Lecture 5

Subgame Perfect Nash Equilibrium and Backward Induction in games of imperfect information

Backward Induction in dynamic games of imperfect information

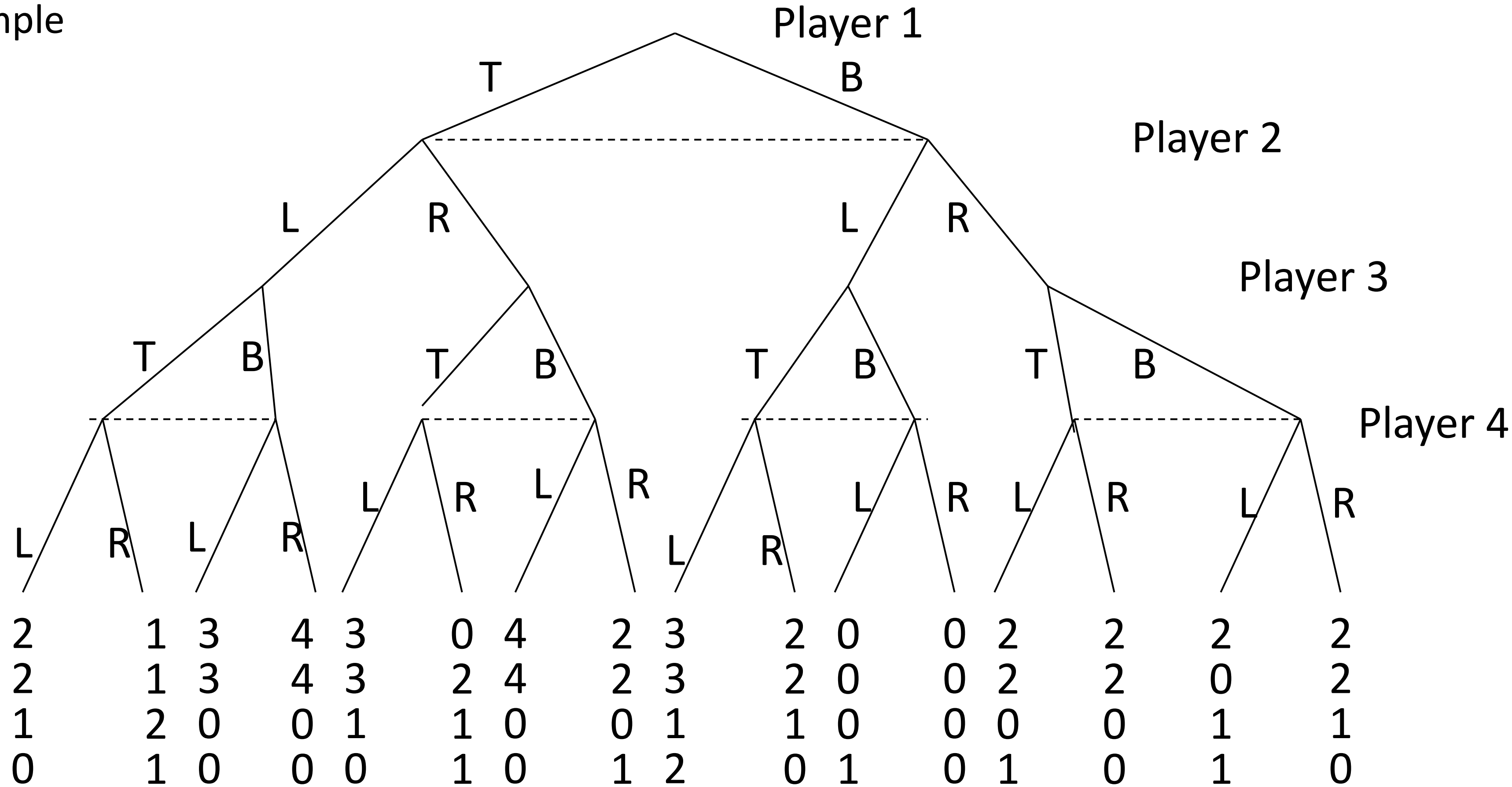
- We start at the end of the trees
- first find the Nash equilibrium (NE) of the last subgame
- then taking this NE as given, find the NE in the second last subgame
- continue working backwards

If in each subgame there is only one NE, this procedure leads to a **Unique Subgame Perfect Nash equilibrium**

Example: two stage game of imperfect information

- **Stage 1**: Players 1 and 2 move simultaneously taking, respectively, actions $a_1 \square A_1$ and $a_2 \square A_2$
- **Stage 2:** Players 3 and 4 observe (a_1, a_2) , then move simultaneously taking, respectively, actions $a_3 \square A_3$ and $a_4 \square A_4$
- **Payoffs:** $u_i(a_1, a_2, a_3, a_4)$ for i = 1, 2, 3, 4
- **Solution:**
  - We solve the simultaneous - move game between players 3 and 4 in the second stage:
  - Players 1 and 2 anticipate the behaviour of players 3 and 4

example



Player 1
T
B
Player 2
L
R
L
R
Player 3
T
B
T
B
T
B
T
B
Player 4
L
R
L
R
L
R
L
R
L
R
L
R
L
R
L
R
2 2 1 0
1 1 2 1
3 3 0 0
4 4 0 0
3 3 1 0
0 2 1 1
4 4 0 0
2 2 0 1
3 3 1 2
2 2 1 0
0 0 0 1
0 0 0 0
2 2 0 1
2 2 0 0
2 0 1 1
2 2 1 0

		Player 2 L	Player 2 R
Player 1	T	G1	G2
	B	G3	G4

G1

		Player 4 L	Player 4 R
Player 3	T	2, 2,1,0	1,1 , 2, 1
	B	3,3, 0,0	4,4, 0, 0

G3

		Player 4 L	Player 4 R
Player 3	T	3,3,1,2	2,2,1, 0
	B	0,0,0,1	0,0,0, 0

G2

		Player 4 L	Player 4 R
Player 3	T	3,3,1,0	0,2,1, 1
	B	4,4,0,0	2,2,0, 1

G4

		Player 4 L	Player 4 R
Player 3	T	2,2,0,1	2,2,0,0
	B	2,0,1,1	2,2,1, 0

		Player 2	
		L	R
Player 1	T	1,1 , 2, 1	0,2,1, 1
	B	3,3,1,2	2,0,1,1

G1

		Player 4	
		L	R
Player 3	T	2, 2,1,0	1,1 , 2, 1
	B	3,3, 0,0	4,4, 0, 0

G2

		Player 4	
		L	R
Player 3	T	3,3,1,0	0,2,1, 1
	B	4,4,0,0	2,2,0, 1

G3

		Player 4	
		L	R
Player 3	T	3,3,1,2	2,2,1, 0
	B	0,0,0,1	0,0,0, 0

G4

		Player 4	
		L	R
Player 3	T	2,2,0,1	2,2,0,0
	B	2,0,1,1	2,2,1, 0

Backward induction outcome:

(B, L, T, L)

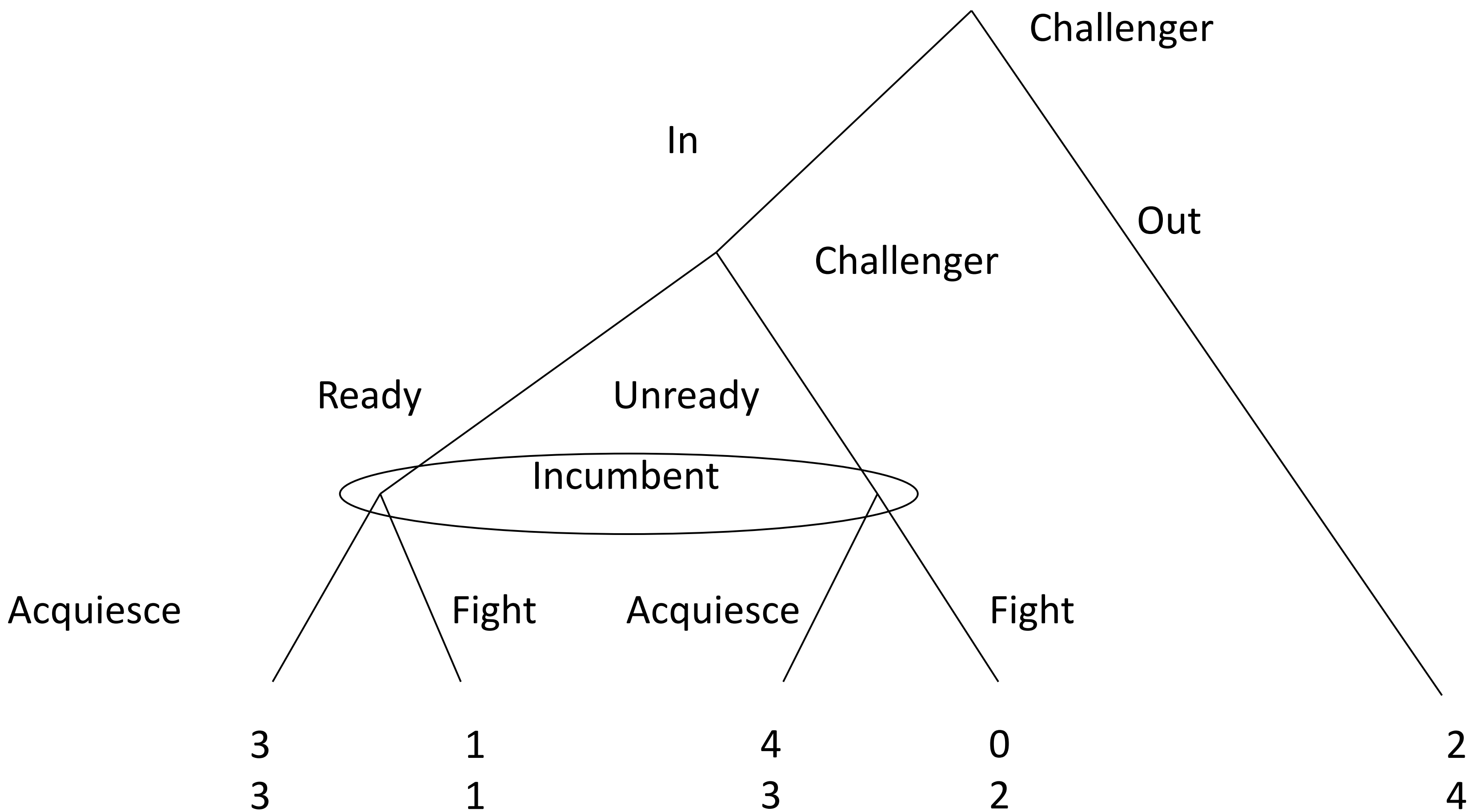
Subgame perfect Nash equilibrium

(B, L, (T, T, T, B), (R, R, L, L))

Example 2

Challenger's strategies: {(Out Ready), (Out Unready) (In ready), (In Unready)}

Incumbent' strategies: Acquiesce, Fight



		Incumbent	
		Acquiesce	Fight
	Out Ready	2, 4	2, 4
	Out Unready	2, 4	2, 4
Challenger	In Ready	3, 3	1, 1
	In Unready	4, 3	0, 2

Three Nash equilibria:
(Out Ready, Fight);
(Out Unready, Fight)
(In unready, Acquiesce)

Consider the subgame starting in the decision node after Challenger's choice ***In***

		Incumbent	
		Acquiesce	Fight
Challenger	Ready	3, 3	1, 1
	Unready	4, 3	0, 2

An unique Nash equilibrium: Unready, Acquiesce
Then, only (In unready, Acquiesce) is **subgame perfect NE**

Example 3

		Incumbent	
		Acquiesce	Fight
Challenger	Ready	3, <u>3</u>	1,1
	Unready	<u>4</u>, <u>3</u>	0, 2
	Out	2, <u>4</u>	<u>2</u>, <u>4</u>

Two Nash equilibria:
(Out, Fight)
(Unready, Acquiesce)

Both are SPNE