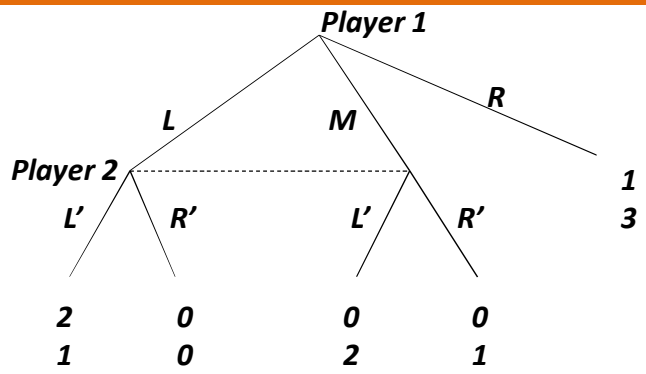
Lecture 9
Perfect Bayesian Equilibrium

An Example



Each player has one information set

Player 1' strategies: $\boldsymbol{S_1 = \{L, M, R\}}$

Player 2' strategies: $\boldsymbol{S_2 = \{L', R'\}}$

One sub-game (the whole game) :

it implies that all NE are SPNE

To find all NE we use the normal form

		Player 2	
		L'	R'
Player 1	L	2, 1	0, 0
	M	0, 2	0, 1
	R	1, 3	1, 3

Two NE: $\{L, L'\}$ and $\{R, R'\}$ Both NE are SPNE

Equilibrium $\{R, R'\}$ is based on a no credible threat:
- Suppose *player 1* plays either *L* or *M*, then *player 2*'s best response is to play *L'*
- It follows that *player 1* cannot be induced to play *R* by *player 2*'s threat to play *R'*.

3

In order to rule out equilibrium $\{R, R'\}$ we impose more requirements

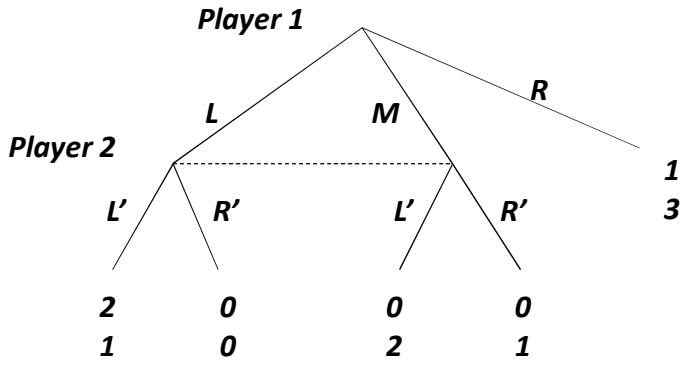
Requirement 1. At each information set , the player with the move must have a ***belief*** about which node has be reached

Requirement 2. Given their beliefs, the players' strategies must be ***sequentially rational***, i.e. At each information set the action taken by the player that has to move must be optimal given:

a) the beliefs at that information set

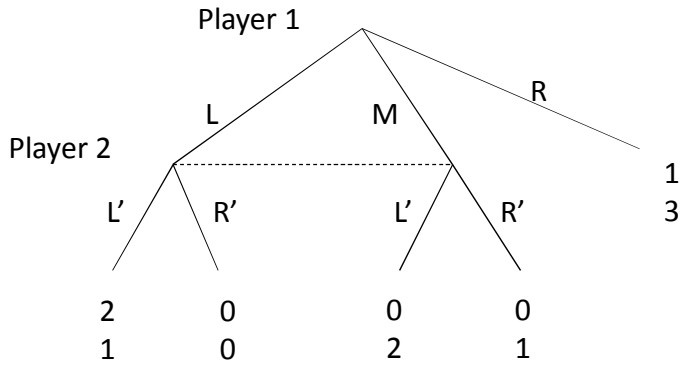
b) the other players' subsequent strategies

4



Requirement 1**:** when player 2 has to move, player 2 must have a ***belief*** about which node has been reached (or about the action of player 1).
Belief of player 2:
*"by probability **p** player 1 has chosen action L"*

5



Requirement 2: when player 2 has to move, he has to choose ***the action with the higher expected value*** (given his beliefs):

$$E(L') = 1\,p + 2\,(1 - p) = 2 - p$$
$$E(R') = 0\,p + 1\,(1 - p) = 1 - p$$

Then Player 2 will choose L' ($E(L') > E(L')\ \forall\ p \in [0,1]$)

6

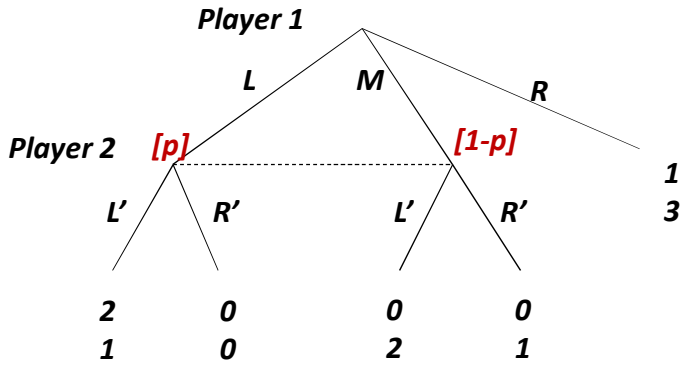
Requirement 1 imposes that players have beliefs, but it does not impose any requirements on these beliefs.

Definition:

a. an information set is ***on the equilibrium path*** if it will be reached with strictly positive probability when the game is played according the equilibrium strategies
b. an information set is ***off the equilibrium path*** if it will be reached with zero probability when the game is played according the equilibrium strategies

Requirement 3: At information sets on the equilibrium path, beliefs are determined by Bayes' rule and equilibrium strategies.

7



Consider NE $\{L, L'\}$, player 2's beliefs is $\boldsymbol{p = 1}$

Suppose it exists an equilibrium where player 1 plays ***L*** by probability $\boldsymbol{q}$, ***M*** by probability $\boldsymbol{r}$ and ***R*** by probability $\boldsymbol{1 - q - r}$, then the belief of player 2 is:

$$\boldsymbol{p = \frac{q}{q + r}}$$

8

Requirement 4: At information sets off the equilibrium path, beliefs are determined by Bayes' rule and equilibrium strategies when possible

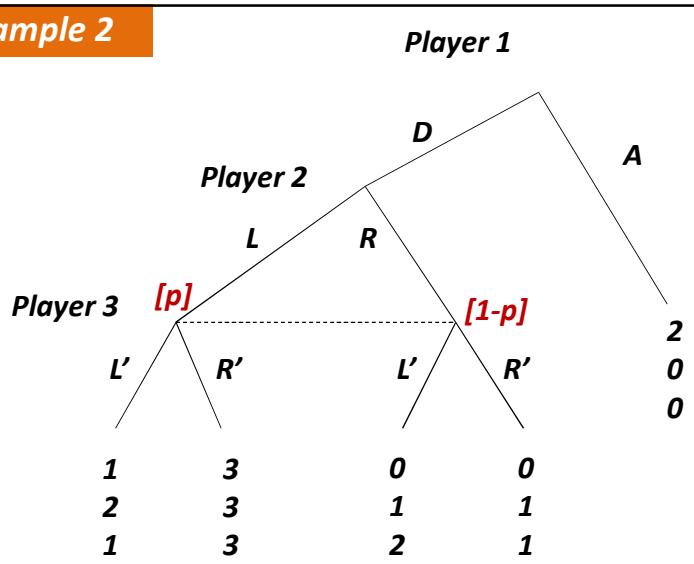
9

Definition of Perfect Bayesian Equilibrium (PBE)

A perfect Bayesian Nash equilibrium consists of strategies and beliefs satisfying requirements 1, 2, 3 and 4.

10

Example 2



11

		Player 1			
		D		A	
		Player 2		Player 2	
		L	R	L	R
Player 3	L'	1, 2, 1	2, 1, 0	0, 0, 2	0, 0, 2
	R'	3, 3, 3	1, 1, 0	0, 0, 2	0, 0, 2

Four Nash equilibria:
$\{D, L, R'\}$, $\{A, L, L'\}$, $\{A, R, L'\}$, $\{A, R, R'\}$

One subgame that begins after player 1 plays D
This subgame has only one NE: $\{L, R'\}$
Then one SPNE: $\{D, L, R'\}$

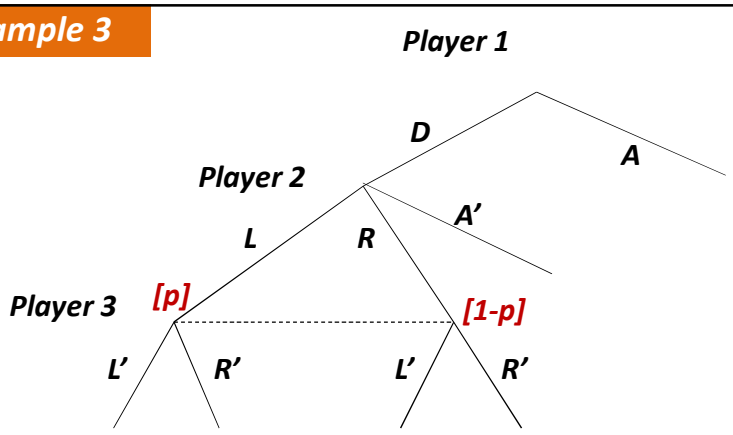
12

SPNE: $\{D, L, R'\}$ satisfies requirement 1, 2 and 3
R1 and R3: Player 3's belief is *p=1*
R2: R' is a best response given the beliefs and others 'strategies
R4 is satisfied because there are no info sets off the equilibrium path

Consider the NE: $\{A, L, L'\}$ with belief p = 0.
Requirement 1, 2 , 3 are satisfied
But p = 0 is inconsistent with player 2's strategy
Then Requirement 4 is not satisfied

13

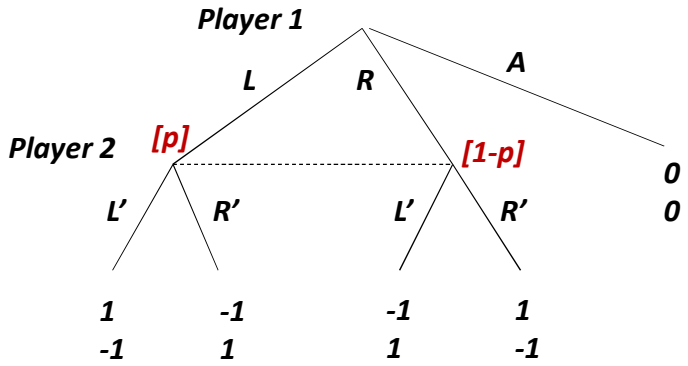
Example 3



Suppose an equilibrium where player 1 and player 2 play, respectively, A and A'.
Player 3' s information set is off the equilibrium path
By requirement 4, player 3's beliefs have no restrictions, because we cannot use Bayes rule to determine p

14

Example 4



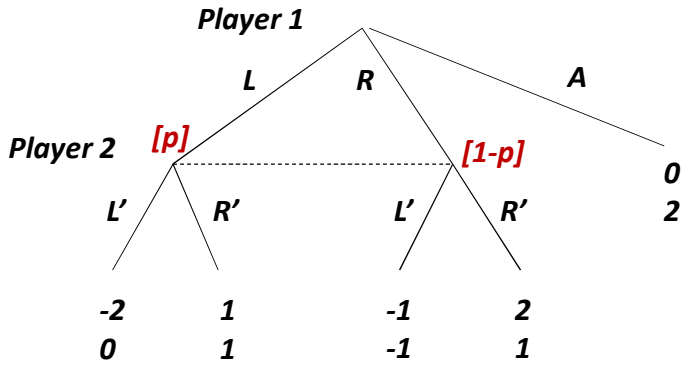
Try to check if the following strategy profiles are PBE

1) $s_1 = \left(\frac{1}{3}\frac{1}{3}\frac{1}{3}\right)$ $s_1 = \left(\frac{1}{2}, \frac{1}{2}\right)$

2) $s_1 = (0, 0, 1)$ $s_1 = \left(\frac{1}{2}, \frac{1}{2}\right)$

15

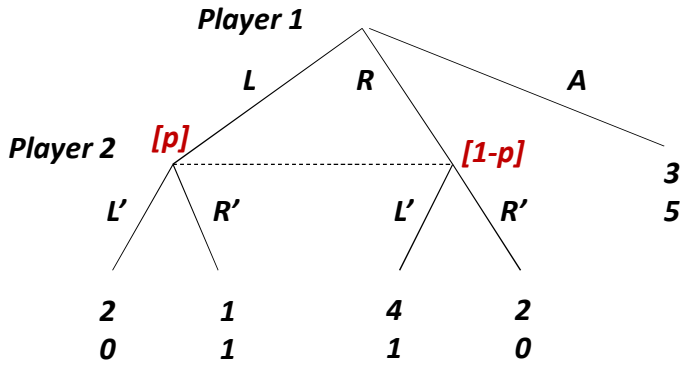
Example 5



Find all Nash equilibria and check they are PBE

16

Example 6



Find all Nash equilibria and check they are PBE

17