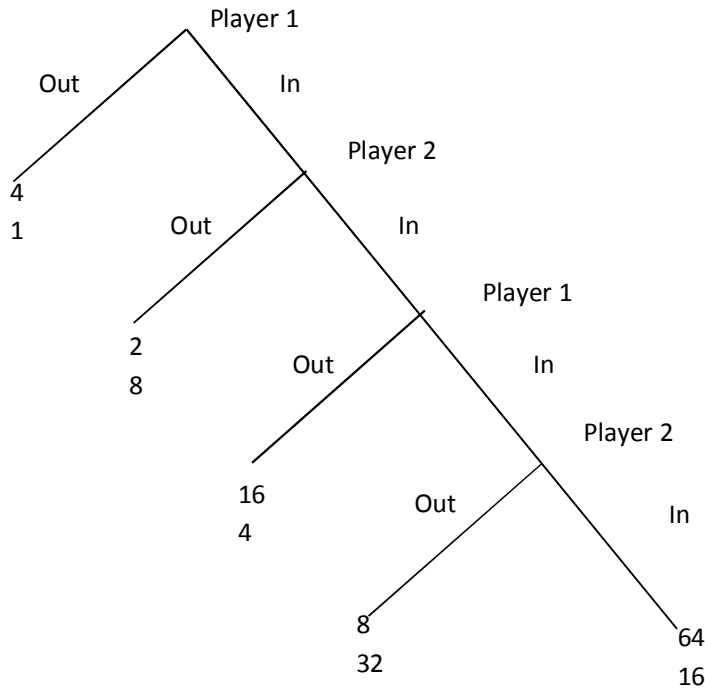**Problem set 4**

Ex 1.

In each decision node players have two possible actions, *In* or *Out*

a) How many information sets for each player?
b) How many subgames?
c) Write all possible strategies for both players
d) Represent this game in normal form and find all Nash equilibria.

Solution

a) Each player has two information sets
b) There are 4 subgames
c) Player 1: {(out, out), (out, in), (in, out), (in, in)} Player 2: {(out, out), (out, in), (in, out), (in, in)}
d)

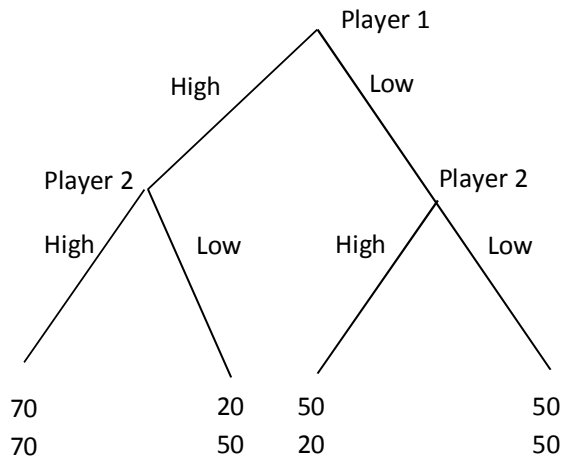|		Player 2			
		Out Out	Out In	In Out	In In
Player 1	Out Out	<u>4</u>, <u>1</u>	<u>4</u>, <u>1</u>	4, <u>1</u>	4, <u>1</u>
	Out In	<u>4</u>, 1	<u>4</u>, <u>1</u>	4, <u>1</u>	4, <u>1</u>
	In Out	2, <u>8</u>	2, <u>8</u>	<u>16</u>, 4	16, 4
	In In	2, 8	2, 8	8, <u>32</u>	<u>64</u>, 16

Nash equilibria:

{(Out, Out), (Out, Out)}   {(Out, Out), (Out, In)}
{(Out, In), (Out, Out)}   {(Out, In), (Out, In)}

Ex 2. Two individuals, A and B, are working on a join project. They can devote it either high effort or low effort. If both players devote high effort, the outcome of the project is of high quality and each one receives 100$. If one or both devote low effort, the outcome of the project is of low quality and each one receives 50$. The opportunity cost to provide high effort is 30. The opportunity cost to provide low effort is 0. Individual A moves first, individual B observes the action of A and then moves.

- i) Represent this situation using the extensive form representation
- ii) for both players write all possible strategies
- iii) Using the normal form, find all Nash equilibriums

Solution

i)

ii) Player 1's strategies: {High, Low}

Player 2's strategies: {(Low, Low), (Low, High), (High, Low), (High, High)} where by (x, y) we denote action x after High, action y after Low

iii)

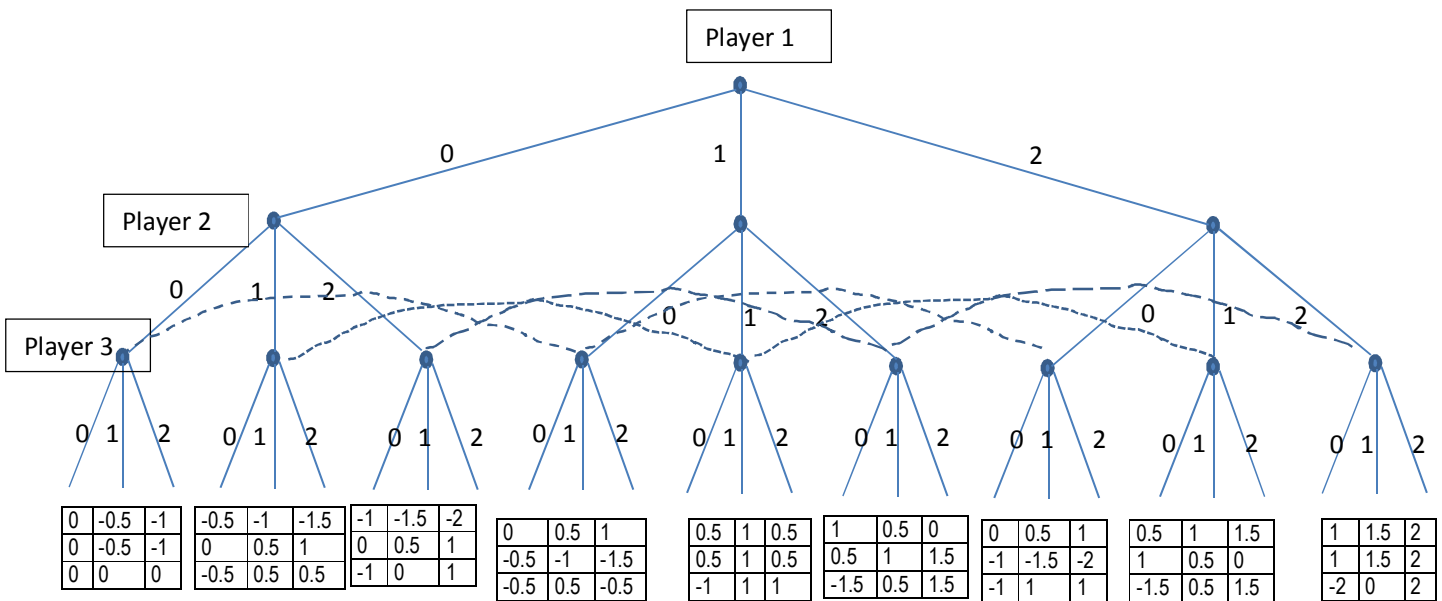|		Player 2			
		High High	High Low	Low High	Low Low
Player 1	High	70, 70	70, 70	20, 50	20, 50
	Low	50, 20	50, 50	50, 20	50, 50

By underscore we denote the best responses.

Nash equilibria:

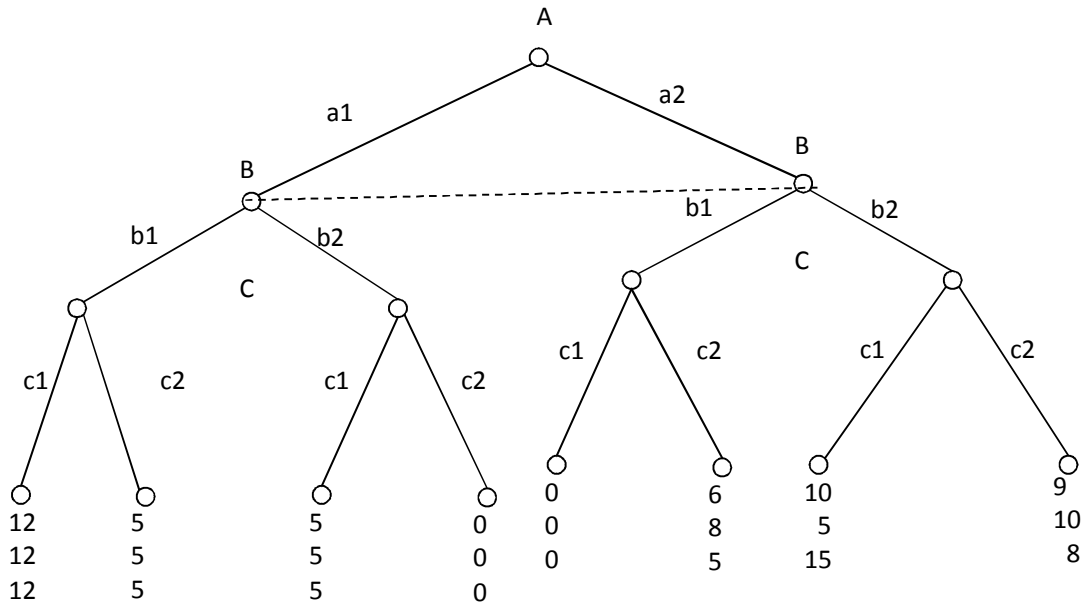{(High), (High, High)} {(High), (High, Low)}

{(Low), (Low, Low)}

Ex. 3 There are 3 players that must state one number from the set {0, 1, 2}. The payoff of each player is given by the stated number minus the absolute difference between his stated number and the average of the numbers stated by the other two players. Players move in a sequence: Player 1 moves first, player 2 observes the choice of player 1 then he moves, player 3 observes the choice of player 2 (not that of player 1) then he moves.

    a) Represent this situation using the extensive form

    b) How many information sets has this game?

    c) How many subgames?

Player 1 · 0 · 1 · 2

Player 2 · 0 · 1 · 2

Player 3 · 0 1 2

Payoff tables:

0	-0.5	-1
0	-0.5	-1
0	0	0

-0.5	-1	-1.5
0	0.5	1
-0.5	0.5	0.5

-1	-1.5	-2
0	0.5	1
-1	0	1

0	0.5	1
-0.5	-1	-1.5
-0.5	0.5	-0.5

0.5	1	0.5
0.5	1	0.5
-1	1	1

1	0.5	0
0.5	1	1.5
-1.5	0.5	1.5

0	0.5	1
-1	-1.5	-2
-1	1	1

0.5	1	1.5
1	0.5	0
-1.5	0.5	1.5

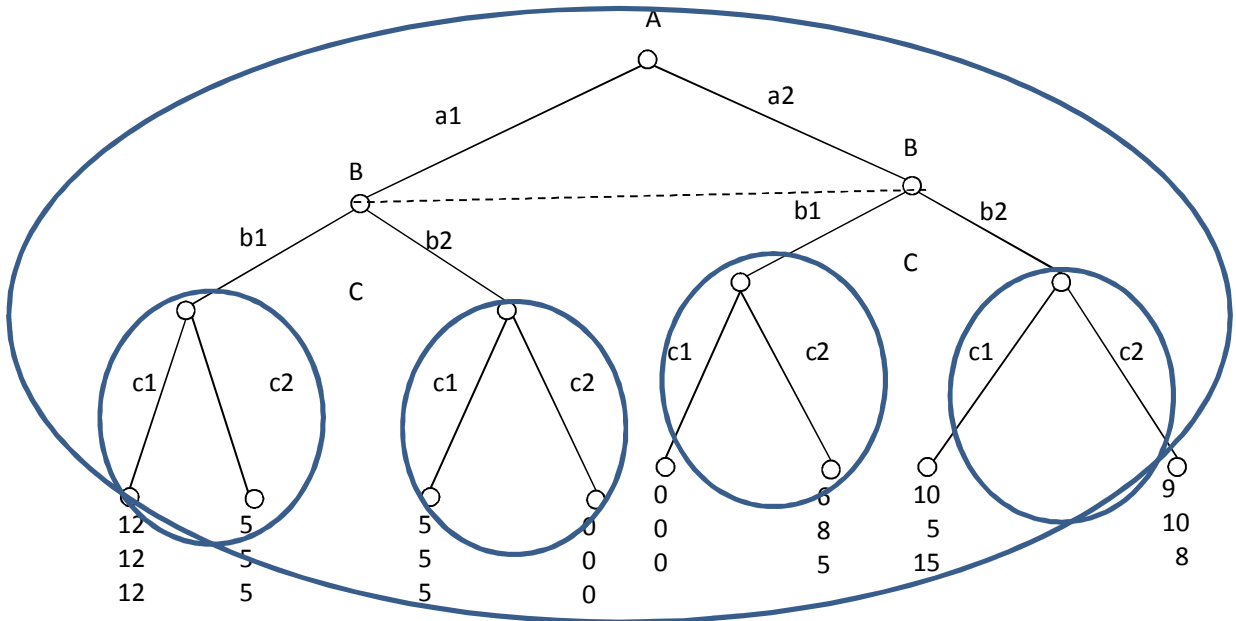| 1	1.5	2
1	1.5	2
-2	0	2

b) 7 information sets

c) 1 subgames

Ex. 4

i.   Find all subgames
ii. How many information sets has player A?and player B? And player C?
iii. For each player describe all possible strategies

Solution

i.   Find all subgames

ii.   How many information sets has player A?and player B? And player C?

Player A has one information set

Player B has one information set

Player C has 4 information sets

iii.   For each player describe all possible strategies

Player 1: $\{(a_1), (a_2)\}$

Player 2: $\{(b_1), (b_2)\}$

Player 3:
$$\left\{\begin{array}{l} (c_1,c_1,c_1,c_1),(c_2,c_1,c_1,c_1),(c_1,c_2,c_1,c_1),(c_1,c_1,c_2,c_1),, \\ (c_1,c_1,c_1,c_2),(c_2,c_2,c_1,c_1),(c_2,c_1,c_2,c_1)(c_2,c_1,c_1,c_2), \\ (c_1,c_2,c_2,c_1),(c_1,c_2,c_1,c_2),(c_1,c_1,c_2,c_2),(c_1,c_2,c_2,c_2), \\ (c_2,c_1,c_2,c_2),(c_2,c_2,c_1,c_2),(c_2,c_2,c_2,c_1),(c_2,c_2,c_2,c_2) \end{array}\right\}$$

Ex. 5 There are 2 players that must state one number from the set {0, 1, 2}. The payoff of each player is given by the stated number minus the absolute difference between his stated number and the number stated by the other player. Players move in a sequence: Player 1 moves first then player 2. When player 2 has to move he is only partially informed about the choice of player 1: he can see if player 1 chosen 2 but he cannot discriminate if player 1 chosen 0 or 1

    a) Represent this situation using the extensive form
    b) How many information sets has this game
    c) How many subgames
    d) Describe all strategies of players 1 and 2
    e) using the normal form find all NE

a)
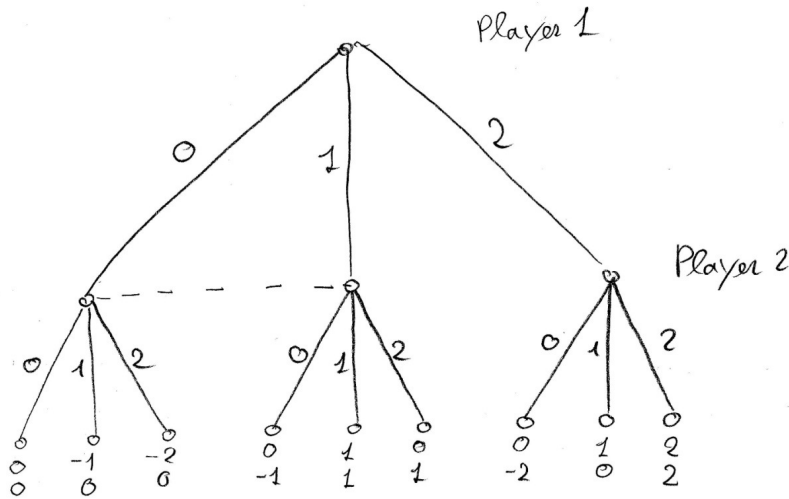
b) 3 information sets
1 for Player 1   2 for Player 2

c) 2 subgames

d) $\{(0), (1), (2)\}$ Player 1

$\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2),$
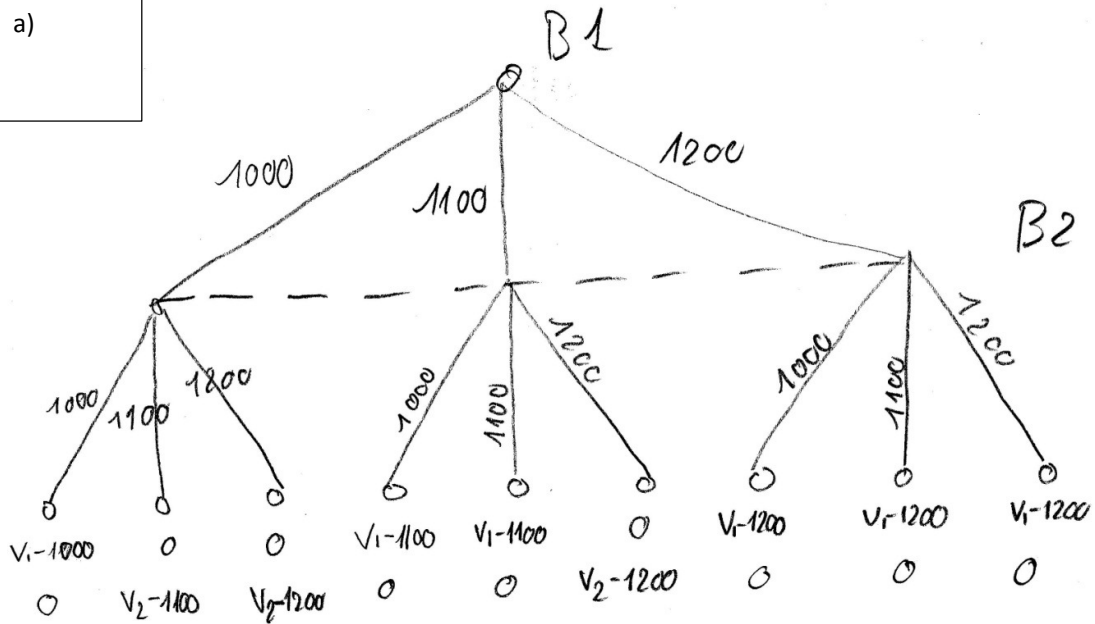$(2,0), (2,1), (2,2)\}$ Player 2.

e)

Pl 1

	0		1		2	
Pl.2						
(0,0)	0	0	-1	0	-2	0
(0,1)	0	0	-1	0	0	1
(0,2)	0	0	-1	0	2	2
(1,0)	0	-1	1	1	-2	0
(1,1)	0	-1	1	1	0	1
(1,2)	0	-1	1	1	2	2
(2,0)	0	-2	1	0	-2	0
(2,1)	0	-2	1	0	0	1
(2,2)	0	-2	1	0	2	2

NE 1) {(0), (0,0)}   4) {(1), (1,1)}
    2) {(2), (0,2)}   5) {(2), (1,2)}
    3) {(1), (1,0)}   6) {(1), (2,0)}
                          7) {(2), (2,2)}

Ex. 6 An individual want to sell a car at a price no lower than £ 1.000. Two buyers, 1 and 2, simultaneously send to the car's seller their offers. Car's seller chooses to sell the car to the buyer that sent the best offer. If the two offers are equal, the car's seller sells the car to buyer 1

   a. Represent this situation using the extensive form assuming that buyers can send only three offers: 1000, 1100, 1200.
   b. Represent the game in point a. using the normal form and find all NE.

a)

b)

B 2

	1000		1100		1200	
B1						
1000	V_1-1000	O	O	V_2-1100	O	V_2-1200
1100	V_1-1100	O	V_1-1100	O	O	V_2-1200
1200	V_1-1200	O	V_1-1200	O	V_1-1200	O

$NE \quad \{(1200), (1200)\}$