- A PERSON MUST DO AN ACTIVITY EXACTLY ONCE
- ACTIVITIES INVOLVE EITHER IMMEDIATE COSTS OR
- IMMEDIATE REWARDS
- PEOPLE ARE **SOPHISTICATED** OR **NAÏVE** ABOUT FUTURE SELF CONTROL

## RESULTS

NAIVE PEOPLE → PROCRASTINATE IMMEDIATE COSTS ACTIVITIES
NAIVE PEOPLE → PREOPERATE IMMEDIATE REWARDS

SOPHISTICATED → MITIGATES PROCRASTINATION
SOPHISTICATED → EXACERBATES PREOPERATION

$\beta\delta$ preferences $\quad 0<\beta<1 \quad 0<\delta<1 \quad \delta = \frac{1}{1+\rho}$

$\beta = 1 \longrightarrow$ exponential discounting

$\beta < 1 \longrightarrow$ present biased preferences

SOPHISTICATED : she knows exactly what her future behavior will be

NAIVE : she believes her future preferences will be identical to her current p.

$T$ periods to do an activity exactly once

REWARDS $\longrightarrow v \equiv (v_1, v_2, \dots\dots v_T)$

COSTS $\longrightarrow c \equiv (c_1, c_2, \dots\dots c_T)$

Let $\tau$ be the period in which she is doing the activity

$U_t(\tau)$ intertemporal utility from the perspective of period $t$ of completing the activity in period $\tau \geq t$

Other assumptions

2) delayed costs or rewards are experienced in $T+1$

2) If a person waits until period $T$, she must do the activity in $T$

# UTILITY

Immediate costs

$$U_t(\tau) = \begin{cases} \beta\delta^{(T+1-t)} \cdot v_\tau - c_\tau & \tau = t \\ \beta\delta^{(T+1-t)} \cdot v_\tau - \beta\delta^{\tau-t} \cdot c_\tau & \tau > t \end{cases}$$

Immediate rewards

if $\delta = 1$ $\begin{cases} \beta v_\tau - c_\tau & \tau = t \\ \beta v_\tau - \beta c_\tau & \tau > t \end{cases}$

$$U_t(\tau) = \begin{cases} v_\tau - \beta\delta^{(T+1-t)} \cdot c_\tau & \tau = t \\ \beta\delta^{\tau-t} \cdot v_\tau - \beta\delta^{(T+1-t)} \cdot c_\tau & \tau > t \end{cases}$$

if $\delta = 1$ $\begin{cases} v_\tau - \beta c_\tau & \tau = t \\ \beta v_\tau - \beta c_\tau & \tau > t \end{cases}$

3 types of individuals

1) Time consistent preferences $\quad \beta = 1 \quad$ $\underline{TC}$

2) } Time inconsistent pref $\quad \beta < 1 \quad$ naive $\quad$ $\underline{N}$

3) } $\quad$ sophisticated $\quad$ $\underline{S}$

## DEFINITION OF STRATEGY

$$S = (s_1, s_2, s_3 \dots s_T)$$

where $s_t \in \{Y, N\}$

$s_t = Y$ means doing in t

$s_t = N$ waiting

$$s_T = Y$$

example $\quad s_i = (Y\,Y\,Y\,Y)$

# PERCEPTION PERFECT STRATEGY (PPS)

A person chooses the optimal action given her current preferences and her perception of her future behavior

- PPS for a TC is a strategy $S = (s_1, s_2, \ldots s_T)$ that satisfies $\forall\ t < T \quad s_t = Y$ if and only if

$$U_t(t) \geq U_t(\tau) \quad \forall\ \tau > t$$

The rule to compute the PPS for a N is equal to PPS's rule of TC

PPS for a $S$ is a strategy $S = (S_1, S_2 \dots S_T)$ that satisfies

$$\forall\, t < T \quad S_t = Y$$

if and only if

$$U_t(t) \geq U_t(\tau') \text{ where } \tau' = \min\{\tau \mid S_\tau = Y\}$$

To COMPUTE PPS for $S$

we use "BACKWARD INDUCTION"

we start from the last period and go back

Ex 1 immediate costs $T=4$ $\beta=\frac{1}{2}$ $\delta=1$

$v=(20, 20, 20, 20)$

$c=(3, 5, 8, 13)$

TC $U_1(1)=20-3=17$ $U_1(2)=15$ $U_1(3)=12$ $U_1(4)=7$ $S_1=Y$

$U_2(2)=15$ $U_2(3)=12$ $U_2(4)=7$ $S_2=Y$

$U_3(3)=12$ $U_3(4)=7$ $S_3=Y$

**TC** $s=(Y\ Y\ Y\ Y)$ $\tau=1$

N $U_1(1)=\frac{1}{2}\cdot 20-3=7$ $U_1(2)=\frac{1}{2}(20-5)=7.5$ $U_1(3)=\frac{1}{2}(20-8)=6$

$U_1(4)=\frac{1}{2}(20-13)=3.5$

$S_1=N$

**N** $s=(N\ N\ N\ Y)$ $\tau=4$

**S** $s=(N, Y, N, Y)$ $\tau=2$

## PROPOSITION 1

IMMEDIATE COSTS : $\tau_n \geq \tau_{TC}$

IMMEDIATE REWARDS : $\tau_n \leq \tau_{TC}$

## PROPOSITION 2

$$\tau_s \leq \tau_n$$

Note : for immediate costs sophistication helps mitigate the tendency to procrastinate

for immediate rewards, sophistication can exacerbate the tendency to preoperate

Because sophisticated are affected by SOPHISTICATION EFFECT in addition TO PRESENT BIAS EFFECT, the qualitative behavior of S relative to TC is complicated

It can be that S do not even exhibit the basic present bias intuition

<u>Ex. n° 3</u> immediate costs, $T=3$, $\beta=\frac{1}{2}$, $\delta=1$

$v=(12, 18, 18)$ $\quad c=(3, 8, 13)$

$\tau_S=1 \qquad \tau_{TC}=2 \qquad \tau_N=3$

## DEFINITION 5

A person obeys dominance if whenever there exists some period $\tau$ with $v_\tau > 0$ and $c_\tau = 0$ the person does not do it in any period $\tau'$ with $c_{\tau'} > 0$ and $v_{\tau'} = 0$

## DEFINITION 6

For any $V = (v_1, v_2, \dots v_T)$ and $C \equiv (c_1, c_2, \dots c_T)$ define

$$V^{-t} = (v_1, v_2, \dots v_{t-1}, v_{t+1} \dots v_T)$$

$$C^{-t} = (c_1, c_2, \dots c_{t-1}, c_{t+1} \dots c_T)$$

A person behavior is independent of irrelevant alternatives if whenever she chooses period $\tau' \neq t$ when facing $V$ and $C$ she also chooses $\tau'$ when facing $v^{-t}$ $c^{-t}$

- A TIME CONSISTENT PERSON WILL NEVER VIOLATE DOMINANCE NOR INDEPENDENCE OF IRRELEVANT ALTERNATIVES

- THIS RESULT DOES NOT HOLD FOR PEOPLE WITH PRESENT BIASED PREFERENCES (N & S)

EXAMPLE: S violate dominance

immediate rewards, $T=3$, $\beta=\frac{1}{2}$, $\delta=1$, $v=(0,5,1)$, $c=(1,8,0)$

Doing in period 1 is dominated by doing it in period 3

But $\tau_S = 1$

Example: N violate dominance

immediate costs, $T=3$, $\beta=\frac{1}{2}$, $\delta=1$, $v=(1,8,0)$, $c=(0,5,1)$

Doing it in period 3 is dominated by doing it in period 1, but $\tau_N = 3$

USING THE SAME EXAMPLE WE CAN SEE VIOLATION OF INDEPENDENT ALTERNATIVES

<u>Ex FOR S.</u>

$\tau = 1$. If you delete $t = 2$ and $\tau = 3$ $\quad V = (0, -, 1) \quad C = (1, -, 0)$

<u>Ex FOR N</u>

$\tau = 3$ If you delete $t = 2$ and $\tau = 1$ $\quad V = (1, -, 0) \quad C = (0, -, 1)$

# MULTI - TASKING

the activity must be performed more than once, $n \geq 1$ times

She can do it at most once in any given period

$\tau^i(n)$ denotes the period in which a person is doing the activity for the $i^{th}$ time

$$\Theta(n) \equiv \{\tau^1(n), \tau^2(n), \dots \tau^n(n)\}$$ is the set of periods in which she does the activity.

## Results

- for each $n \in [1, 2, \dots T-1]$ $\quad \Theta_{TC}(n) \subset \Theta_{TC}(n+1)$

$$\Theta_N(n) \subset \Theta_N(n+1)$$

- immediate rewards : $\tau^i_N(n) \leq \tau^i_{TC}(n)$
- immediate costs : $\tau^i_N(n) \geq \tau^i_{TC}(n)$

EXAMPLE MULTI TASKING

immediate rewards $T = 3$ $\beta = \frac{1}{2}$ $\delta = 1$

$v = (6, 11, 21)$ $c = (0, 0, 0)$

if $n = 1$ $\tau_S = 1$, $\tau_N = 2$, $\tau_{TC} = 3$

if $n = 2$ $\Theta_S(2) = \{2, 3\}$ $\Theta_N(2) = \{1, 2\}$

$\Theta_{TC}(2) = \{2, 3\}$