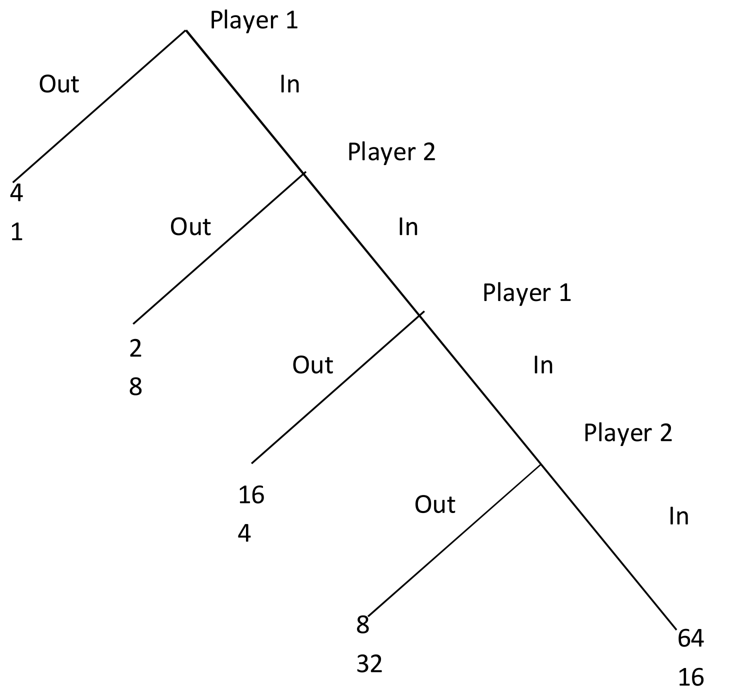
Solution Problem set 5

Ex 1.



In each decision node players have two possible actions, *In* or *Out*

a) Represent this game in normal form and find all Nash equilibria.
b) Find all Subgame perfect Nash equilibria.

Solution

a)

		Player 2			
		Out Out	Out In	In Out	In In
Player 1	Out Out	4, 1	4, 1	4, 1	4, 1
	Out In	4, 1	4, 1	4, 1	4, 1
	In Out	2, 8	2, 8	16, 4	16, 4
	In In	2, 8	2, 8	8, 32	64, 16

Nash equilibria:

{(Out, Out), (Out, Out)} {(Out, Out), (Out, In)}

{(Out, In), (Out, Out)} {(Out, In), (Out, In)}

b) Start from the end of the tree.
the optimal action for player 2 in the last decision node is "Out" (32>16)
Now consider player 1 in his last decision node. If he plays "out", he will get 16. If he plays "In", player 2 plays out and he will get 8. Then player 1 prefers "Out".

Now consider player 2 in his first decision node. His preferred action is “Out” because he gets 8, while playing “In” will produce a payoff of 4 (in such case Player 1 will play “Out”)
Consider player 1 at the start of the game. He prefers to play “Out” (Payoff of 4) because playing “in” produces a payoff of 2 (Player 2 plays “Out”)
In each decision node there is only one optimal decision, so there is an unique Subgame perfect Nash equilibria that is:
Player 1 plays strategy {out, out} and Player 2 plays strategy {out, out}

Ex 2. Two individuals, A and B, are working on a join project. They can devote it either high effort or low effort. If both players devote high effort, the outcome of the project is of high quality and each one receives 100$. If one or both devote low effort, the outcome of the project is of low quality and each one receives 50$. The opportunity cost to provide high effort is 30. The opportunity cost to provide low effort is 0. Individual A moves first, individual B observes the action of A and then moves.

i) Using the normal form, find all Nash equilibriums
ii) Find all Subgame perfect Nash equilibria.

Solution

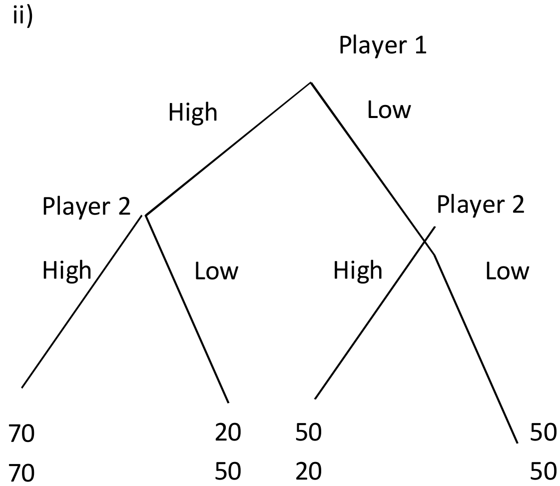
i)

		Player 2			
		High High	High Low	Low High	Low Low
Player 1	High	70, 70	70, 70	20, 50	20, 50
	Low	50, 20	50, 50	50, 20	50, 50

Nash equilibria:

{(High), (High, High)}
{(High), (High, Low)}
{(Low), (Low, Low)}

ii)



In the subgame on the left, for player 2 is optimal to play High, in the subgame on the right is optimal to play Low. Then the optimal strategy for player 2 is {High, Low}.

Player 1 anticipates the strategy of player 2. Then for player 1 is optimal to play High.

In each decision node there is only one optimal decision, so there is an unique Subgame perfect Nash equilibria that is:

Player 1 plays strategy {High} and Player 2 plays strategy {High, Low}

Ex. 3 There are 2 players that must state one number from the set {0, 1, 2). The payoff of each player is given by the stated number minus the absolute difference between his stated number and the number stated by the other player. Players move in a sequence: Player 1 moves first then player 2. When player 2 has to move he is only partially informed about the choice of player 1: he can see if player 1 chosen 2 but he cannot discriminate if player 1 chosen 0 or 1

a) using the normal form find all NE
b) suppose that player 2 can observe all choices of player 1. Find all Subgame perfect Nash equilibria.

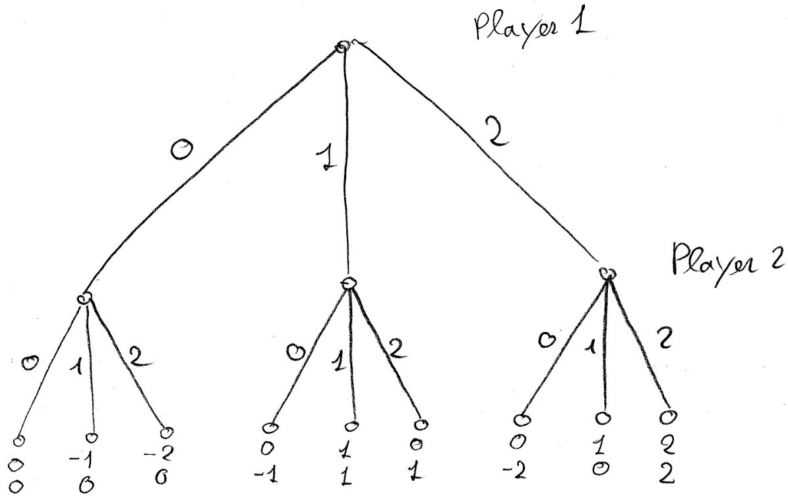
Solution

a)

Pl.2 \ Pl 1	0		1		2	
(0,0)	0	0	-1	0	-2	0
(0,1)	0	0	-1	0	0	1
(0,2)	0	0	-1	0	2	2
(1,0)	0	-1	1	1	-2	0
(1,1)	0	-1	1	1	0	1
(1,2)	0	-1	1	1	2	2
(2,0)	0	-2	1	0	-2	0
(2,1)	0	-2	1	0	0	1
(2,2)	0	-2	1	0	2	2

NE
1) {(0), (0,0)}
2) {(2), (0,2)}
3) {(1), (1,0)}
4) {(1), (1,1)}
5) {(2), (1,2)}
6) {(1), (2,0)}
7) {(2), (2,2)}

b) Here we assume perfect info



Player's 2 best response are:

- To play either 0 or 1 or 2 after player 1 plays 0
- To play either 1 or 2 after player 1 plays 1
- To play either 2 after player 1 plays 2

Then from the point of view of player 1 all the following player 1' strategies are plausible

Player 2 strategy **(x, y, z)** means player 2 plays **x** after player 1 plays 0, plays **y** after player 1 plays 1, plays **z** after player 1 plays 2

1) (0, 1, 2)
2) (0, 2, 2)
3) (1, 1, 2)
4) (1, 2, 2)
5) (2, 1, 2)
6) (2, 2, 2)

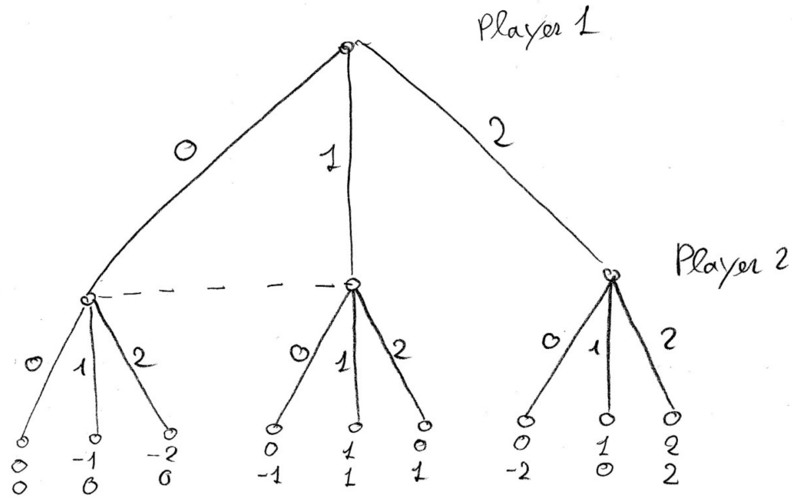
For player 1 anticipating any one of these strategy the best is to play 2

Therefore we have six SPNE:

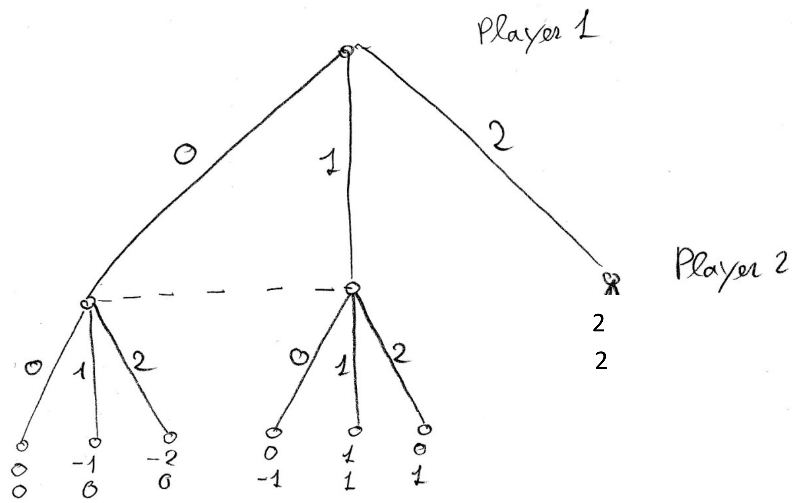
(strategy player 1) (strategy player 2)

1) (2) (0, 1, 2)
2) (2) (0, 2, 2)
3) (2) (1, 1, 2)
4) (2) (1, 2, 2)
5) (2) (2, 1, 2)
6) (2) (2, 2, 2)

c) THIS IS AN EXTENSION: HERE WE FIND THE SUBGAME PERFECT NASH EQUILIBRIAINTHE GAME WITH IMPERFECT INFO (POINT a)



In the subgame starting with the decision node of player 2 (on the right) the optimal action is 2. The reduced game is



We solve it using the normal form

		Player 2		
		0	1	2
Player 1	0	0, 0	-1, 0	-2, 0
	1	0, -1	1, 1	0, 1
	2	2, 2	2, 2	2, 2

Then in the reduced game there are 3 NE

1. Player 1 plays 2 and player 2 plays 0
2. Player 1 plays 2 and player 2 plays 1

3. Player 1 plays 2 and player 2 plays 2

In the whole game there are 3 subgame perfect Nash equilibria:

1. Player 1 plays 2 and player 2 plays (0, 2)
2. Player 1 plays 2 and player 2 plays (1, 2)
3. Player 1 plays 2 and player 2 plays (2, 2)

Ex. 4 An individual want to sell a car at a price no lower than £ 1.000. Two buyers, 1 and 2, simultaneously send to the car's seller their offers. Car's seller chooses to sell the car to the buyer that sent the best offer. If the two offers are equal, the car's seller sells the car to buyer 1

Find all Subgame perfect Nash equilibria assuming that buyers can send only three offers: 1000, 1100, 1200.

Solution

There is only one subgame, then all Nash equilibria are subgame perfect.

Let B1 be the buyer 1's evaluation of the car

Let B2 be the buyer 2's evaluation of the car

c)

B1 \ B2	1000	1100	1200
1000	$V_1 - 1000$, 0	0 , $V_2 - 1100$	0 , $V_2 - 1200$
1100	$V_1 - 1100$, 0	$V_1 - 1100$, 0	0 , $V_2 - 1200$
1200	$V_1 - 1200$, 0	$V_1 - 1200$, 0	$V_1 - 1200$, 0

NE {(1200), (1200)}