Problem set 8

1) Consider the following game:
   a. 2 players, 1 and 2
   b. $A_1 = \{T, B\}$, $A_2 = \{L, R\}$
   c. $t_1 \in \{1, 2\}$ $t_2 \in \{1, 2\}$
   d. *Utilities are:*

		Player 2, t_2= 1	
		L	R
Player 1 t_1= 1	T	2, 2	0, 0
	B	0, 0	1, 1

		Player 2, t_2= 2	
		L	R
Player 1 t_1= 1	T	2, 1	0, 0
	B	0, 0	1, 2

		Player 2, t_2= 1	
		L	R
Player 1 t_1= 2	T	1, 2	0, 0
	B	0, 0	3, 1

		Player 2, t_2= 2	
		L	R
Player 1 t_1= 2	T	1, 2	0, 0
	B	0, 0	3, 2

Assuming that t_1= *1 by probability* $\frac{1}{4}$, t_2= *1 by probability* $\frac{3}{4}$; t_1 and t_2 are i.i.d.

Suppose that Player 1 plays a strategy (T, B) and Player 2 plays a strategy (R, L)

Note: (X, Y) means she plays X when t=1 and Y when t=2.

a. Compute the probability to observe Player 1 plays B and Player 2 plays R
b. Compute the probability to observe Player 1 plays B and Player 2 plays L
c. Compute the Player 2's expected payoff in this strategy profile
d. Repeat the computation in the previous points assuming that
   - t_1= *1 and* t_2= *1 by probability* $\frac{4}{10}$
   - t_1= *2 and* t_2= *2 by probability* $\frac{4}{10}$
   - t_1= *2 and* t_2= *1 by probability* $\frac{1}{10}$
   - t_1= *1 and* t_2= *2 by probability* $\frac{1}{10}$

2) Find all Bayesian Nash equilibria of the previous game, Assuming that $t_1 = 1$ *by probability* $\frac{1}{4}$, $t_2 = 1$ *by probability* $\frac{3}{4}$; t_1 and t_2 are i.i.d.

3) Two firms, 1, 2 produce an homogeneous good. Firms have no fixed cost and produce at constant marginal cost of 1. By q1 and q2 we denote the quantities produced, respectively, by firm 1 and 2. The inverse demand function is P(Q) = 100 – Q by probability $\frac{1}{3}$ and P(Q) = 50 – Q by probability $\frac{2}{3}$ where Q = q1 + q2. Firms 1 and 2 simultaneously choose the quantities to produce. Firm 1 knows if the demand is high or low but firm 2does not. What is the Bayesian Nash equilibrium of the game?
(read the example3.1.A in the textbook.)

4) Consider the following game.
Nature determines if payoffs are as in G1 or in G2 by equal probability

		Player 2		
	G1	L	C	R
Player 1	T	2,0	1, 1	4, 2
	M	3,4	1, 2	2, 3
	B	1, 3	0, 2	3, 0

		Player 2		
	G2	L	C	R
Player 1	T	2,0	1, 1	2, 0
	M	3,4	1, 2	2, 3
	B	1, 3	0, 2	3, 0

Players 1 and 2 move simultaneously
Players 2 knows which game ***Nature*** has chosen, but Player 1 does not.

a) Describe all possible strategies of players 1 and 2
b) Find the Bayesian Nash equilibrium