Problems:

1) Represent the following situation using the normal form representation. Find all Nash equilibria.

   Two individuals are working on a join project. They can devote it either high effort or low effort. If both devote high effort the outcome of the project is of high quality and they each one receives 100$. If one or both devote low effort, the outcome of the project is of low quality and each one will receive 50$. The opportunity cost to provide high effort is 30. The opportunity cost to provide low effort is 0.

   Solution:

   Players: individual 1 and individual 2

   Strategies: S_1={High effort, Low effort}

   Payoff:

		Individual 2	
		Low effort	High effort
Individual 1	Low effort	50, 50	50, 20
	High effort	20, 50	70, 70

   Define the best responses

		Individual 2	
		Low effort	High effort
Individual 1	Low effort	<u>50</u>, <u>50</u>	50, 20
	High effort	20, 50	<u>70</u>, <u>70</u>

   Nash Equilibria:

   1. {Low effort, Low effort}

   2. {High effort, High effort}

2) Consider the following game:

		Player 2	
		L	R
Player 1	T	1, 1	2, 0
	B	0, 2	2, 2

a) Find all Nash Equilibria and say if they are strict or not

b) Show that Iterated elimination of no strictly dominated strategies eliminates a Nash equilibrium

Solution:

a) Define the best responses

		Player 2	
		L	R
Player 1	T	$\underline{1}, \underline{1}$	$\underline{2}, 0$
	B	$0, \underline{2}$	$\underline{2}, \underline{2}$

Nash equilibria:

1. {T, L}
2. {B, R}

b) Consider Player 1. Strategy B is (weakly) dominated, deleting it we get:

		Player 2	
		L	R
Player 1	T	$\underline{1}, \underline{1}$	$\underline{2}, 0$

Then for player 2 strategy R is strictly dominated, therefore we can reduce the game

		Player 2
		L
Player 1	T	$\underline{1}, \underline{1}$

The prediction of the outcome of this game is {T, L} and the process has eliminated the equilibrium {B, R}.

Therefore the elimination of weakly dominated strategy cannot be used to predict the outcome of the game.

3) In the following normal form game, what strategies survive iterated elimination of strictly dominated strategies? What are the Nash equilibria?

	L	C	R
T	2,0	1, 1	4, 2
M	3,4	1, 2	2, 3
B	1, 3	0, 2	3, 0

Solutions:
B is dominated by T, the reduced game is:

	L	C	R
T	2,0	1, 1	4, 2
M	3,4	1, 2	2, 3

C is dominated by R, the reduced game is:

	L	R
T	2,0	4, 2
M	3,4	2, 3

There are no dominated strategies to eliminate, Then we find the best responses:

	L	R
T	2,0	4, 2
M	3,4	2, 3

The Nash equilibria are:
1. {T, R}
2. {M, L}

4) Two players (1 and 2) have to divide £10 between themselves using the following procedure:
Each player names a integer number between 0 and 10. Denote by a_1, a_2 the number stated respectively by player 1 and 2.
If $a_1 + a_2 \leq 10$, they get a_1, a_2.
If $a_1 + a_2 > 10$ and $a_1 \leq 5$, player 1 gets a_1 and player 2 gets $10 - a_1$.
If $a_1 + a_2 > 10$ and $a_2 \leq 5$, player 2 gets a_2 and player 1 gets $10 - a_2$.
If $a_1 > 5$ and $a_2 > 5$, they both get 5.

a) Determine the best response of each player to each of the other player's actions
b) Plot the best responses of both players in a diagram where a_1 is on the horizontal axis and a_2 is on the vertical axis.
c) Find all Nash equilibria.

Solution

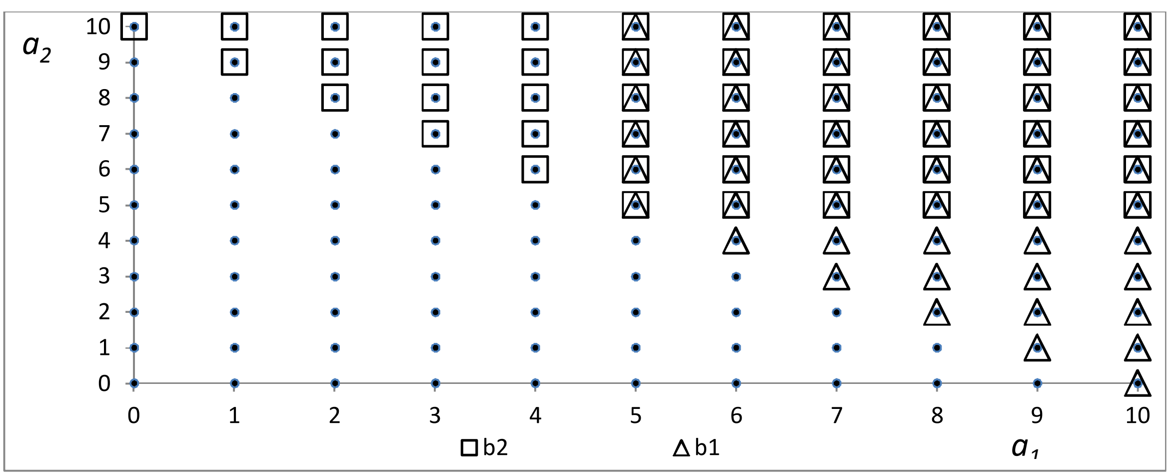
a) Given that the game is symmetric we consider the best response of player 1.
   a. When $a_2 \leq 5$, player 1's best response is $a_1 \geq 10 - a_2$
   b. When $a_2 > 5$, player 1's best response is $a_1 \geq 5$

   best response of player 2:

   a. When $a_1 \leq 5$, player 2's best response is $a_2 \geq 10 - a_1$
   b. When $a_1 > 5$, player 2's best response is $a_2 \geq 5$

b)

b1 denotes the best response of player 1, b2 denotes the best response of player 2



c) The Nash eqiuilibria are all the pairs of actions that are best response to the other action. Then all possible pairs between $a_1 \geq 5$ and $a_2 \geq 5$. In total 36 Nash equilibriums.
{5,5} {5,6} {5,7}...... {5,10}
{6,5} {6,6} {6,7}...... {6,10}
.
.
{10,5} {10,6} {10,7}......{10,10}