Problem set 6

1) Three oligopolists operate in a market with inverse demand function given by $P(Q) = a - Q$ where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 > 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3. Find the subgame perfect outcome.

Solution

The problem of firm 3 is

$$max_{q_3}\ (a - q_1 - q_2 - q_3)q_3 - cq_3$$

The FOC is:

$$(a - q_1 - q_2 - 2q_3) - c = 0$$

The reaction function of firm 3 is:

$$q_3 = \frac{(a - q_1 - q_2 - c)}{2}$$

Similarly the reaction function (best responses) of firm 2 is:

$$q_2 = \frac{(a - q_1 - q_3 - c)}{2}$$

Using symmetry ($q_2 = q_3$) we find that the Nash equilibrium of the simultaneous game between firms 2 and 3 is:

$$q_2 = q_3 = \frac{(a - q_1 - c)}{3}$$

We go to find the optimal behaviour of firm 1.

$$max_{q_1}\ (a - q_1 - q_2 - q_3)q_1 - cq_1$$

But given that Firm 1 anticipates the behaviour of firms 2 and 3 its problem is:

$$max_{q_1}\ \frac{a - q_1 - c}{3} q_1$$

The FOC is

$$\frac{a - 2q_1 - c}{3} = 0$$

Then the optimal choice for firm 1 is:

$$q_1 = \frac{a - c}{2}$$

Replacing q_1 in the solution of the subgame between firms 2 and 3 we have

$$q_2 = q_3 = \frac{a-c}{6}$$

The backward induction outcome is:

$q_1 = \frac{a-c}{2}$ $q_2 = q_3 = \frac{a-c}{6}$

2) Consider the following normal form game where Player 1 chooses the row (either T or B), Player 2 chooses the column (either r or l), Player 3 chooses the table (either R or L)

Player 3

		L (Player 2)		R (Player 2)	
		l	r	l	r
Player 1	T	1, 1, 1	0, 0, 0	0, 0, 0	0, 0, 0
	B	0, 0, 0	0, 0, 0	0, 0, 0	4, 4, 4

a) find all Nash equilibria in pure strategies
b) assume that player 1 moves first, then player 2 and finally player 3; every player, before to play, observes the choices of the predecessors.
   a. Represent the game using the extensive form
   b. Find all subgame perfect Nash equilibria
c) Assume that player 3 is not able to see the choice of player 2
   a. Represent the game using the extensive form
   b. Find all subgame perfect Nash equilibria

Solution

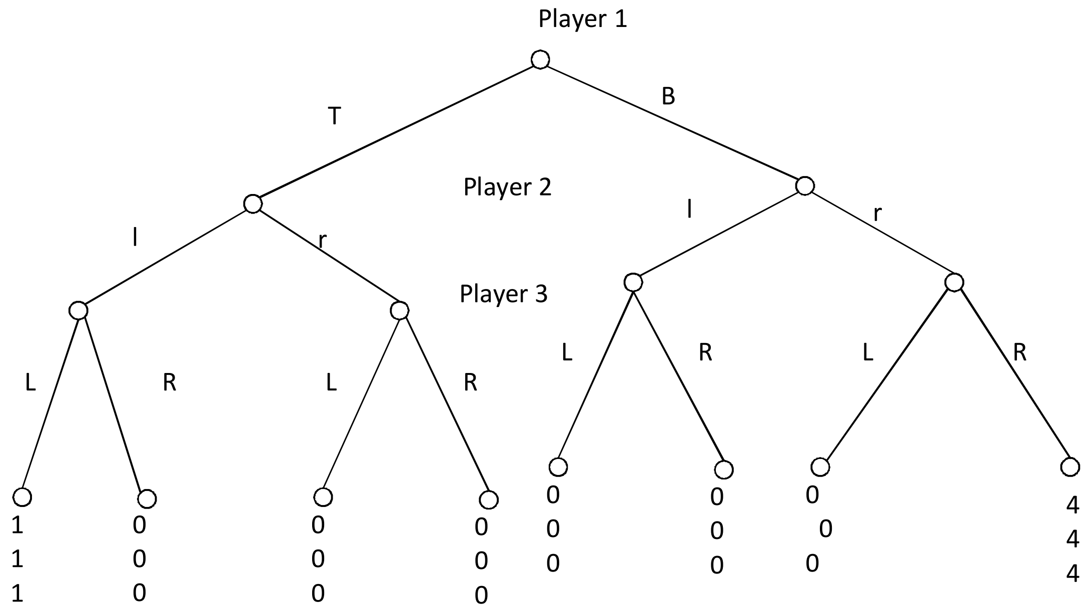
a)

Player 3

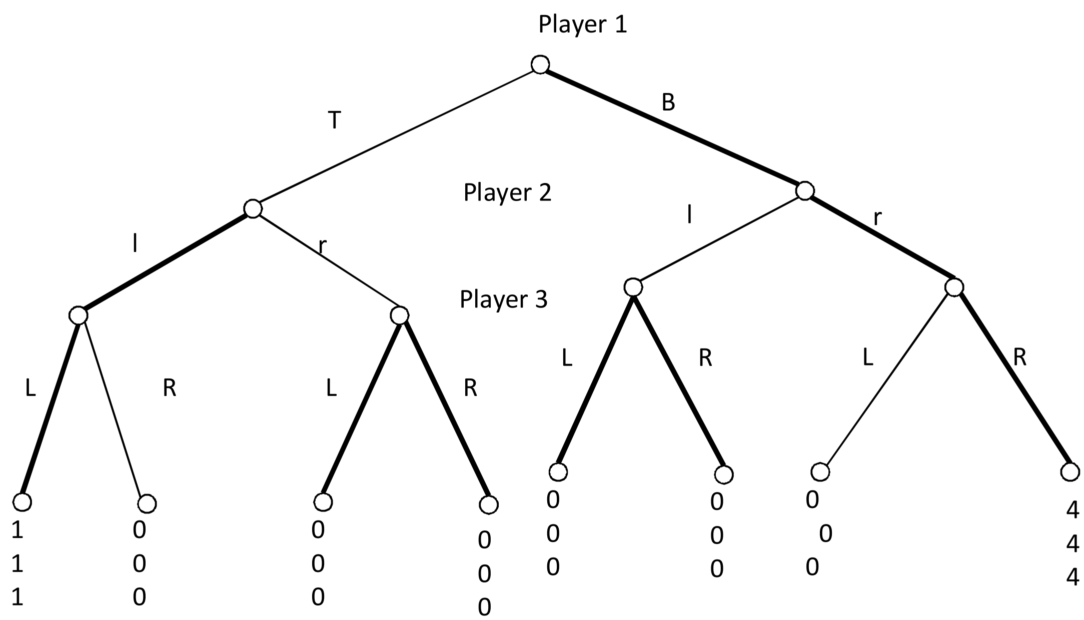
		L (Player 2)		R (Player 2)	
		l	r	l	r
Player 1	T	1, 1, 1	0, 0, 0	0, 0, 0	0, 0, 0
	B	0, 0, 0	0, 0, 0	0, 0, 0	4, 4, 4

Two Nash equilibria: (T, l , L) (B, r, R)

b) Extensive form representation



We use backward induction (in bold the best responses)



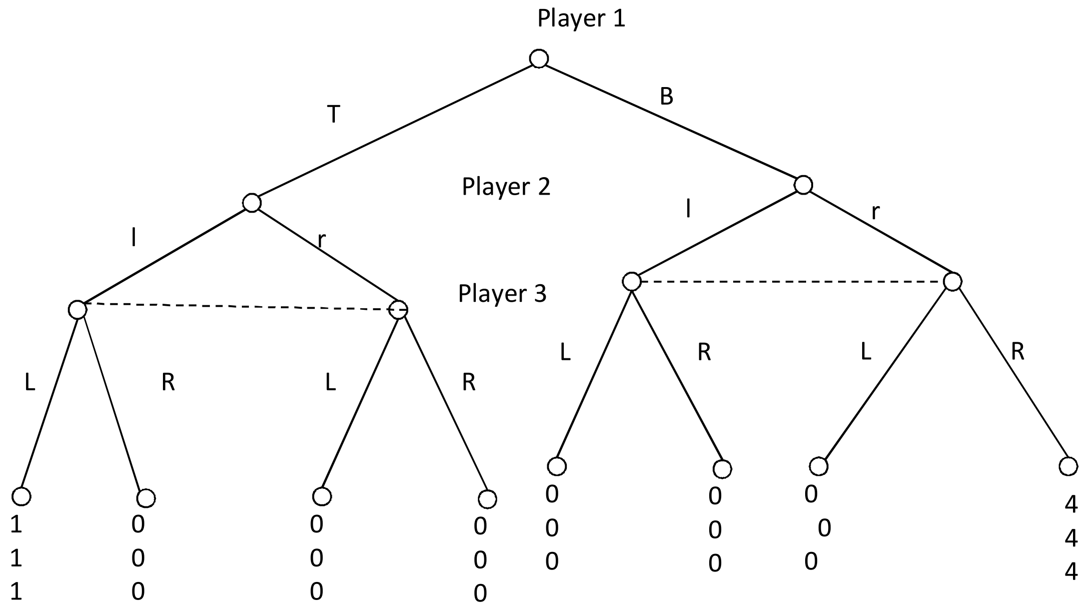
Backward induction outcome: Player 1 plays B, Player 2 plays r, Player 3 plays R

Subgame perfect Nash equilibria

i. {(B), (l, r), (L, L, L, R)}

ii. {(B), (l, r), (L, L, R, R)}
iii. {(B), (l, r), (L, R, L, R)}
iv. {(B), (l, r), (L, R, R, R)}

d) Extensive form representation



There are 3 subgames: the whole game, the game between players 2 and 3 after T, the game between players 2 and 3 after B

After T the subgame is:

Player 3

Player 2		L	R
	l	1, 1	0, 0
	r	0, 0	0, 0

Two Nash equilibria: {l, L} and {r, R}

After L the subgame is:

Player 3

Player 2		L	R
	l	0, 0	0, 0
	r	0, 0	4, 4

Two Nash equilibria: {l, L} and {r, R}

We have to look for the best choices of player 1 for each possible combination of Nash equilibria in the two subgames between players 2 and 3

1) {(T), (l, l), (L, L)}
2) {(L), (l, r), (L, R)}
3) {(T), (r, l), (R, L)}
4) {(L), (r, l), (R, L)}
5) {(L), (r, r), (R, R)}

3) Three periods sequential bargaining. Two players, 1 and 2, are bargaining over \$1 using the following bargaining procedure (alternating offers):
Period 1: Player 1 proposes to take a share s1 of the dollar, leaving 1 – s1 for player 2; Player 2 either accepts (game ends) or rejects (Play goes to period 2)
Period 2: Player 2 proposes a share s2 of the dollar for player 1, leaving 1 – s2 for player 2; Player 1 either accepts (game ends) or rejects (Play goes to period 3)
Period 3: Player 1 receives a share s of the dollar, player 2 receives 1 – s.
Players discount future payoffs by factor δ per period, $0 < \delta < 1$.
Find the backward induction outcome and describe the subgame perfect Nash equilibrium

- The problem of player 1 in period 2 is a choice between
  - to have s2 immediately or
  - s one period later.

  The best response of Player 1 is to accept s2

  if $s2 \geq \delta s$, otherwise reject ($s2 < \delta s$)

- The problem of Player 2 in period 2 is a choice between:
  - to offer $s2 = \delta s$ (player 1 accepts) and receive immediately $1 - \delta s$ or
  - to offer less (player 1 rejects) and receive $1 - s$ one period later

  The best response of Player 2 is to propose

  $s2 = \delta s$, because $1 - \delta s > \delta (1 - s)$

- The problem of player 2 in period 1 is a choice between:
  - To accept s1 and receive $1 - s1$ immediately
  - To reject and receive $(1 - \delta s)$ one period later

  The best response of Player 2 in period 1 is to accept s1 if and only if $1 - s1 \geq \delta(1 - \delta s)$,

  i.e. $s1 \leq 1 - \delta(1 - \delta s)$

- The problem of Player 1 in period 1 is a choice between:
  - To offer $s1 = 1 - \delta(1 - \delta s)$ (player 2 accepts) and receive $1 - \delta(1 - \delta s)$ immediately
  - To offer less (player 2 rejects) and receive δs one period later

  The best response of Player 1 in period 1 is to propose $s1 = 1 - \delta(1 - \delta s)$ because

  $1 - \delta(1 - \delta s) > \delta^2 s$

4) Tariffs and imperfect international competition. There are two identical countries denoted by i = 1, 2. One homogeneous good is produced in each country by a firm, firm *i* in country *i.* A share h_i of this product is sold in the home market and a share e_i is exported in the other country. Governments choose tariffs, i.e. a tax on the import. Government of country *i* chooses tariff t_i

   In country i the inverse demand function is $P_i(Q_i) = a - Q_i$ where $Q_i = h_i + e_j$.
   The firm's payoff (profits) is $\pi_i = [a - h_i - e_j]h_i + [a - h_j - e_i]e_i - c[h_i + e_i] - t_j\, e_i$ where c>0 is the marginal cost. The government's payoff is $W_i = 0.5\, Q_i^2 + \pi_i + t_i\, e_j$
   Timing: Governments simultaneously choose tariffs (t_1,t_2); Firms observe (t_1,t_2) and simultaneously choose quantities (h_1 , e_1) (h_2 , e_2) .
   Find the backward induction outcome and describe the subgame perfect Nash equilibrium

   (Hint: suppose that governments have chosen tariffs (t_1 , t_2) and find the optimal behaviour of firms as function of (t_1 , t_2). Assume that governments correctly predict the optimal behaviour of firms for each possible combination of (t_1 , t_2) and find the optimal tariff rates)

We suppose that governments have chosen tariffs (t_1 , t_2) and we find the optimal behaviour of firms as function of (t_1 , t_2).

Max $_{(h1\,,\, e1)}\, \pi_1$
where
$\pi_1 = [a - h_1 - e_2]h_1 + [a - h_2 - e_1]e_1 - c[h_1 + e_1] - t_2\, e_1$

Firm 1's FOCs:
$[a - 2h_1 - e_2] - c = 0$
$[a - h_2 - 2e_1] - c - t_2 = 0$
→
$h_1 = (a - e_2 - c) / 2$
$e_1 = (a - h_2 - c - t_2) / 2$
For Firm 2:

Max $_{(h2\,,\, e2)}\, \pi_2$
where
$\pi_2 = [a - h_2 - e_1]h_2 + [a - h_1 - e_2]e_2 - c[h_2 + e_2] - t_1\, e_2$

Firm 2's FOCs:
$[a - 2h_2 - e_1] - c = 0$
$[a - h_1 - 2e_2] - c - t_1 = 0$
→
$h_2 = (a - e_1 - c) / 2$
$e_2 = (a - h_1 - c - t_1) / 2$

We have to solve a system of 4 equations in 4 unknowns:

1. $h_1 = (a - e_2 - c) / 2$
2. $e_1 = (a - h_2 - c - t_2) / 2$

3. $h_2 = (a - e_1 - c) / 2$
4. $e_2 = (a - h_1 - c - t_1) / 2$

Solutions:

1. $h_1^* = (a - c + t_1) / 3$
2. $e_1^* = (a - c - 2t_2) / 3$
3. $h_2^* = (a - c + t_2) / 3$
4. $e_2^* = (a - c - 2t_1) / 3$

We assume that governments correctly predict the optimal behaviour of firms for each possible combination of (t_1, t_2) and we find the optimal tariff rates.

The problem of country 1's government is:
Max $_{(t1)}$ $W_1 = 0.5 (Q_1^*)^2 + \pi_1^* + t_1 e_1^*$
where
$Q_1^* = h_1^* + e_2^* = (a - c + t_1) / 3 + (a - c - 2t_1) / 3$
$= (2a - 2c - t_1) / 3$
$\pi_1^* = [a - h_1^* - e_2^*] h_1^* + [a - h_2^* - e_1^*] e_1^* - c[h_1^* + e_1^*] - t_2 e_1^*$

Using algebra:
$W_1 = (2(a-c) - t_1)^2/18 + (a - c + t_1)^2/9 + (a - c - 2t_2)^2/9 + t_1 (a - c - 2t_1)/3$

Similarly we can write the problem of country 2's government
We compute the governments' FOCs and we find:
$t_1^* = (a - c)/3$ $t_2^* = (a - c)/3$

Then
Firm 1 will produce:
$h_1^* = 4(a - c)/9$ $e_1^* = (a - c)/9$
Firm 2 will produce:
$h_2^* = 4(a - c)/9$ $e_2^* = (a - c)/9$

Backward Induction outcome
$t_1^* = (a - c)/3$ $t_2^* = (a - c)/3$
$h_1^* = 4(a - c)/9$ $e_1^* = (a - c)/9$
$h_2^* = 4(a - c)/9$ $e_2^* = (a - c)/9$

Subgame Perfect Nash Equilibrium (SPNE):
Note:
One info set for governments
infinite number of info set for firms, i.e. each possible combination of t_1 t_2
$t_1^* = (a - c)/3$ $t_2^* = (a - c)/3$
$h_1^* = (a - c + t_1) / 3$
$e_1^* = (a - c - 2t_2) / 3$
$h_2^* = (a - c + t_2) / 3$
$e_2^* = (a - c - 2t_1) / 3$