3.3 STABILITY CONSIDERATIONS

The stability of an amplifier, or its resistance to oscillate, is a very important consideration in a design and can be determined from the S parameters, the matching networks, and the terminations. In a two-port network, oscillations are possible when either the input or output port presents a negative resistance. This occurs when $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$, which for a unilateral device occurs when $|S_{11}| > 1$ or $|S_{22}| > 1$. For example, a unilateral transistor is a transistor where $S_{12} = 0$ (or its effect so small that it can be set equal to zero). If $S_{12} = 0$, it follows from (3.2.5) and (3.2.6) that $|\Gamma_{IN}| = |S_{11}|$ and $|\Gamma_{OUT}| = |S_{22}|$. Hence, if $|S_{11}| > 1$ the transistor presents a negative resistance at the input, and if $|S_{22}| > 1$ the transistor presents a negative resistance at the output.

The two-port network shown in Fig. 3.3.1 is said to unconditionally stable at a given frequency if the real parts of Z_{IN} and Z_{OUT} are greater than zero for all passive load and source impedances. If the two-port is not unconditionally stable, it is potentially unstable. That is, some passive load and source terminations can produce input and output impedances having a negative real part.

In terms of reflection coefficients, the conditions for unconditional stability at a given frequency are

$$|\Gamma_s| < 1 \tag{3.3.1}$$

$$|\Gamma_L| < 1 \tag{3.3.2}$$

$$|\Gamma_{IN}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1 \tag{3.3.3}$$

and

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| < 1 \tag{3.3.4}$$

where, of course, all coefficients are normalized to the same characteristic impedance Z_o.

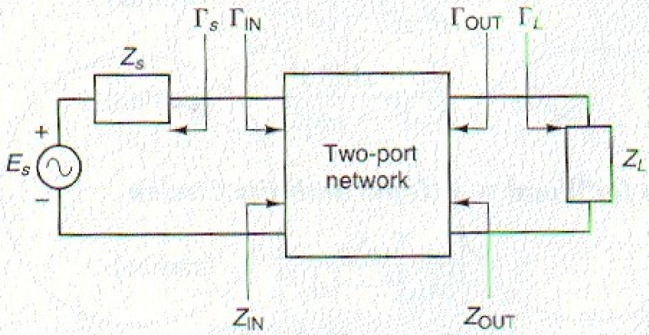


Figure 3.3.1 Stability of two-port networks.

Equations (3.3.1) and (3.3.2) state that the source and load are passive, while (3.3.3) and (3.3.4) state that the input and output impedances must also be passive (i.e., no negative resistance is associated with their real parts).

The solutions of (3.3.1) to (3.3.4) give the required conditions for the two-port network to be unconditionally stable. However, before we discuss the intricacies of the necessary and sufficient conditions for unconditional stability, a graphical analysis of (3.3.1) to (3.3.4) is presented. The graphical analysis is especially useful in the analysis of potentially unstable transistors.

When the two-port in Fig. 3.3.1 is potentially unstable, there may be values of Γ_s and Γ_L (i.e., source and load impedances) for which the real parts of Z_{IN} and Z_{OUT} are positive. These values of Γ_s and Γ_L (i.e., regions in the Smith chart) can be determined using the following graphical procedure.

First, the regions where values of Γ_L and Γ_s produce $|\Gamma_{IN}| = 1$ and $|\Gamma_{OUT}| = 1$ are determined, respectively. Setting the magnitude of (3.3.3) and (3.3.4) equal to 1 and solving for the values of Γ_L and Γ_s shows that the solutions for Γ_L and Γ_s lie on circles (called *stability circles*) whose equations are given by

$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \tag{3.3.5}$$

and

$$\left|\Gamma_s - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| \tag{3.3.6}$$

where

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

The derivations of (3.3.5) and (3.3.6) are given in Appendix A, Section A.2. For completeness, the reader can find in Appendix A, Section A.1, a review of circle equations and bilinear transformations.

The radii and centers of the circles where $|\Gamma_{IN}| = 1$ and $|\Gamma_{OUT}| = 1$ in the Γ_L plane and Γ_s plane, respectively, are obtained from (3.3.5) and (3.3.6), namely

Γ_L values for $|\Gamma_{IN}| = 1$ (Output Stability Circle):

$$r_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \quad \text{(radius)} \tag{3.3.7}$$

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad \text{(center)} \tag{3.3.8}$$

Γ_s values for $|\Gamma_{OUT}| = 1$ (Input Stability Circle):

$$r_s = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| \quad \text{(radius)} \tag{3.3.9}$$

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad \text{(center)} \tag{3.3.10}$$

With the S parameters of a two-port device at one frequency, the expressions (3.3.7) to (3.3.10) can be calculated and plotted on a Smith chart, and the set of values of Γ_L and Γ_s that produce $|\Gamma_{IN}| = 1$ and $|\Gamma_{OUT}| = 1$ can be easily observed. Figure 3.3.2 illustrates the graphical construction of the stability circles where $|\Gamma_{IN}| = 1$ and $|\Gamma_{OUT}| = 1$. On one side of the stability circle boundary, in the Γ_L plane, we will have $|\Gamma_{IN}| < 1$ and on the other side $|\Gamma_{IN}| > 1$. Similarly, in the Γ_s plane on one side of the stability circle boundary, we will have $|\Gamma_{OUT}| < 1$ and on the other side $|\Gamma_{OUT}| > 1$.

Next we need to determine which area in the Smith chart represents the stable region—in other words, the region where values of Γ_L (where $|\Gamma_L| < 1$) produce $|\Gamma_{IN}| < 1$ and where values of Γ_s (where $|\Gamma_s| < 1$) produce $|\Gamma_{OUT}| < 1$. To this end, we observe that if $Z_L = Z_o$, then $\Gamma_L = 0$ and from (3.2.5) $|\Gamma_{IN}| = |S_{11}|$. If the magnitude of S_{11} is less than 1, then $|\Gamma_{IN}| < 1$ when $\Gamma_L = 0$. That is, the center of the Smith chart in Fig. 3.3.2a represents a stable operating point, because for $\Gamma_L = 0$ it follows that $|\Gamma_{IN}| < 1$. On the other hand, if $|S_{11}| > 1$ when $Z_L = Z_o$, then $|\Gamma_{IN}| > 1$ when $\Gamma_L = 0$ and the center of the Smith chart represents an unstable operating point. Figure 3.3.3 illustrates the two cases discussed. The shaded area represents the values of Γ_L that produce a stable operation. Similarly, Fig. 3.3.4 illustrates stable and unstable regions for Γ_s.

For unconditional stability any passive load or source in the network must produce a stable condition. From a graphical point of view, for $|S_{11}| < 1$ and $|S_{22}| < 1$, we want the stability circles shown in Figs. 3.3.3a and 3.3.4a to fall completely outside (or to completely enclose) the Smith chart. The case in which the stability circles fall completely outside the Smith chart is illustrated in Fig. 3.3.5. Therefore, the conditions for unconditional stability for all passive sources and loads can be expressed in the form

$$\left||C_L| - r_L\right| > 1 \qquad \text{for } |S_{11}| < 1 \tag{3.3.11}$$

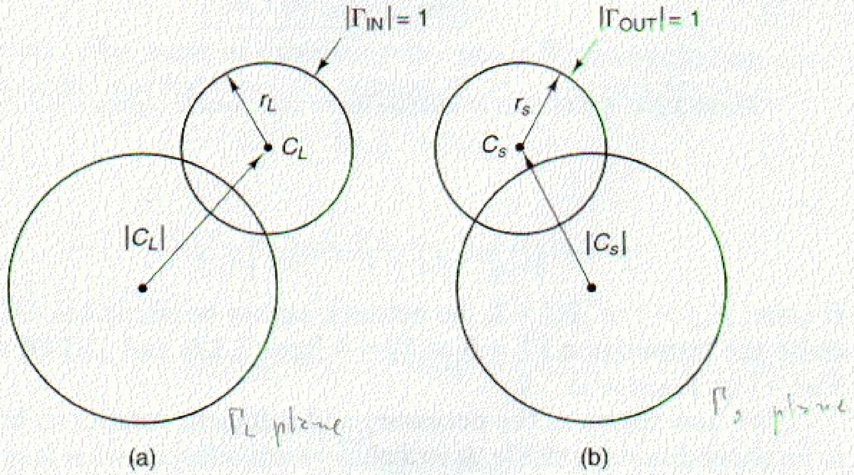


Figure 3.3.2 Stability circle construction in the Smith chart: (a) Γ_L plane; (b) Γ_s plane.

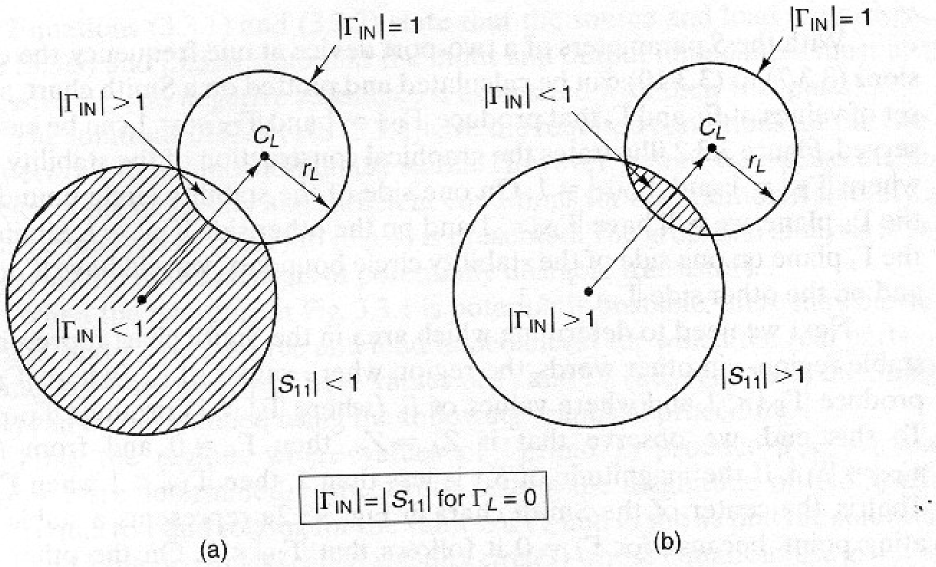


Figure 3.3.3 Smith chart illustrating stable and unstable regions in the Γ_L plane.

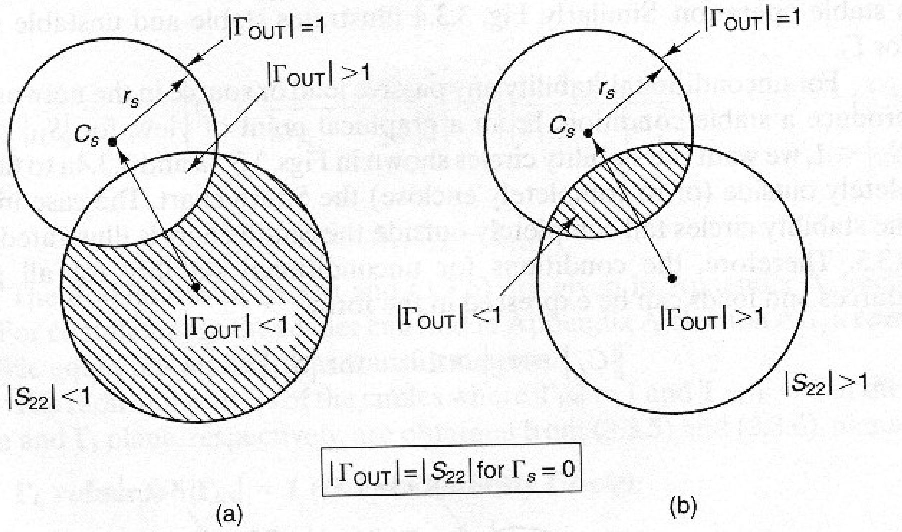


Figure 3.3.4 Smith chart illustrating stable and unstable regions in the Γ_s plane.

and

$$\left||C_s| - r_s\right| > 1 \qquad \text{for } |S_{22}| < 1 \tag{3.3.12}$$

If either $|S_{11}| > 1$ or $|S_{22}| > 1$, the network cannot be unconditionally stable because the termination $\Gamma_L = 0$ or $\Gamma_s = 0$ [see (3.3.3) and (3.3.4)] will produce $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$.

We now return to the necessary and sufficient conditions for a two-port to be unconditionally stable. A straightforward but somewhat lengthy manipu-

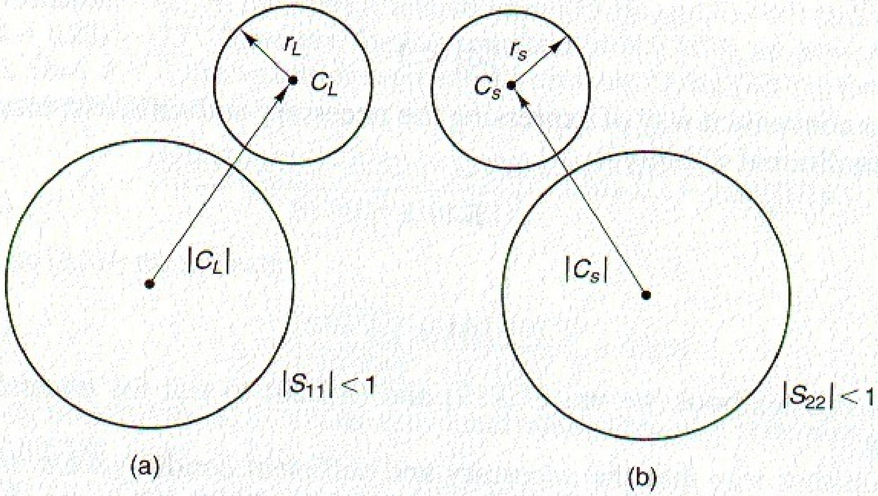


Figure 3.3.5 Conditions for unconditional stability: (a) Γ_L plane; (b) Γ_s plane.

lation of (3.3.1) to (3.3.4) results in the following necessary and sufficient conditions for unconditional stability (see Appendix B):

$$K > 1 \tag{3.3.13}$$

and

$$1 - |S_{11}|^2 > |S_{12}S_{21}| \tag{3.3.14}$$

$$1 - |S_{22}|^2 > |S_{12}S_{21}| \tag{3.3.15}$$

where

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \tag{3.3.16}$$

and

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \tag{3.3.17}$$

There are other ways of expressing the necessary and sufficient conditions for unconditional stability [3.1]. Adding (3.3.14) and (3.3.15) gives

$$2 - |S_{11}|^2 - |S_{22}|^2 > 2|S_{12}S_{21}| \tag{3.3.18}$$

Since

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| \le |S_{11}S_{22}| + |S_{12}S_{21}|$$

we use (3.3.18) to obtain

$$|\Delta| < |S_{11}S_{22}| + 1 - \frac{1}{2}|S_{11}|^2 - \frac{1}{2}|S_{22}|^2$$

$$|\Delta| < 1 - \frac{1}{2}(|S_{11}| - |S_{22}|)^2$$

or simply

$$|\Delta| < 1$$

Hence, a convenient way of expressing the necessary and sufficient conditions for unconditional stability is

$$K > 1 \tag{3.3.19}$$

and

$$|\Delta| < 1 \tag{3.3.20}$$

In this textbook we use (3.3.19) and (3.3.20) to test for unconditional stability.

Another way that the necessary and sufficient conditions for unconditional stability are found in the literature is (see Appendix C)

$$K > 1$$

and

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \tag{3.3.21}$$

From a theoretical point of view, a two-port network can have any value of K and $|\Delta|$. From a practical point of view, most microwave transistors produced by manufacturers are either unconditionally stable or potentially unstable with $K < 1$ and $|\Delta| < 1$. In fact, in potentially unstable transistors most practical values of K are such that $0 < K < 1$. These potentially unstable transistors have source and load stability circles that intersect the boundary of the Smith chart (e.g., see Figs. 3.3.3a and 3.3.4a).

Negative values of K in the range $-1 < K < 0$ result in most of the Smith chart being unstable. Some transistor configurations (e.g., some CB configurations) used in oscillator designs are potentially unstable with negative values of K.

Example 3.3.1

The S parameters of a BJT at $V_{CE} = 15$ V and $I_C = 15$ mA at $f = 500$ MHz, 1 GHz, 2 GHz, and 4 GHz are as follows:

f (GHz)	S_{11}	S_{12}	S_{21}	S_{22}
0.5	$0.761\angle{-151°}$	$0.025\angle{31°}$	$11.84\angle{102°}$	$0.429\angle{-35°}$
1	$0.770\angle{-166°}$	$0.029\angle{35°}$	$6.11\angle{89°}$	$0.365\angle{-34°}$
2	$0.760\angle{-174°}$	$0.040\angle{44°}$	$3.06\angle{74°}$	$0.364\angle{-43°}$
4	$0.756\angle{-179°}$	$0.064\angle{48°}$	$1.53\angle{53°}$	$0.423\angle{-66°}$

Determine the stability. If the transistor is potentially unstable at a given frequency, draw the input and output stability circles.

Solution. At $f = 500$ MHz it follows from (3.3.16) and (3.3.17) that $K = 0.482$ and $\Delta = 0.221\underline{|-123^\circ}$. Therefore, for this transistor at 500 MHz we have $K < 1$ and $|\Delta| < 1$. Since $K < 1$, the transistor is potentially unstable. Using (3.3.10), the center of the input stability circle is

$$C_s = \frac{(0.761\underline{|-151^\circ} - 0.221\underline{|-123^\circ}(0.429\underline{|35^\circ}))^*}{(0.761)^2 - (0.221)^2} = 1.36\underline{|157.6^\circ}$$

and from (3.3.9) the radius is

$$r_s = \left|\frac{0.025\underline{|31^\circ}(11.84\underline{|102^\circ})}{(0.761)^2 - (0.221)^2}\right| = 0.558$$

Similarly, from (3.3.8) and (3.3.7) the center and radius of the output stability circles are $C_L = 2.8\underline{|57.86^\circ}$ and $r_L = 2.18$.

At $f = 1$ GHz, we find that $K = 0.857$ and $\Delta = 0.173\underline{|-162.9^\circ}$. Therefore, since $K < 1$, the transistor is potentially unstable at 1 GHz. The center and radius of the input stability circle at 1GHz are $C_s = 1.28\underline{|169^\circ}$ and $r_s = 0.315$; and for the output stability circle $C_L = 2.62\underline{|51.3^\circ}$ and $r_L = 1.71$.

At $f = 2$ GHz, we find that $K = 1.31$ and $\Delta = 0.174\underline{|160^\circ}$. Since $K > 1$ and $|\Delta| < 1$, it follows from (3.3.19) and (3.3.20) that the transistor is unconditionally stable at 2 GHz.

At $f = 4$ GHz, we find that $K = 1.535$ and $\Delta = 0.226\underline{|121^\circ}$. Therefore, since $K > 1$ and $|\Delta| < 1$, the transistor is unconditionally stable at 4 GHz.

The stability circles are plotted in Fig. 3.3.6 at $f = 500$ MHz and $f = 1$ GHz.

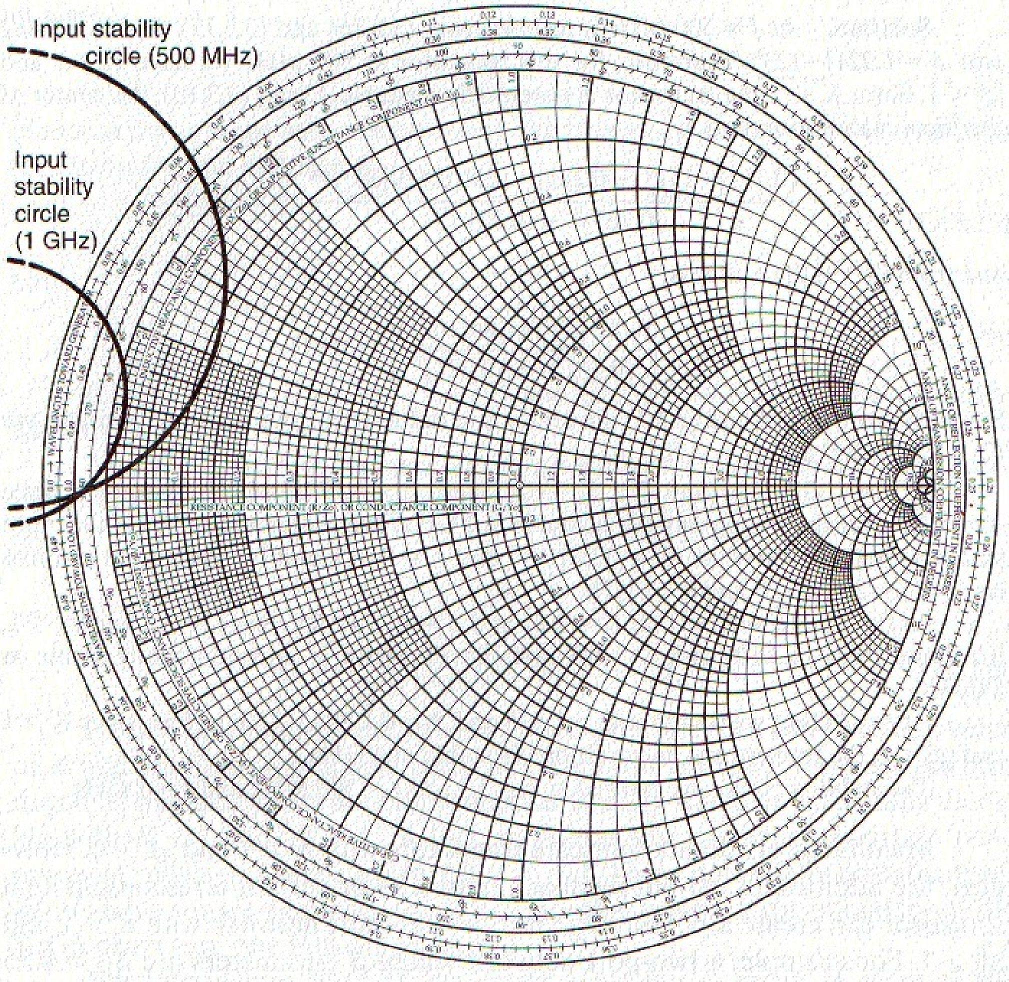
Manufacturers do not fabricate transistors with $K > 1$ and $|\Delta| > 1$. However, the addition of certain feedback networks or certain terminations to a transistor can create a potentially unstable two-port network with $K > 1$ and $|\Delta| > 1$. For example, a two-port network whose S parameters are $S_{11} = 0.75\underline{|-60^\circ}$, $S_{21} = 6\underline{|90^\circ}$, $S_{22} = 0.5\underline{|60^\circ}$, and $S_{12} = 0.3\underline{|70^\circ}$ has $K = 1.344$ and $|\Delta| = 2.156$. This two-port network has $K > 1$; however, it is potentially unstable because $|\Delta| > 1$. In fact, the input and output stability circles, from (3.3.7) to (3.3.10), are located at $C_s = 0.1\underline{|107.4^\circ}$, $r_s = 0.44$, $C_L = 0.26\underline{|-36.3^\circ}$, and $r_L = 0.41$, which can be drawn inside the Smith chart to show the unstable regions. The evaluation of B_1 in (3.3.21) gives $B_1 = -3.34$. Hence, when $|\Delta| > 1$ it follows that $B_1 < 0$. This shows that when the condition (3.3.20) for stability is not satisfied, neither is the condition (3.3.21).

A two-port network is said to be unilateral when $S_{12} = 0$. In a unilateral two-port network, $\Gamma_{IN} = S_{11}$ and $\Gamma_{OUT} = S_{22}$. Hence, we have unconditional stability if $|S_{11}| < 1$ and $|S_{22}| < 1$ for all passive source and load terminations. In fact, from (3.3.16) and (3.3.17), with $S_{12} = 0$ we have $K = \infty$ and $\Delta = S_{11}S_{22}$, and it follows from (3.3.19) that

$$1 - |S_{11}|^2 - |S_{22}|^2 + |S_{11}S_{22}|^2 > 0$$

or

$$(1 - |S_{11}|^2)(1 - |S_{22}|^2) > 0$$



(a)

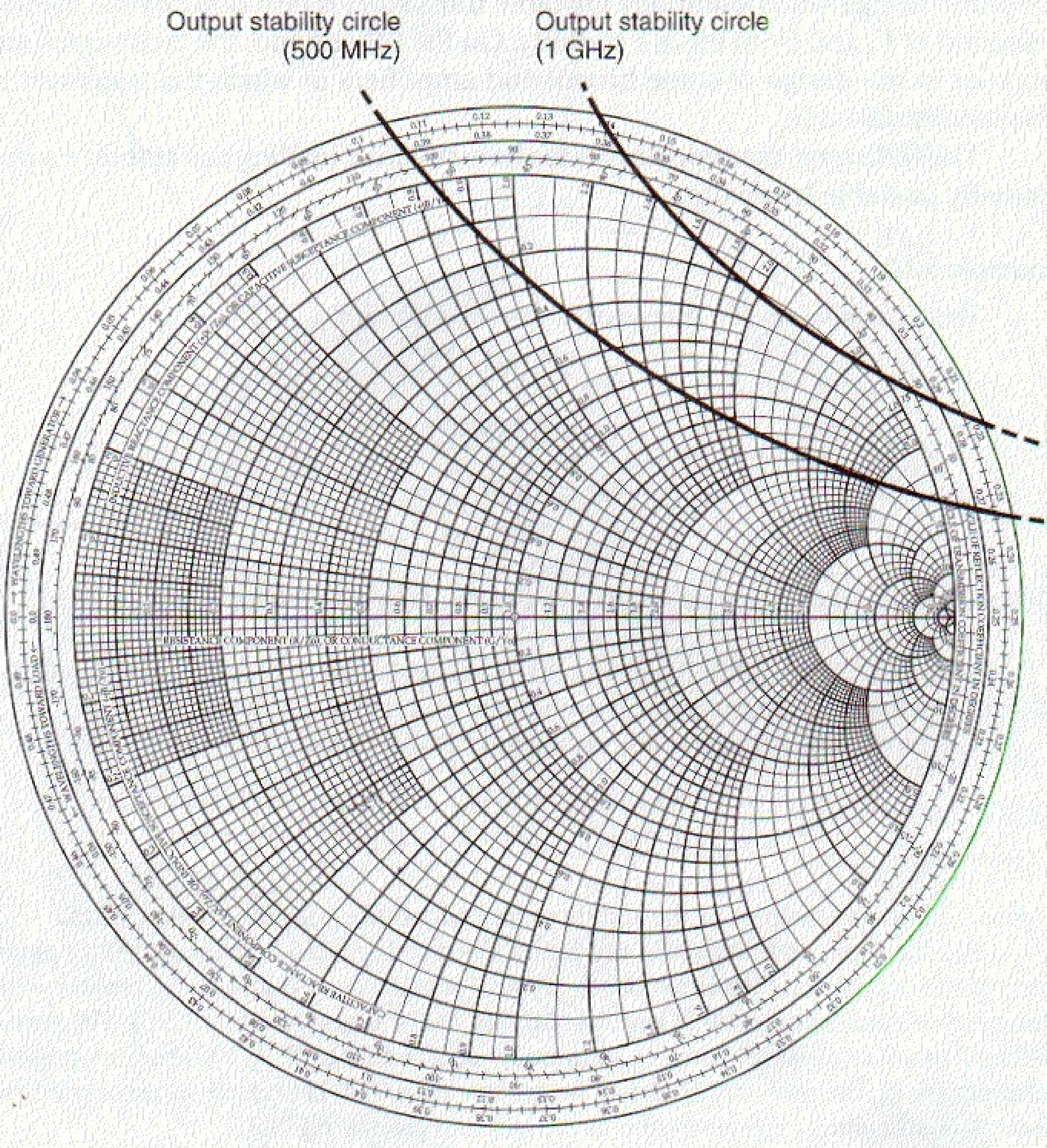
Figure 3.3.6 (a) Input stability circles for Example 3.3.1; (b) output stability circles.

The preceding inequality requires that $|S_{11}| < 1$ and $|S_{22}| < 1$ for unconditional stability of a unilateral two-port.

It is interesting to see what happens to the stability circles in the limit as $S_{12} \to 0$. The reader is referred to Problem 3.10 for this analysis.

In the potentially unstable situation illustrated in Figs. 3.3.3 and 3.3.4, the real part of the input and output impedances can be negative for some source and load reflection coefficients. In this case, selecting Γ_s and Γ_L in the stable region produces a stable operation.

Even when the selection of Γ_L and Γ_s produces $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$, the circuit can be made stable if the total input and output loop resistance in Fig. 3.3.1 is positive. In other words, the circuit is stable if



(b)

Figure 3.3.6 Continued

$$\mathrm{Re}\,(Z_s + Z_{\mathrm{IN}}) > 0$$

and

$$\mathrm{Re}\,(Z_L + Z_{\mathrm{OUT}}) > 0$$

A potentially unstable transistor can be made unconditionally stable by either resistively loading the transistor or by adding negative feedback. These techniques are not recommended in narrowband amplifiers because of the resulting degradation in power gain, noise figure, and VSWRs. Narrowband amplifier design with potentially unstable transistors is best done by the proper selection of Γ_s and Γ_L to ensure stability. On the other hand, the techniques are popular in the design of some broadband amplifiers in which the transistor is potentially unstable.

The following example illustrates how resistive loading can stabilize a potentially unstable transistor.

Example 3.3.2

The S parameters of a transistor at $f = 800$ MHz are

$$S_{11} = 0.65\angle{-95^\circ}$$
$$S_{12} = 0.035\angle{40^\circ}$$
$$S_{21} = 5\angle{115^\circ}$$
$$S_{22} = 0.8\angle{-35^\circ}$$

Determine the stability and show how resistive loading can stabilize the transistor.

Solution. From (3.3.16) and (3.3.17) we find that $K = 0.547$ and $\Delta = 0.504\angle{249.6^\circ}$. Since $K < 1$, the transistor is potentially unstable at $f = 800$ MHz.

The input and output stability circles are calculated using (3.3.7) to (3.3.10):

$$C_s = 1.79\angle{122^\circ} \qquad C_L = 1.3\angle{48^\circ}$$
$$r_s = 1.04 \qquad r_L = 0.45$$

Figure 3.3.7 shows the plot of the stability circles, together with the stable region.

For the input stability circle the Smith chart in Fig. 3.3.7 represents the Γ_s plane and for the output stability circle the Γ_L plane. It can be seen that a series resistor with the input of approximately 9 Ω assures stability at the input (see Fig. 3.3.8). The series addition of a 9-Ω resistor produces an impedance Z_s equal to $Z_s' + 9\ \Omega$. For any passive termination Z_s', the real part of Z_s will be greater than 9 Ω. Therefore, its associated reflection coefficient Γ_s will always be in the stable region in Fig. 3.3.7.

Also, a shunt resistor with the input of approximately 0.7/50 = 14 mS (or 71.5 Ω) produces stability at the input. Looking at the output stability circle, it follows that either a series resistor of approximately 29 Ω or a shunt resistor of approximately 500 Ω at the output produces stability at the output. The four choices of resistive loading are shown in Fig. 3.3.9. Usually, stabilizing one port of a transistor results in an unconditionally stable device.

All four choices of resistive loading affect the gain performance of the amplifier. In addition, from a practical point of view, resistive loading at the input (as shown in Figs. 3.3.9a and 3.3.9b) is not used because it produces a significant deterioration in the noise performance of the amplifier (see Chapter 4).

In some potentially unstable designs of broadband amplifiers, the shunt resistor loading at the output, as shown in Fig. 3.3.9d, produces a trade-off between gain and stability that is quite acceptable, resulting in a stable two-port with reasonable gain over a wide bandwidth (see Example 4.4.3).

For the stabilized shunt resistor configuration in Fig. 3.3.9d (i.e., with a 500-Ω shunt resistor), the resulting S parameters are

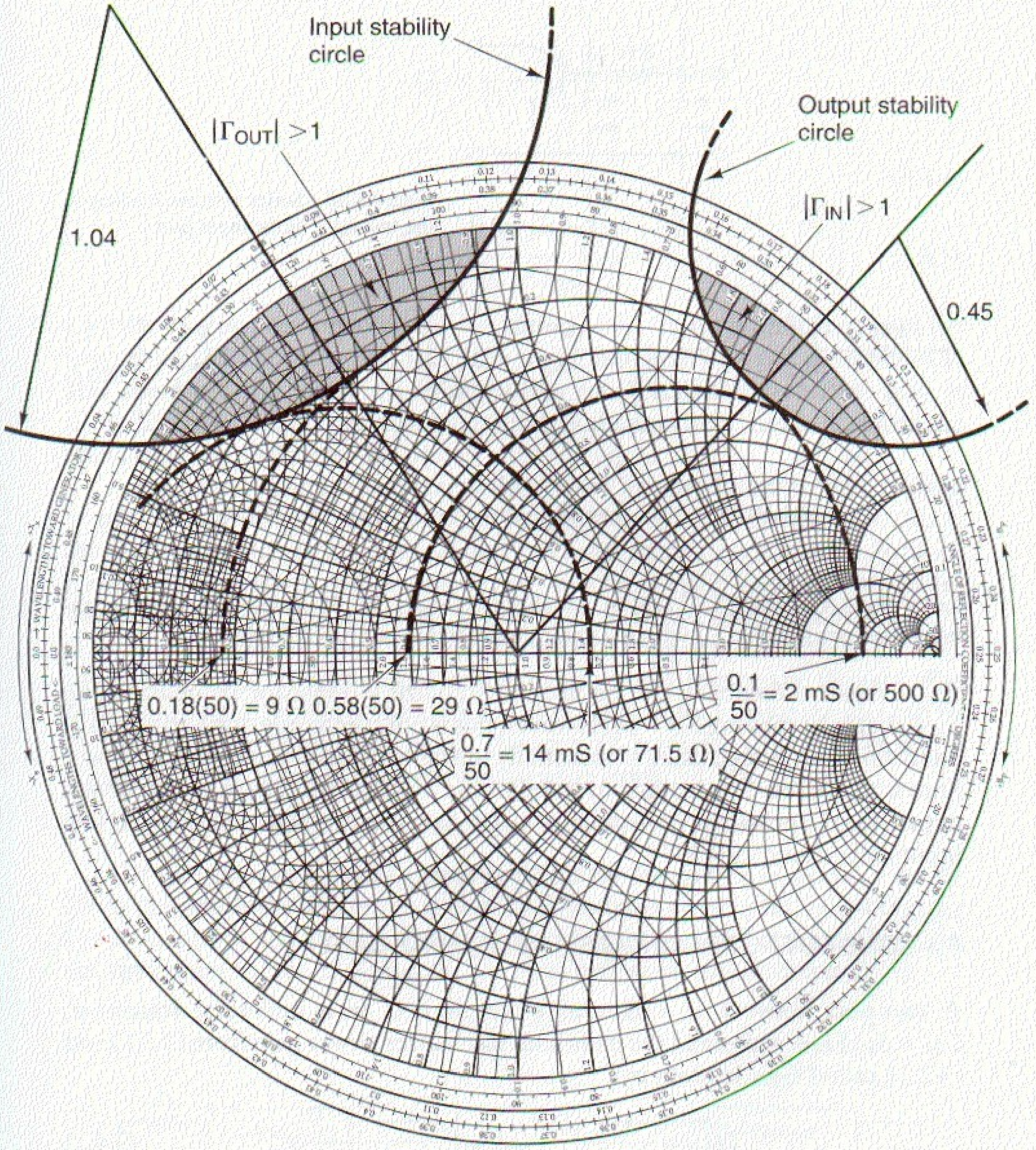


Figure 3.3.7 Input and output stability circles.

$$S_{11} = 0.65\underline{|-94^\circ}$$
$$S_{12} = 0.032\underline{|41.2^\circ}$$
$$S_{21} = 4.62\underline{|116.2^\circ}$$
$$S_{22} = 0.66\underline{|-36^\circ}$$

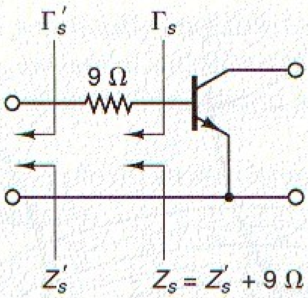


Figure 3.3.8 Series resistive loading of the transistor at the input port.

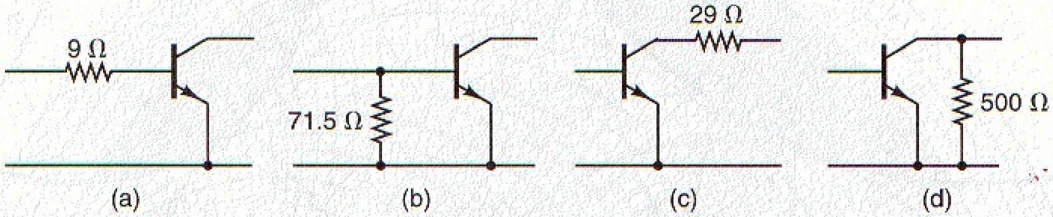


Figure 3.3.9 Four types of resistive loading to improve stability.

and from (3.3.16) and (3.3.17) $K = 1.04$ and $\Delta = 0.409\underline{|250.13°}$, which show that the stabilized network in Fig. 3.3.9d is unconditionally stable at $f = 800$ MHz.

Negative feedback can be used to stabilize a transistor by neutralizing S_{12}—that is, by making $S_{12} = 0$. However, this is not commonly done. In a broadband amplifier design using a potentially unstable transistor, a common procedure is to use resistive loading to stabilize the transistor and negative feedback to provide the proper ac performance—that is, to provide constant gain and low input and output VSWR.

3.4 CONSTANT-GAIN CIRCLES: UNILATERAL CASE

A two-port network is unilateral when $S_{12} = 0$. In a unilateral transistor, $\Gamma_{IN} = S_{11}, \Gamma_{OUT} = S_{22}$, and the unilateral transducer power gain from (3.2.1) and (3.2.2), called G_{TU}, is given by

$$G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{3.4.1}$$

The first term in (3.4.1) depends on the S_{11} parameter of the transistor and the source reflection coefficient. The second term, $|S_{21}|^2$, depends on the transistor scattering parameter S_{21}; and the third term depends on the S_{22} parameter of the transistor and the load reflection coefficient. We can think of (3.4.1) as being composed of three distinct and independent gain terms. Therefore, we can write (3.4.1) in the form

$$G_{TU} = G_s G_o G_L \tag{3.4.2}$$