1) a) x = number of Tails

$P_2(x)$ probability of x Tails

$$P_2(x) = \frac{3!}{x!\,(3-x)!}\,0{,}5^3$$

$$E = \sum_{x=0}^{3} P_2(x)\left[100\cdot x + 50(3-x)\right]$$

$$= \frac{1}{8}\cdot 300 + \frac{3}{8}\,250 + \frac{3}{8}\,200 + \frac{1}{8}\,150 = 225$$

b)

$$E = \int_0^{100} x\left(\frac{1}{50} - \frac{1}{5000}x\right) dx =$$

$$= \int_0^{100} \frac{x}{50} - \frac{x^2}{5000}\, dx = \left|\frac{x^2}{100} - \frac{x^3}{15000}\right|_0^{100} = 33{,}\overline{3}$$

2) $q = \left(200, \frac{1}{4} ; 100, \frac{1}{2}\right)$

$$E(q) = 200 \cdot \frac{1}{4} + 100 \frac{1}{2} = 100$$

3) $r = (150, 0.2 ; 100, 0.4)$

$$W = \left(q, \frac{1}{2} ; r, \frac{1}{2}\right) = \left(200, \frac{1}{8} ; 150, 0.1 ; 100, 0.45\right)$$

4) Independence

Consider 2 prospects q, r such that $q \succsim r$

then

$$(q, p ; s, (1-p)) \succsim (r, p ; s, (1-p))$$

$\forall p \in [0,1]$ and every prospect s

i take $s = q$, The independence condition is:

(A) $q \succsim (r, p ; q, (1-p)) \quad \forall p \in [0,1]$

ii take $s = r$, the independence condition is:

(B) $(q, p ; r, (1-p)) \succsim r \quad \forall p \in [0,1]$

(A) and (B) are the conditions of betweenness.

5)

$a = \left(300, \frac{1}{8} ; 100, \frac{3}{8} ; -100, \frac{3}{8} ; -300, \frac{1}{8}\right)$

$b = \left(200, \frac{1}{4} ; -200, \frac{1}{4}\right)$

$c = \left(-300, \frac{1}{4} ; 300, \frac{1}{4}\right)$

Let $F_y(x)$ the probability to get at maximum x in project y

Among the 3 projects we have 7 outcomes:

$-300, -200, -100, 0, 100, 200, 300$

	-300	-200	-100	0	100	200	300
$F_a(x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{7}{8}$	1
$F_b(x)$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	1
$F_c(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1

c cannot stochastically dominate b or a because

$F_c(-300) > F_b(-300)$ and $F_c(-300) > F_a(-300)$

b cannot stochastically dominate a or c because

$F_b(200) > F_a(200)$ and $F_b(200) > F_c(200)$

a cannot stochastically dominate b or c because

$F_a(-100) > F_b(-100)$ and $F_a(-100) > F_c(-100)$

6) it depends on the utility function

Assume $U(x) = \ln(x+1)$

$W = 10$ the prospect is $q = (20, \frac{2}{3})$

$U(q) = \frac{2}{3} \ln 21 = 2.03$ $U(10) = \ln 11 = 2.40$

the prospect is not acceptable

$W = 100$ the prospect is $q = (90, \frac{1}{3} ; 110, \frac{2}{3})$

$U(q) = \frac{1}{3} \ln 91 + \frac{2}{3} \ln 111 = 4.64$ $U(100) = \ln 101 = 4.62$

The prospect is acceptable

$W = 1000$ the prospect is $q = (990, \frac{1}{3} ; 1010, \frac{2}{3})$

$U(q) = \frac{1}{3} \ln 991 + \frac{2}{3} \ln 1011 = 6.912$ $U(1000) = \ln 1001 = 6.909$

the prospect is acceptable

7) $q = (200, \frac{1}{4}; 100, \frac{1}{2})$

$E(q) = 100$

$U(q) = -e^{-0.01 \cdot 200} \cdot \frac{1}{4} - e^{-0.01 \cdot 100} \cdot \frac{1}{2} - e^{-0.01 \cdot 0} \cdot \frac{1}{4} = -0.4678$

$CE(q) = -\frac{\ln(0.4678)}{0.01} = 75.9771$

Risk premium

$R = 100 - 75.98 = 24.02$

8)

a. $U(x) = -e^{-0.1x}$

$$\frac{dU(x)}{dx} = 0.1e^{-0.1x}$$

$$\frac{d^2U(x)}{dx^2} = -0.01e^{-0.1x}$$

The second derivative is negative, then the utility fct is concave $\longrightarrow$ risk averse

$$A(x) = -\frac{-0.01e^{-0.1x}}{0.1e^{-0.1x}} = 0.1$$

$$R(x) = x \cdot 0.1$$

b. $U(x) = e^{0.1x}$

$$\frac{dU(x)}{dx} = 0.1e^{0.1x} \qquad \frac{d^2U(x)}{dx^2} = 0.01e^{0.1x}$$

the second derivative is positive, then the utility fct is convex $\longrightarrow$ risk seeking

$$A(x) = -\frac{0.01e^{0.1x}}{0.1e^{0.1x}} = -0.1$$

$$R(x) = -0.1x$$

9) $q' = (100, \frac{1}{3}; 50, \frac{1}{3})$ $\qquad U(0) = 0$

$$U(q') = \frac{1}{3} U(100) + \frac{1}{3} U(50) = \bar{U}$$

Let be $q = 1 - p - w$

$$U(q) = p\, U(100) + (1 - p - w)\, U(50)$$

The indifference curve is given by the solution of

$$\bar{U} = p\, U(100) + (1 - p - w)\, U(50)$$

$$p = \frac{\bar{U} - (1-w)\, U(50)}{U(100) - U(50)} = \frac{\bar{U} - U(50)}{U(100) - U(50)} + \frac{U(50)}{U(100) - U(50)} \cdot w$$

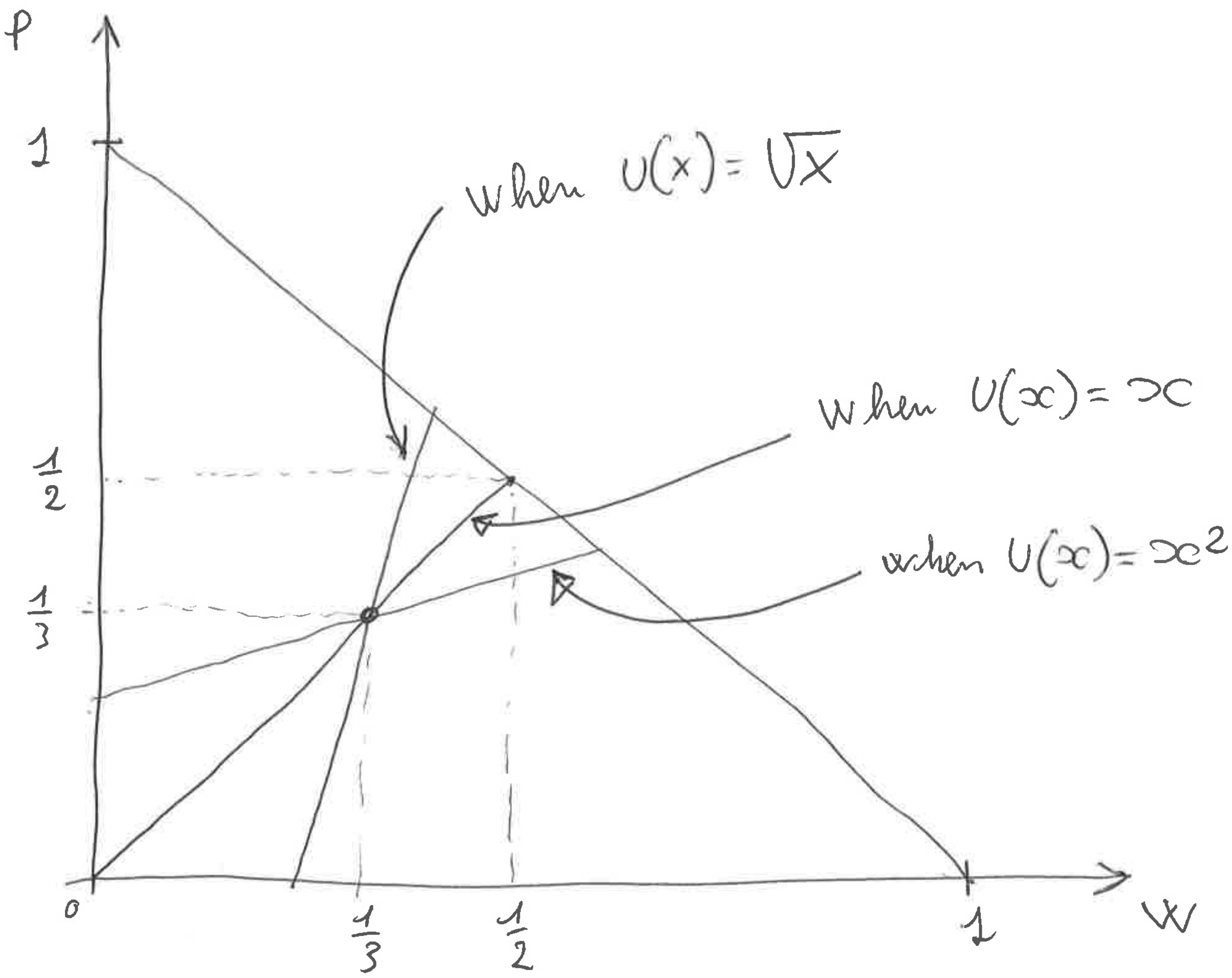
$U(x) = x$ $\qquad \bar{U} = 50$ $\qquad \underline{p = w}$

$U(x) = \sqrt{x}$ $\qquad \bar{U} = 5.69$ $\qquad U(50) = 7.07$ $\qquad U(100) = 10$

$$\underline{p = -0.47 + 2.41\, w}$$

$U(x) = x^2$ $\qquad \bar{U} = 4166.7$ $\qquad U(50) = 2500$ $\qquad U(100) = 10.000$

$$\underline{p = 0.22 + 0.33\, w}$$



p
1
when $U(x)=\sqrt{x}$
when $U(x)=x$
$\frac{1}{2}$
when $U(x)=x^2$
$\frac{1}{3}$
0
$\frac{1}{3}$
$\frac{1}{2}$
1
W