Matrix Appendix, a.a. 2010-11

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Financial Econometrics course
Trieste, February 14th 2011

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Outline of the talk

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Matrices, vectors and scalars-1

A *matrix* is a collection or array of numbers

$$\begin{pmatrix} 1 & -2 & 4 \\ 1.2 & 3 & 5 \end{pmatrix}$$

2 Rows by 3 Columns (2x3) : *size* of the matrix
-This is a *rectangular* matrix

A *square* matrix: number of Rows = number of Columns = *dimension*

$$\underset{(2x2)}{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$$

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Matrices, vectors and scalars-2

$$\underset{(2x3)}{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$$

Element (i,j) of matrix A: a_{ij}

i=1,2 (row index); j=1,..,3 (column index).

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Matrices, vectors and scalars–3

Column vector

$$\underset{(2x1)}{a} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$$

or more simply

$$\underset{(2x1)}{a} = \begin{pmatrix} a_{1} \\ a_{2} \end{pmatrix}$$

Row vector $\underset{(1x3)}{a} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \end{pmatrix}$

or more simply

$$\underset{(1x3)}{a} = \begin{pmatrix} a_{1} & a_{2} & a_{3} \end{pmatrix}$$

A Scalar: $\underset{(1x1)}{c} \in R$

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Outline of the talk

Zero, Symmetric, Diagonal Matrices

A zero matrix

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A symmetric (square) matrix: $a_{ij} = a_{ji}$

$$\begin{pmatrix} 1 & 3 & -1 \\ 3 & 0 & 5 \\ -1 & 5 & -2 \end{pmatrix}$$

A diagonal (square) matrix: $a_{ij} = 0$ for $i \neq j$

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Scalar and Identity Matrix

A scalar matrix: a diagonal matrix with $a_{ii} = c \ \forall i$

e.g. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

The identity matrix: a scalar matrix with $a_{ii} = 1 \ \forall i$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is the matrix equivalent of the number one!

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Addition and Subtraction of Matrices-1

In order to perform operations with matrices, matrices must be *conformable*!
Specifically, for Addition and Subtraction matrices must have the same size or dimension. Then the operation is performed element by element.

Therefore, given matrices A and B, both of size (RxC), $A + B = D$ means that the sum of A and B is equal to the matrix D of size (RxC), where the element

$d_{ij} = a_{ij} + b_{ij} \; \forall ij$

$A - B = D$
means that the difference between A and B is equal to the matrix D of size (RxC), where the element

$d_{ij} = a_{ij} - b_{ij} \; \forall ij$

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Addition and Subtraction of Matrices-2

If $A = \begin{pmatrix} 0.3 & 0.6 \\ -0.1 & 0.7 \end{pmatrix}$

and $B = \begin{pmatrix} 0.2 & -0.1 \\ 0 & 0.3 \end{pmatrix}$ then:

$$A + B = \begin{pmatrix} 0.5 & 0.5 \\ -0.1 & 1.0 \end{pmatrix}$$

and $A - B = \begin{pmatrix} 0.1 & 0.7 \\ -0.1 & 0.4 \end{pmatrix}$

Addition and Subtraction of Matrices-2

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Multiplication of a matrix by a scalar-1

Given a scalar c and a Matrix A, multiplying A by c means

$cA = D$
where

$d_{ij} = ca_{ij} \; \forall ij$

$$2\begin{pmatrix} 0.5 & 0.5 \\ -0.1 & 1.0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -0.2 & 2 \end{pmatrix}$$

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Properties-1

$$A + B = B + A$$

$$cA = Ac$$

$$c(A + B) = cA + cB$$

Multiplying two matrices together-1

Matrices A and B must be conformable:

$$\underset{(mxn)}{A}\underset{(nxp)}{B} = \underset{(mxp)}{D}$$

i.e. the number of columns of the first matrix must be equal to the number of rows of the second matrix

-The resulting matrix has size given by the number of rows of the first matrix and number of columns of the second matrix

Multiplying two matrices together-2

Notice that in general

$AB \neq BA$

so the order of the factors is important!!

Multiplying two matrices together-3

The product of matrices is also called "row by column" product.
Why?
Assume for the moment that the two matrices A and B are two vectors s.t.

$$\underset{(1xn)}{a}\underset{(nx1)}{b} = \underset{(1x1)}{d}$$

so that d is a scalar.
The product ab is obtained by computing the "row by column" product as follows:

$$\underset{(1xn)}{\begin{pmatrix} a_{11} & a_{12} & . & . & a_{1n} \end{pmatrix}} \underset{(nx1)}{\begin{pmatrix} b_{11} \\ b_{21} \\ . \\ . \\ b_{n1} \end{pmatrix}} = \underset{(1x1)}{\left(\sum_{k=1}^{n} a_{1k} b_{k1} \right)} = \underset{(1x1)}{(d_{11})} = d$$

Multiplying two matrices together-4

For the general case

$$\underset{(mxn)}{A}\underset{(nxp)}{B} = \underset{(mxp)}{D}$$

Each element d_{ij} of D is obtained by considering the $i-th$ row of A and the $j-th$ column of B and computing the "row by column" product as follows:

$$d_{ij} = \underset{(1xn)}{\begin{pmatrix} a_{i1} & a_{i2} & . & . & a_{in} \end{pmatrix}} \underset{(nx1)}{\begin{pmatrix} b_{1j} \\ b_{2j} \\ . \\ . \\ b_{nj} \end{pmatrix}} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Multiplying two matrices together-5

For example

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} (2x1 + 3x5) & (2x2 + 3x3) \\ (1x1 + 4x5) & (1x2 + 4x3) \\ (3x1 + 1x5) & (3x2 + 1x3) \end{pmatrix}$$

Transpose of a matrix-1

Given

$$\underset{(nxm)}{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & . & . & a_{1m} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2m} \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & a_{nm} \end{pmatrix}$$

the transpose of matrix A, written A' or A^T:

$$\underset{(mxn)}{A'} = \begin{pmatrix} a_{11} & a_{21} & a_{31} & . & . & a_{n1} \\ a_{12} & a_{22} & a_{32} & . & . & a_{n2} \\ . & . & . & . & . & . \\ a_{1m} & a_{2m} & a_{3m} & . & . & a_{nm} \end{pmatrix}$$

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Transpose of a matrix-2

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \\ 3 & 1 \end{pmatrix}$$

$$\underset{(2x3)}{A'} = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 4 & 1 \end{pmatrix}$$

Properties of the transpose

$(A')' = A$

$(A + B)' = A' + B'$

$(AB)' = B'A'$ Notice that the order is reversed!

Linear combination of vectors-1

Given m column vectors of dimension $(nx1)$, $a_1, a_2, ..., a_m$, and scalars $\alpha_j \in R$, for $j = 1, ..., m$,

the **linear combination** of the m column vectors with weights α_j is the following:

$$\alpha_1 a_1 + \alpha_2 a_2 + ... + \alpha_m a_m = \underset{(nx1)}{c}$$

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Linearly independent vectors-1

The m vectors are **linearly independent** if and only if the only way to obtain that

$$\alpha_1 a_1 + \alpha_2 a_2 + ... + \alpha_m a_m = \underset{(nx1)}{c} = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}$$

is by setting all the weights equal to zero ($\alpha_j = 0$, for $j = 1, ..., m$).

Linearly independent vectors-2

Notice that:

-if one of the vectors a_j is a **zero vector** then the m vectors, $a_1, a_2, ..., a_m$, are not linearly independent (they are **linearly dependent**);

-if two vectors are equal or proportional to each other then the two vectors are linearly dependent (and so are all the m vectors).

Linearly dependent vectors-1

If the m vectors, $a_1, a_2, ..., a_m$, are **linearly dependent** then

at least one of them can be written as a linear combination of the remaining ones.

In fact, assume that

$$\alpha_1 a_1 + \alpha_2 a_2 + ... + \alpha_m a_m = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}$$

with e.g. $\alpha_1 \neq 0$ then

$$a_1 = -(\alpha_2/\alpha_1)a_2 - ... - (\alpha_m/\alpha_1)a_m$$

Linear combination and independency-3

It worth noticing that we might have considered row vectors instead of column ones. In fact, the concepts of linear combination and linear dependency/independency of a set of vectors will apply to row vectors exactly in the same way.

Outline of the talk

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Rank of a Matrix-1

Given a rectangular (nxm) matrix A, we could ask ourselves what is the maximum number of the row vectors of the matrix that are linearly independent? This number is known as the row rank of the matrix, and it is at most equal to n.

The maximum number of column vectors of the matrix A that are linearly independent is called the column rank of the matrix.

There is an important theorem that states that the row rank and the column rank of a matrix are equal.

So, it's defined *rank* of a matrix the maximum number of column (row) vectors of the matrix that are linearly independent.

From the definition it follows that

$rank(A) <= min(n, m)$

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Rank of a Matrix-2

If the rank is equal to $min(n, m)$ then the matrix is said to be of *full rank*, otherwise it is said to be *singular* (or of not full rank).

A *nxn* square matrix is of full rank if and only if all its column vectors and all its row vectors are linearly independent.

Some properties of the rank:

- $rank(A) = rank(A')$
- $rank(AB) <= min(rank(A), rank(B))$
- $rank(A'A) = rank(AA') = rank(A)$

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Rank of a Matrix-3

- By convention the rank of a zero matrix is equal to zero.
- A vector with at least one element different from zero has rank equal to one.
- $rank \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = 2$
- $rank \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix} = 1$

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Outline of the talk

Inverse of a matrix-1

The inverse of a matrix A, denoted A^{-1}, where defined, is that matrix which, when pre-multiplied or post-multiplied by A, will result in the identity matrix I, i.e.

$$AA^{-1} = A^{-1}A = I$$

The inverse of a matrix exists if and only if the matrix is square and it is non-singular (i.e. it is of full rank). If the two conditions are satisfied, the matrix is said to be invertible.

The inverse of a 2x2 non-singular matrix A whose elements are

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

will be given by $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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Inverse of a matrix-2

The calculation of the inverse of a *nxn* matrix for $n > 2$ is more complex and beyond the scope of the text.

Properties of the inverse of a matrix include:

- $I^{-1} = I$
- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1}A^{-1}$ if both A and B are invertible.

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The calculation of the inverse of a *nxn* matrix for $n > 2$ is more complex and beyond the scope of the text.

Properties of the inverse of a matrix include:

- $I^{-1} = I$
- $(A^{-1})^{-1} = A$
- $(A')^{-1} = (A^{-1})'$
- $(AB)^{-1} = B^{-1}A^{-1}$ if both A and B are invertible.

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