Seeking
(100, 0.05)	Gain, low probability	5	14	Seeking
(-100, 0.05)	Loss, low probability	-5	-8	Averse

CE > EV implies risk seeking

CE < EV implies risk aversion

Ratio between the CE and the no zero outcome

$$\frac{CE}{\text{no zero outcome}}$$

Can be interpreted as the decision weight

In our example the no zero outcome is 100 so the first lottery of the table $\frac{CE}{100} = 0.78$

For gains

$$\frac{CE}{\text{no zero outcome}} < p \text{ implies risk aversion}$$

$$\frac{CE}{\text{no zero outcome}} = p \text{ implies risk neutrality}$$

$$\frac{CE}{\text{no zero outcome}} > p \text{ implies risk seeking}$$

For losses the situation is reversed