Problem set 2

1) Suppose there are n firms in the Cournot oligopoly model. The inverse demand function is $P(Q) = 100 - Q$ where Q is the aggregate quantity on the market. All firms are equal and face the following cost function: $c(q_i) = 2q_i$. Firms choose their quantities simultaneously.
   a. Find the Nash equilibrium
   b. Find the strategy profile where the aggregate quantity is equal to the monopoly quantity and firms produce the same quantity.
   c. Show that the strategy profile at point b) is not an equilibrium (use best responses)
   d. Show that firms prefer the strategy profile at point b) respect to the Nash equilibrium (compare profits)
   e. Let $n = 2$ and suppose firms can choose to produce the Nash quantity or the quantity you find in point b. No other quantities are feasible. Represent this situasion as a normal form game using a payoff table.
   f. Let $n = 2$ and $c(q_1) = 2q_1$ $c(q_2) = 3q_2$ (firms have different cost functions). Find the Nash equilibrium.

2) Consider the Bertrand duopoly model with homogeneous product. The demand function of firm 1 is $q_1 = \begin{cases} 100 - p_1 \, if \, p_1 < p_2 \\ \frac{100 - p_1}{2} \;\; if \; p_1 = p_2 \\ 0 \;\; if \; p_1 > p_2 \end{cases}$; that of firm 2 is $q_2 = \begin{cases} 100 - p_2 \, if \, p_2 < p_1 \\ \frac{100 - p_2}{2} \;\; if \; p_2 = p_1 \\ 0 \;\; if \; p_2 > p_1 \end{cases}$.
   The two firms are equal and face the following cost function: $c(q_i) = c \cdot q_i$
   Show that the unique Nash equilibrium is $p_2 = p_1 = c$.

3) Consider the model of final offer arbitration. Find the Nash equilibrium when
   a. $F(x) = \frac{x^2}{10000}$ for $0 \leq x \leq 100$ and $F(x) = 1$ for $x > 100$
   b. $F(x) = 0.01 \cdot x$ with $0 \leq x \leq 100$
4) Consider the Problem of the Commons. Assume that $n = 3$and that $v(x) = 120 - x$. Compute the Nash equilibrium, the total number of goats in the Nash equilibrium and the number of goats that maximize the social welfare.

5) Represent by a table a traveler's dilemma game with two players. They can choose integer numbers between 1 and 4 and R=2. Find the Nash equilibrium

6) Represent a beauty contest game with two players. They can choose integer numbers between 1 and 4 :
   a. when p=0.5
   b. when p=1
   c. when p=2

   In all cases find the Nash equilibria