Problem set 6

1) Three oligopolists operate in a market with inverse demand function given by $P(Q) = a - Q$ where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 > 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3. Find the subgame perfect outcome.

2) Consider the following normal form game where Player 1 chooses the row (either T or B), Player 2 chooses the column (either r or l), Player 3 chooses the table (either R or L)

		Player 3			
		L		R	
		Player 2		Player 2	
		l	r	l	r
Player 1	T	1, 1, 1	0, 0, 0	0, 0, 0	0, 0, 0
	B	0, 0, 0	0, 0, 0	0, 0, 0	4, 4, 4

a) find all Nash equilibria in pure strategies
b) assume that player 1 moves first, then player 2 and finally player 3; every player, before to play, observes the choices of the predecessors.
   a. Represent the game using the extensive form
   b. Find all subgame perfect Nash equilibria
c) Assume that player 3 is not able to see the choice of player 2
   a. Represent the game using the extensive form
   b. Find all subgame perfect Nash equilibria

3) Three periods sequential bargaining. Two players, 1 and 2, are bargaining over \$1 using the following bargaining procedure (alternating offers):
<u>Period 1:</u> Player 1 proposes to take a share s1 of the dollar, leaving 1 – s1 for player 2; Player 2 either accepts (game ends) or rejects (Play goes to period 2)
<u>Period 2:</u> Player 2 proposes a share s2 of the dollar for player 1, leaving 1 – s2 for player 2; Player 1 either accepts (game ends) or rejects (Play goes to period 3)
<u>Period 3:</u> Player 1 receives a share s of the dollar, player 2 receives 1 – s.
Players discount future payoffs by factor δ per period, $0 < \delta < 1$.
Find the backward induction outcome and describe the subgame perfect Nash equilibrium

4) Tariffs and imperfect international competition. There are two identical countries denoted by i = 1, 2. One homogeneous good is produced in each country by a firm, firm *i* in country *i.* A share h_i of this product is sold in the home market and a share e_i is exported in the other country. Governments choose tariffs, i.e. a tax on the import. Government of country *i* chooses tariff t_i

   In country i the inverse demand function is $P_i(Q_i) = a - Q_i$ where $Q_i = h_i + e_j$.
   The firm's payoff (profits) is $\pi_i = [a - h_i - e_j]h_i + [a - h_j - e_i]e_i - c[h_i + e_i] - t_j e_i$ where c>0 is the marginal cost. The government's payoff is $W_i = 0.5\, Q_i^2 + \pi_i + t_i e_j$
   Timing: Governments simultaneously choose tariffs (t_1, t_2); Firms observe (t_1, t_2) and simultaneously choose quantities (h_1, e_1) (h_2, e_2) .
   Find the backward induction outcome and describe the subgame perfect Nash equilibrium

   (Hint: suppose that governments have chosen tariffs (t_1, t_2) and find the optimal behaviour of firms as function of (t_1, t_2). Assume that governments correctly predict the optimal behaviour of firms for each possible combination of (t_1, t_2) and find the optimal tariff rates)