

All the exercises

1. Five different objects are given to three persons. How many possible distributions are there if each person receives at least one object?
2. Four different objects are given to two persons. How many possible distributions are there if each person receives at least one object?
3. How many numbers are there of five figures (from 10000 to 99999) with an odd figure appearing twice and another different odd figure appearing three times (example 13133)?
4. How many even numbers are there of 5 figures (from 10000 to 99999) with 4 odd figures appearing in increasing order (e.g., 35790)?
5. How many numbers of five figures (from 00000 to 99999) are there with three even figures and with two figures equal to 3?
6. How many numbers can be considered of four figures (from 1000 to 9999) if we require that exactly three different figures appear in nondecreasing order (example: 2377)?
7. How many numbers of five figures (from 00000 to 99999) are there with exactly three even figures which are required to be all equal each other?
8. How many numbers are there of 5 figures (from 10000 to 99999) beginning with 1 and with exactly 3 even figures appearing in increasing order (e.g., 14368)?
9. How many ways are there of distributing 7 different objects to 3 persons in a such a way that one person receives three objects and each of the remaining two persons receives two objects?
10. How many possibilities are there of rolling a sum of 7 with four dice?
11. How many numbers of four figures are there that begin with an odd figure and that have three even figures appearing in increasing order (example: 7048)?
12. How many numbers of three figures (from 000 to 999) are there with exactly two odd figures if in addition we require that the sum of the odd figures is equal to 6?
13. How many ways are there of colouring 5 regions by using three colours (white, red and blue) so that white is used three times and red once is used once as well as blue?
14. How many numbers are there of 5 figures (from 10000 to 99999) with 4 odd figures appearing in increasing order and beginning with an even number (e.g., 21379)?
15. How many numbers are there of 5 figures (from 10000 to 99999) with 4 odd figures appearing in increasing order and beginning with an even number (e.g., 21379)?
16. How many possibilities are there of rolling a sum of 7 with three dice?
17. How many numbers are there of four figures (from 10000 to 99999) with exactly two odd figures in increasing order and two even figures equal each other (example 1272)?
18. How many numbers can be considered of three different figures (from 100 to 999) in increasing order (example: 237)?
19. How many numbers are there of four figures (from 1000 to 9999) with exactly two even figures that are equal each other (example 3252)?

20. How many numbers of four figures all different from zero are there that have exactly two figures equal to 1 and with the remaining figures different from each other (example: 6131)?
21. How many ways are there of coloring 5 boxes with three colors (black, white and red) in such a way that at least two colors are used?
22. How many numbers are there of 4 figures all different from zero with sum of the figures equal to 6 (e.g., 1131)?
23. How many ways may be considered of coloring four boxes with two colors (red and blue) if we require that both colors are used?
24. How many numbers are there consisting of 4 positive digits whose sum is equal to 8?
25. How many ways can a group of 10 children be divided into three teams of four, three and three children respectively?
26. How many numbers can be considered of four figures (from 1000 to 9999) if we require that exactly three different figures appear in the number and that they are in nondecreasing order (example: 2447)?
27. We consider 7 different objects and 4 persons. How many distributions of the objects are there if we require that 3 persons receive 2 objects each and 1 person receives 1 object?

□

28. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \operatorname{arctang} 2x & \text{if } x < 0 \\ x^2 + 1 & \text{if } x \geq 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

29. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 - 1 & \text{if } x < 0 \\ \operatorname{arctang} x & \text{if } x \geq 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

30. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -\operatorname{arctang} x & \text{if } x \leq 0 \\ \operatorname{arctang} x - \frac{\pi}{2} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

31. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ \frac{2}{\pi} \operatorname{arctang} x & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

32. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 & \text{if } x \leq 0 \\ \frac{1}{x^3} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

33. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \log(x+1) & \text{se } x \geq 0 \\ 2x & \text{se } x < 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

34. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 & \text{if } x \leq 0 \\ e^{-x} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

35. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 + 1 & \text{if } x \leq 0 \\ \left(\frac{1}{2}\right)^x + 1 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

36. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 - 1 & \text{if } x \leq 0 \\ x^3 - 1 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

37. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \log(x+1) & \text{if } -1 < x \leq 0 \\ e^{-x} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

38. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -(x+2)^2 & \text{if } x \leq -2 \\ \frac{1}{x+2} & \text{if } x > -2 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

39. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ -2x - 1 & \text{if } x \geq 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

40. Consider the real-valued function defined as follows:

$$y = f(x) = \sqrt{\arctan x}.$$

Determine the inverse function $x = f^{-1}(y)$.

41. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \log x & \text{se } x \geq 1 \\ \frac{1}{x-1} & \text{se } x < 1 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

42. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^3 - 1 & \text{if } x < 0 \\ \sqrt{x} + 1 & \text{if } x \geq 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

43. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \log_{\frac{1}{2}}(x+1) & \text{if } -1 < x \leq 0 \\ -(x-1)^2 & \text{if } 0 < x < 1 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

44. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x + 2 & \text{if } x \leq 0 \\ \frac{1}{x} + 2 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

45. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ e^{-x} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

46. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \sqrt{-x} & \text{if } x \leq 0 \\ \log x & \text{if } 0 < x < 1 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

47. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ -x^3 - 2 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

48. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} e^{2x} & \text{if } x \leq 0 \\ \frac{2}{x} + 1 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

49. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

50. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -x^3 + 1 & \text{if } x \leq 0 \\ -x^2 - 1 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

51. Consider the real-valued function defined as follows:

$$y = f(x) = \text{arctang}(\log x).$$

Determine the inverse function $x = f^{-1}(y)$.

52. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \sqrt{-x} & \text{if } x \leq 0 \\ -x^2 & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

53. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} e^x - 1 & \text{if } x \leq 0 \\ \frac{1}{x^2} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function $x = f^{-1}(y)$.

□

54. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 + x^2}}{\sin^2 x} .$$

55. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{x - \text{tang } x}{\sin^3 x} .$$

56. Determine the following limit:

$$\lim_{x \rightarrow \infty} \frac{1 - \cos \frac{1}{x}}{\log(1 + \frac{2}{x^2})} .$$

57. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\log(x^2 + 1)}{1 - \cos x} .$$

58. Determine the following limit:

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{\sin(x - 1)} .$$

59. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\sin(x^2)}{1 - \cos x} .$$

60. Determine the following limit:

$$\lim_{x \rightarrow +\infty} \frac{2x}{x + e^{-x}} .$$

61. Determine the following limit:

$$\lim_{x \rightarrow +\infty} \frac{e^{\frac{1}{x}} - 1}{\sin \frac{2}{x}}.$$

62. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\log(\cos x)}{x^2}.$$

63. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{e^x - e}{x^2 - 1}.$$

64. Determine the following limit:

$$\lim_{x \rightarrow 0^+} \frac{e^{2\sqrt{x}} - 1}{\sqrt{x}}.$$

65. Determine the following limit:

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{x^2 - 1}}{\sin(x - 1)}.$$

66. Determine the following limit:

$$\lim_{x \rightarrow +\infty} \frac{x}{2x + \arctan x}.$$

67. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{1 - \sqrt{x^2 + 1}}.$$

68. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \sqrt{x^2 + 1}}.$$

69. Determine the following limit:

$$\lim_{x \rightarrow 0^+} \frac{e^{2\sqrt{x}+1}}{\sin \sqrt{x}}.$$

70. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{e^x + 1}}{\log(1 + x)}.$$

71. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{e^{x^2+1} - e}{x^2}.$$

72. Determine the following limit:

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)^2}{1 - \cos(x - 1)}.$$

73. Determine the following limit:

$$\lim_{x \rightarrow 0^+} \frac{e^{x^2-x} - 1}{x}.$$

74. Determine the following limit:

$$\lim_{x \rightarrow 1} \frac{\sin(\sqrt{x} - 1)}{x - 1}.$$

75. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sqrt{1 - \cos x}}.$$

76. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\cos \sqrt{x} - 1}.$$

77. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{x \sin x}{\cos x - 1}.$$

78. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{x - \log(1 + x)}{\sin^2 x}.$$

□

79. Study the following function and draw its graph:

$$f(x) = x e^{-\sqrt{x}}.$$

80. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log(x^2 - 1).$$

81. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \sqrt{\frac{x}{x^2 - 1}}.$$

82. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{x - 1}{x + 1}\right).$$

83. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{x}{x - 1}\right).$$

84. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \sqrt{\frac{1 - x}{x^2}}.$$

85. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = x \log\left(\frac{1}{x}\right).$$

86. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{1}{x}} + \frac{1}{x}.$$

87. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{1}{x^2 + 1}\right).$$

88. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{x+1}{\sqrt{x}}}.$$

89. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{x^2 + 4}{x}\right).$$

90. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \sqrt{x}e^{-\sqrt{x}}$$

91. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{x}{x-1}}.$$

92. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{x^2 - 1}{x}\right).$$

93. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{x}{x^2+1}}.$$

94. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{-\frac{x}{x^2-1}}.$$

95. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log(x - \sqrt{x}).$$

96. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log\left(\frac{x^2 + 2}{x^2 + 1}\right).$$

97. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{1}{x-1}}$$

98. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = x^2 \log x.$$

99. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log \left(\frac{x}{\sqrt{x} - 2} \right).$$

100. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log(\log x).$$

101. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \log \left(\frac{x - 1}{\sqrt{x} + 1} \right).$$

102. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \frac{\sqrt{x}}{1 + x}.$$

□

103. Solve the inequality $2x \geq \sqrt{|x^2 - x|}$.

104. Solve the inequality $\sqrt{|x^2 - 1|} \geq x$.

105. Solve the inequality $\frac{2x^2 - x - 1}{4x + 1} \geq 0$.

106. Solve the inequality $x + 1 \geq \sqrt{6x - x^2}$.

□

107. Determine the domain of the function $f(x) = \sqrt{\frac{1}{\arccos(1-2x)}}$.

108. Determine the domain of the function $f(x) = \arcsin\left(\frac{x}{x+3}\right)$.

109. Determine the domain of the function $f(x) = \sqrt{\arcsin(2x - 1)}$.

□

110. Determine the following indefinite integral:

$$\int \sqrt{x} \log x.$$

111. Determine the following indefinite integral:

$$\int \frac{\sin(\tan x)}{\cos^2 x} dx.$$

112. Determine the following indefinite integral:

$$\int \cos x \sin(\sin x) dx.$$

113. Determine the following indefinite integral:

$$\int \cos x \sin^2 x dx.$$

114. Determine the following indefinite integral:

$$\int \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx.$$

115. Determine the following indefinite integral:

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx.$$

116. Determine the following indefinite integral:

$$\int \frac{e^x}{1 + e^{2x}} dx.$$

117. Determine the following indefinite integral:

$$\int \frac{\sqrt{\tan x}}{\cos^2 x} dx.$$

118. Determine the following indefinite integral:

$$\int \frac{\sin(\arctan x)}{1 + x^2} dx.$$

119. Determine the following indefinite integral:

$$\int \frac{1}{\sqrt{x} \cos^2 \sqrt{x}} dx.$$

120. Determine the following indefinite integral:

$$\int \frac{\cos \sqrt[3]{x}}{\sqrt[3]{x^2}} dx.$$

121. Determine the following indefinite integral:

$$\int \frac{\cos \sqrt[3]{x}}{\sqrt[3]{x^2}} dx.$$

122. Determine the following indefinite integral:

$$\int \frac{\sin x}{2 + \cos x} dx.$$

123. Determine the following indefinite integral:

$$\int x^2 \log x dx.$$

124. Determine the following indefinite integral:

$$\int \sin x \sqrt[3]{\cos x} dx.$$

125. Determine the following indefinite integral:

$$\int \frac{x}{1+x^4} dx.$$

126. Determine the following indefinite integral:

$$\int x^3 \log x dx.$$

127. Determine the following indefinite integral:

$$\int \frac{1}{(1+x^2)\sqrt{\arctan x}} dx.$$

128. Determine the following indefinite integral:

$$\int \frac{\sin x}{1+\cos^2 x} dx.$$

129. Determine the following indefinite integral:

$$\int \frac{\tan \sqrt{x}}{\sqrt{x}} dx.$$

130. Determine the following indefinite integral:

$$\int \frac{\tan \sqrt{x}}{\sqrt{x}} dx.$$

131. Determine the following indefinite integral:

$$\int x \sin x dx.$$

132. Determine the following indefinite integral:

$$\int \frac{\sin e^{-x}}{e^x} dx.$$

133. Determine the following indefinite integral:

$$\int x \sin 2x dx.$$

134. Determine the following indefinite integral:

$$\int \frac{\cos x}{2+\sin x} dx.$$

□

135. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = x^2 e^{xy}.$$

136. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = x \sin(x^2 y).$$

137. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \frac{x^2}{x + y}.$$

138. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \log(xy^2).$$

139. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sin(xy^2).$$

140. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \cos(xy^2).$$

141. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \log(1 + e^{x^2 y}).$$

142. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \frac{x}{x - y}.$$

143. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = x\sqrt{xy}.$$

144. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \arcsin(xy^2).$$

145. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = e^{-xy^2}.$$

146. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sin(x(y - 1)).$$

147. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = x^2 \sin(xy).$$

148. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sin\left(\frac{x}{y^2}\right).$$

149. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sqrt{x - y^2}.$$

150. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \log(x^2 + e^y).$$

151. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = xe^{x-y}.$$

152. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sqrt[3]{x^2y}.$$

153. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = xe^{xy^2}.$$

154. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = e^{-x\sqrt{y}}.$$

155. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \log(x\sqrt{y}).$$

156. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = xe^{-xy}.$$

157. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = e^{\sqrt{x^2y}}.$$

158. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = e^{x\sin y}.$$

159. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = x\log(xy).$$

□

160. Determine the intervals on which the following function is increasing:

$$f(x) = 3\arctan x + \frac{2}{x}.$$