The HOT BIG BANG

The ***Hot Big Bang*** model, i.e. the standard (cosmological) model, and its time evolution rests on **3 pillars**:

1. **the expansion of the Universe**
2. **the microwave background radiation** at *2.73 K* (*CMB*), which reveals the existence of a phase in the life of the universe during which the there was **thermodynamic equilibrium**
3. The prediction of the abundances of the light elements (D, ^3He, ^4He, ^7Li), in particular helium; this **cosmological nucleosynthesis** requires also that there was an era in which $T \approx 10^9 K$

To these facts it may be added that the predicted **age** for the universe is comparable to the age estimated directly for some types of cosmic objects (globular clusters, …), and that it is possible to give a reasonable theoretical explanation for the **formation of cosmic structures** through their gravitational collapse, starting from the perturbations in the microwave background (CMB).

We also mention the **problems** of **flatness** and **horizon** (+ the **monopoles** problem, see below) which we have already mentioned, and whose solution is not found in the standard model of cosmic evolution, but which are solved through the mechanism of inflation.

The Standard Model of Particle Physics and beyond

We describe here some aspects of Particle Physics which are connected to cosmology and to particular epochs in the evolution of the Universe.

In the Standard Model (*SM*) of particle physiscs, described by Quantum Field Theory (*QFT*), only three interactions are considered: electromagnetism, weak and strong interactions. Gravitation is much weaker and is not considered, at least at the energy scales involved in present experimental projects. But, as we imagine to go back in time, the temperature and the energy of particles increases and new aspects have to be taken into account. As we shall see, cosmology can set useful constraints to Particle Physics, beyond the *SM*.

In *QFT* it is useful to use dimensionless quantities to estimate the strength of these interactions, the dimensionless couplings, like the fine structure constant

$$\alpha_{EM} = \frac{e^2}{\hbar c} \cong \frac{1}{137}$$

for electromagnetism. For weak interactions one can use the Fermi weak coupling constant G_F [$G_F/(\hbar c)^3 = 1.166\times10^{-5}\ GeV^{-2}$] and the dimensionless coupling for a typical hadronic mass, the proton mass m_p, is given by

$$\frac{G_F m_p^2 c^4}{(\hbar c)^3} \approx 1.03\times10^{-5},$$

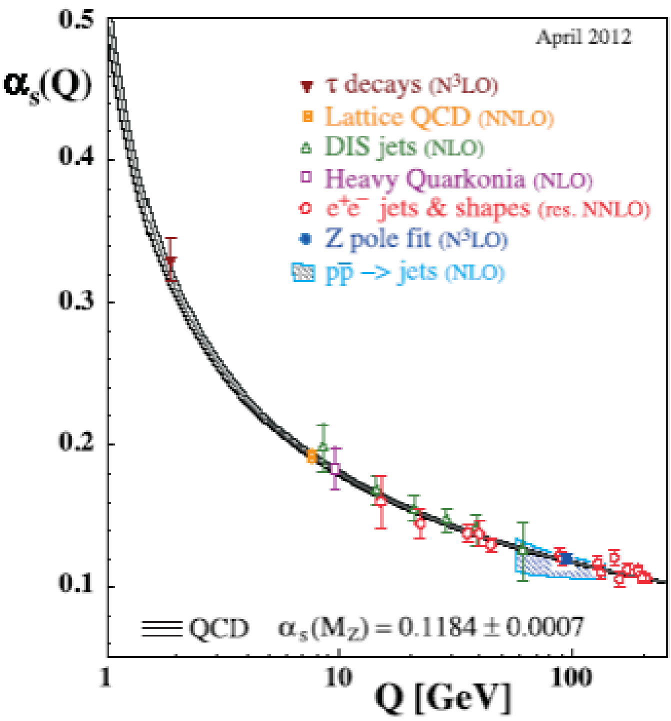
The weakness of weak interactions is due to the improbability of the emission of the very massive bosons W^+, W^-, Z^0. The dimensionless coupling, according to Weinberg-Salam theory, is linked to G_F by the relation

$$\frac{G_F}{(\hbar c)^3} = \frac{\pi}{\sqrt{2}}\frac{\alpha_W}{{M_W}^2 c^4}$$

where $M_W \sim 80\ GeV/c^2$. For strong interactions (Quantum Chromo Dynamics, *QCD*) a dimensionless coupling α_S can be defined.

In *QFT* these couplings are not constant, but are "running", i.e. change their values with the energy scale, linked to a distance scale $r \sim \hbar c/E \sim \hbar/mc$ ($E=mc^2$). For instance, in *QCD*,

$$\alpha_s(E) \sim \frac{0.73}{\ln\left(\frac{E}{\Lambda_{QCD}}\right)} \sim \frac{0.73}{\ln\left(\frac{\hbar c}{\Lambda_{QCD} r}\right)} \qquad \Lambda_{QCD} \equiv \Lambda_C$$



Hence $\alpha_s \to 0$ as $r \to 0$, which gives rise to the so-called "asymptotic freedom," i.e., the fact that quarks and gluons inside a hadron behave like free particles when very close together. On the other hand, α_s apparently diverges as $r \to R_C \equiv \hbar c/\Lambda_C$, where Λ_C is the hadronic energy scale. This divergence simply heralds the breakdown of perturbation theory, of course, but nonetheless it leads us to expect that the strength of the force between quarks will increase if they are pulled apart. As a result, quarks and gluons are "confined" inside hadrons whose size is of order R_C.

The crucial parameter characterizing the strong interaction is the energy scale Λ_C that appears in (1.12). It turns out to be

$$\Lambda_C \simeq 200\ \mathrm{MeV} \tag{1.13}$$

and R_C~10^{-13} *cm* (1 *fm*), the size of hadrons.

The interesting point, as shown in the following figure, is that the couplings tend to converge to one single value at energies on the order of 10^{15} *GeV*, or higher. From this comes the idea that at high energy there is only one interaction, whose symmetry is broken at lower energies, as the electroweak interaction splits into weak interaction and electromagnetism at energies below ~ *100 GeV*. One speaks of Grand Unified Theories (*GUTs*).

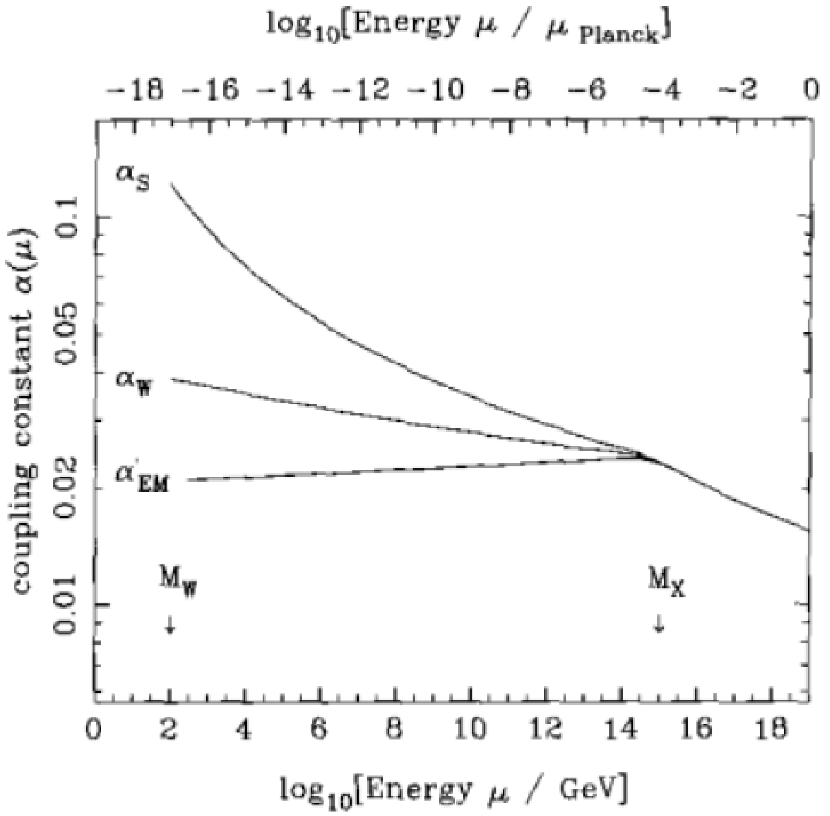
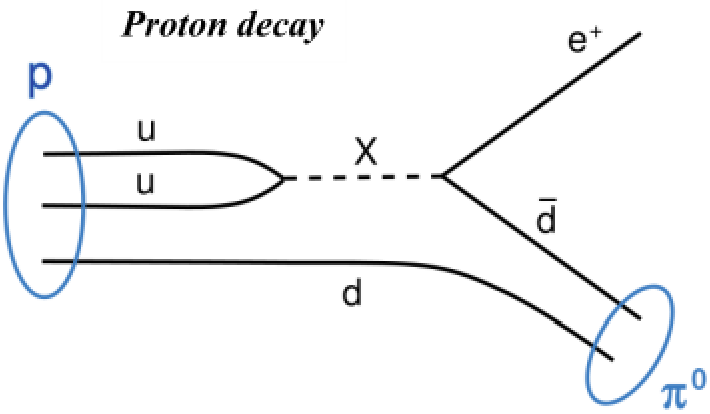


Figure 8.6. The predicted variation of the coupling constants α_i with energy scale (here denoted by μ), according to the $SU(5)$ GUT. All three interactions reach comparable strength at a scale around $M_X \sim 10^{15}$ GeV (adapted from figure 2 of Iliopoulos 1990).

In GUTs there are new gauge bosons *X* which link quarks and leptons and mediate interactions that violate Baryon number *B* and Lepton number *L*. These new interactions must be wery weak since they have eluded detection so far, which means that the *X* bosons must be very massive ($M_X c^2 \sim E_{GUT} \sim 10^{15}$-$10^{16}$ *GeV*). Even if *B* and *L* conservation are violated, in some GUTs *B-L* is conserved.

The *B*-violating interactions would make the proton unstable. Since no proton decay has been observed so far, there are lower limits on proton lifetime $\tau_P > 10^{31}$-10^{32} years.



The Planck era

What about **gravity**? A natural chice for a dimensionless, gravitational coupling is given by

$$\alpha_G = Gm_p^2 / \hbar c \approx 6 \times 10^{-39}$$

which is extremely small. But $m=E/c^2$ and

$$\alpha_G = GE^2 / \hbar c^5 \approx 1 \text{ if } E = E_{Pl} \sim \left(\frac{\hbar c^5}{G} \right)^{1/2} \sim M_{Pl} c^2$$

$E_{Pl} \approx 2 \times 10^{16}$ *erg* $\approx 1.2 \times 10^{19}$*GeV*; $M_{Pl} \approx 2 \times 10^{-5}$*g*. From the relations $\Delta E \times \Delta t \sim \hbar$ and $\Delta t \sim l/c$, **Planck Energy** E_{Pl} corrisponds to a scale (**Planck lenghth**)

$$l_{Pl} \sim \frac{\hbar c}{E_{Pl}} \sim \left(\frac{\hbar G}{c^3} \right)^{1/2} \sim 1.6 \times 10^{-33}\ cm$$

and to a **Planck time**

$$t_{Pl} \sim \frac{l_{Pl}}{c} \sim \left(\frac{\hbar G}{c^5} \right)^{1/2} \sim 5 \times 10^{-44}\ s$$

So at energies of the order or above E_{Pl} gravitation becomes strong, and cannot be neglected in comparison to the other interactions. We need to link *QFT* and *GR*, but such a theory is not available at the moment (String theory could be such a theory). This means that all our extrapolations of the known and experimentally tested Physics have to stop at the Planck scale.

At E_{Pl} the age of the Universe was $t \sim t_{Pl}$, the particle horizon was $\sim l_{Pl}$, the density was

$$\rho_{Pl} \sim \frac{1}{Gt_{Pl}^2} \sim \frac{c^5}{\hbar G^2} \sim 5\times 10^{93}\ g\, cm^{-3}$$

and the mass within the horizon was $M_H \sim \rho_{Pl}\, l_{Pl}^{\ 3} \sim M_{Pl}$.

Moreover, E_{Pl} , l_{Pl} and t_{Pl} are the only possible results if one combines $\hbar$ (Quantum Mechanics), c (Special Relativity) and G (Gravitation) to obtain an energy-mass, a length and a time, and they are the most *natura*l choice.

SUPERSYMMETRY (SUSY)

Supersymmetry (SUSY) is a generalizetion of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa. In particular, it is possible that supersymmetry will ultimately explain the origin of the large hierarchy of energy scales from the W and Z masses to the GUTs and Planck scales.

If supersymmetry were an exact symmetry of nature, then particles and their superpartners (which differ in spin by half a unit) would be degenerate in mass. Since superpartners have not (yet) been observed, supersymmetry must be a broken symmetry. Nevertheless, the stability of the gauge hierarchy can still be maintained if the supersymmetry breaking is soft, and the corresponding supersymmetry-breaking mass parameters are no larger than a few TeV.

In the Minimal Supersymmetric extension of the Standard Model (MSSM) *B-L* is conserved. As a consequence of $B-L$ invariance, the MSSM possesses a multiplicative ***R-parity*** invariance, where $R = (-1)^{3(B-L)+2S}$ for a particle of spin S. Note that this implies that all the ordinary Standard Model particles have even R parity, whereas the corresponding supersymmetric partners have odd R parity[1].

The conservation of R parity in scattering and decay processes has a crucial impact

[1] In the SM: for leptons L=1, B=0, S=1/2; for quarks L=0, B=1/3, S=1/2; for bosons B=L=0 and S is an integer. So R turns out to be always +1. For the superpartners B and L are the same, but S=0 for fermionic partners and S=1 for bosonic partners, so R is always -1.

on supersymmetric phenomenology. For example, starting from an initial state involving ordinary (R-even) particles, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay into lighter states. However, R-parity invariance also implies that the lightest supersymmetric particle (LSP) is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle. In order to be consistent with cosmological constraints, a stable LSP is almost certainly electrically and color neutral. Consequently, the LSP in an R-parity-conserving theory is weakly interacting with ordinary matter, i.e., it behaves like a stable heavy neutrino and will escape collider detectors without being directly observed. So **the LSP is a promising candidate for dark matter**.

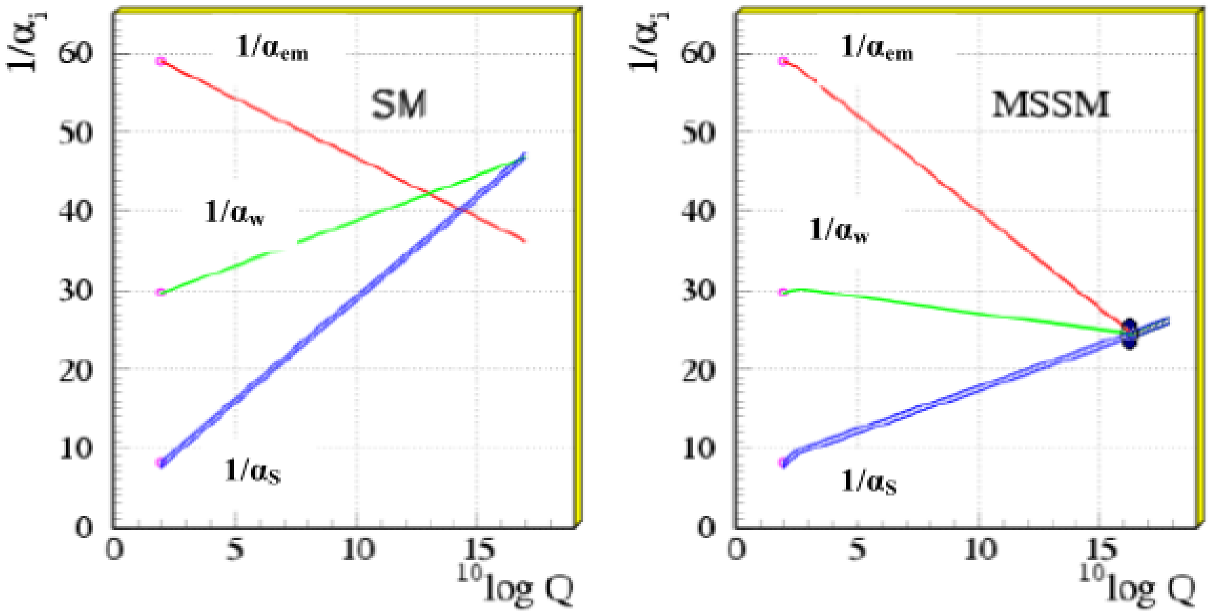


Figure 15.1: Gauge coupling unification in non-SUSY GUTs on the left vs. SUSY GUTs on the right using the LEP data as of 1991. Note, the difference in the running for SUSY is the inclusion of supersymmetric partners of standard model particles at scales of order a TeV (Fig. taken from Ref. 24). Given the present accurate measurements of the three low energy couplings, in particular $\alpha_s(M_Z)$, GUT scale threshold corrections are now needed to precisely fit the low energy data. The dark blob in the plot on the right represents these model dependent corrections.

Axions

In *QCD* the vacuum is a superposition of degenerate states. This introduces a new arbitrary parameter Θ in the theory which leads to an additional term in the *QCD* Lagrangian. However, the existence of this term violates CP, T and P and leads to a neutron electric dipole moment of $d_n/e \sim 5\times10^{-16}\ \Theta\ cm$. Observations give an upper limit $d_n/e \sim 10^{-25}\ cm$, so $\Theta \le 10^{-10}$. Why is Θ so small? This is the ***strong CP problem*** of *QCD*.

In 1977 Peccei and Quinn showed that Θ could be driven to zero by introducing in the Lagrangian a new symmetry which is spontaneously broken at an energy scale f_{PQ}. This induces the existence of a new boson, the ***axion***, which is not massless, but has a mass of the order

$$m_A c^2 \sim 0.6\ \frac{10^7\ GeV}{f_{PQ}(GeV)}\, eV$$

In their original paper Peccei and Quinn assumed that f_{PQ} was on the order of the vacuum expectation value $\boldsymbol{v}$ of the Electroweak phase transition ($\boldsymbol{v} \sim 250\ GeV$). In this case m_A would be $\sim 100\ keV$, excluded by experiments. But the value of f_{PQ} can be anywhere between $250\ GeV$ and $10^{19}\ GeV$, and m_A spans a huge range of values.

Limits on m_A are given also by stellar evolution. Detection techniques to find out evidence of the existence of axions are based on the conversion of axions into microwave photons in the presence of a very strong magnetic field. The contribution of axions to the **dark matter** is given, if they exist, by

$$\Omega_A h^2 \approx 0.3 \left(\frac{f_{PQ}(GeV)}{10^{12} GeV}\right)^{7/6}$$

which means that, in order to represent a major contribution to dark matter, the mass of the axions must be $m_A \approx 10\ \mu eV$.

Thermodynamics of the Early Universe

Going back in time temperature T and density ρ grow and it is expected that the particles reach the thermodynamic equilibrium through rapid interactions. The rate of interaction $\Gamma = n\sigma v$ (n = number density, σ = cross section, v = particle velocity) grows more rapidly, with the temperature, than the rate of expansion H, so $\Gamma \gg H$ at high T. This means that, with regard to the interactions, the expansion is quasi-static and there is enough time for the universe to continuously restore thermodynamic equilibrium.

This allows a very simple treatment of the distribution functions of the particles. In thermodynamic equilibrium, the number density n of particles of a given species, with momentum between P and $P + dP$ is

$$dn = \frac{g}{2\pi^2\hbar^3}\frac{P^2 dP}{e^{\frac{E-\mu}{kT}} \pm 1}$$

where $E^2=P^2c^2+m^2c^4$, μ is the chemical potential, and g is the spin-degeneracy factor , which counts the number of degrees of freedom, taking into account the spins and colors of particles (for spin states $g=1$ if $m=0, s=0$; $g=2$ if $m=0$, $s\neq 0$; $g=2s+1$ if $m\neq 0$; $g_\gamma=2$, $g_e=2$, *but* $g_\nu=1$ since neutrinos are only *left handed*; for each quark flavour $g=6$, a factor *2* for the spin and a factor *3* for the colors) The + or - sign corresponds to fermions (f) and bosons (b).

For photons μ is naturally zero since they have a planckian distribution with temperature $T_\gamma(t)$; if a species A is in thermal equilibrium with photons ($\Gamma_{A\gamma} \gg H$), $T_A=T_\gamma$ and the same holds for all species in equilibrium. So we use the photon temperature as reference: $T_\gamma \equiv T_{\text{Universe}}=T$.

In thermodynamic equilibrium the number density n_i and the energy density $\rho_i c^2$ of "i" particles are given by

$$n_i = \int_0^\infty \frac{\mathrm{d}n}{\mathrm{d}P}\mathrm{d}P = \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{P^2}{e^{\frac{E-\mu}{kT}} \pm 1}\mathrm{d}P$$

$$\rho_i c^2 = \int_0^\infty E\frac{\mathrm{d}n}{\mathrm{d}P}\mathrm{d}P = \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{EP^2}{e^{\frac{E-\mu}{kT}} \pm 1}\mathrm{d}P$$

For the pressure p, from $p = \frac{n}{3} < \vec{P}\cdot\vec{v} >$, and

$$P = \gamma m\vec{v} \Longrightarrow \vec{P}\cdot\vec{v} = \vec{P}\cdot\vec{P}/\gamma m = P^2c^2/\gamma mc^2 = P^2c^2/E$$

$$p_i = \int_0^\infty \frac{1}{3}\left(\vec{P}\cdot\vec{v}\right)\frac{\mathrm{d}n}{\mathrm{d}P}\mathrm{d}P = \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{1}{3}\frac{P^2c^2}{E}\frac{P^2}{e^{\frac{E-\mu}{kT}} \pm 1}\mathrm{d}P$$

$$p_i = \frac{g_i c^2}{6\pi^2\hbar^3}\int_0^\infty \frac{1}{E}\frac{P^4}{e^{\frac{E-\mu}{kT}} \pm 1}\mathrm{d}P$$

In the Early Universe, for various reasons, the chemical potentials are negligible (fermions are non-degenerate, bosons do not form a Bose condensate). The main argument comes from the fact that the net chemical potential in the early universe can be set to zero, because the asymmetry between particles and antiparticles is very small. From chemical thermodynamics, for a reaction $1 + 2 \leftrightarrow 3 + 4$, the relation among chemical potentials is $\mu_1 + \mu_2 = \mu_3 + \mu_4$. From a reaction like ($\gamma+\gamma \leftrightarrow A + \bar{A}$), since $\mu_\gamma=0$, then $\mu_A = -\mu_{\bar{A}}$. So, for number densities, $n_A - n_{\bar{A}} \neq 0$ and this gives a nonzero value for the quantum numbers (electric charge, baryon number, color charge, …) associated to particle *A*. But electric charge, color charge, …, of the Universe seem to be consistent with zero; moreover, the number density of baryons is much smaller than that of photons: $(n_B - n_{\bar{B}})/n_\gamma \leq 10^{-9}$. So, in the Early Universe, it is usually assumed that $n_B \cong n_{\bar{B}}$ and chemical potentials are set to zero.

The above relations for number density, energy density, and pressure are general. It is easy to evaluate these integrals in two extreme cases: ***ultrarelativistic, non-degenerate particles*** and ***non-relativistic particles***.

- **Ultrarelativistic case: *$kT \gg m_i c^2$; $E^2 \sim P^2c^2$, $|\mu| \ll kT$*** (use *$Pc/kT=u$*)

$$n_i \cong \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty \frac{P^2dP}{e^{Pc/kT}\pm 1} \cong \frac{g_i}{2\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \underbrace{\int_0^\infty \frac{u^2du}{e^u \pm 1}}$$

ζ(x): Riemann Zeta function
ζ(3)=1.202
ζ(4)=π⁴/90=1.082

(+)fermions =3/2·ζ(3)

(-)bosons =2·ζ(3)

For photons ($g_\gamma=2$)

$$n_\gamma = \frac{2}{\pi^2}\zeta(3)\left(\frac{kT}{\hbar c}\right)^3$$

and for bosons and fermions:

$$n_{b,i} = \frac{g_i}{\pi^2}\zeta(3)\left(\frac{kT_i}{\hbar c}\right)^3 = \frac{g_i}{2}\left(\frac{T_i}{T}\right)^3 n_\gamma(T)$$

$$n_{f,i} = \frac{3}{4}\cdot\frac{g_i}{\pi^2}\zeta(3)\left(\frac{kT_i}{\hbar c}\right)^3 = \frac{3}{8}\cdot g_i\left(\frac{T_i}{T}\right)^3 n_\gamma(T)$$

For the energy density

$$\rho_i c^2 \cong \frac{g_i c}{2\pi^2\hbar^3}\int_0^\infty \frac{P^3dP}{e^{Pc/kT}\pm 1} \cong \frac{g_i (kT)^4}{2\pi^2\hbar^3c^3} \underbrace{\int_0^\infty \frac{u^3du}{e^u \pm 1}}$$

(+)fermions =7/8·6·ζ(4)

(-)bosons =6·ζ(4)

Stefan-Boltzmann constant:

$$a_B = \frac{\pi^2 k^4}{15\hbar^3c^3} = 7.5659\times 10^{-15}\, erg\, cm^{-3} K^{-4}$$

Remember that $\zeta(4)= \pi^4/90$ and so, using *Stefan-Boltzmann* constant,

$$\rho_{f,i}\, c^2 = \frac{7}{8} 6 \frac{\pi^4}{90} \frac{g_i k^4 T^4}{2\pi^2 \hbar^3 c^3} = \frac{7}{16} \frac{\pi^2 k^4}{15 \hbar^3 c^3} g_i T_i^4 = \frac{7}{16} a_B\, g_i T_i^4$$

$$\rho_{b,i}\, c^2 = \frac{1}{2} g_i a_B T_i^4$$

The mean energy per particle is $< E_i >= \rho_i c^2/n_i$:

$$\langle E \rangle_b = \frac{\pi^4}{30\zeta(3)} kT = 2.70\, kT \qquad \langle E \rangle_f = \frac{7\pi^4}{180\zeta(3)} kT = 3.15\, kT$$

which can be approximated by $< E_i > \approx 3kT$.

For the pressure p it is easy to realize that, if $E \sim Pc$,

$$p_i = 1/3\ \rho_i\, c^2.$$

- **Non relativistic case: $kT << m_i c^2$ $(Pc << m_i c^2)$;**

$E^2 = P^2c^2 + m_i^2 c^4 = m_i^2 c^4 [1 + P^2/m_i^2 c^2] \Rightarrow E \sim m_i c^2 [1 + P^2/2m_i^2 c^2]$; $E \sim m_i c^2 + P^2/2m$

$e^{E/kT} \gg 1 \Rightarrow$ *no difference between fermions and bosons*. (use $P/\sqrt{mkT} \equiv u$)

$$n_i \cong \frac{g_i}{2\pi^2 \hbar^3} \int_0^\infty P^2 e^{-\frac{m_i c^2 - \mu_i}{kT}} e^{-\frac{P^2}{2m_i kT}} dP \cong$$

$$\cong \frac{g_i}{2\pi^2 \hbar^3} (m_i kT)^{3/2} e^{-\frac{m_i c^2 - \mu_i}{kT}} \underbrace{\int_0^\infty u^2 e^{-u^2/2} du}_{= \frac{\Gamma(3/2)}{2\left(\frac{1}{2}\right)^{3/2}} = \frac{\sqrt{2\pi}}{2}}$$

and we have:

$$n_i \cong \frac{g_i}{\hbar^3} \left(\frac{m_i kT}{2\pi} \right)^{3/2} e^{-\frac{m_i c^2 - \mu_i}{kT}}$$

Note the ***strong exponential cut***, since $kT << m_i c^2$. This cut is due to ***annihilation*** of particles with their antiparticles. When particles are ultrarelativistic (***$kT \gg m_i c^2$***), annihilation is balanced by pair production, but for ***$kT << m_i c^2$*** pair production is ineffective and annihilation prevails.

In a similar way we get

$$\rho_i c^2 \cong n_i m_i c^2 + \tfrac{3}{2} n_i kT \sim n_i m_i c^2$$
$$p_i \cong n_i kT << \rho_i c^2$$

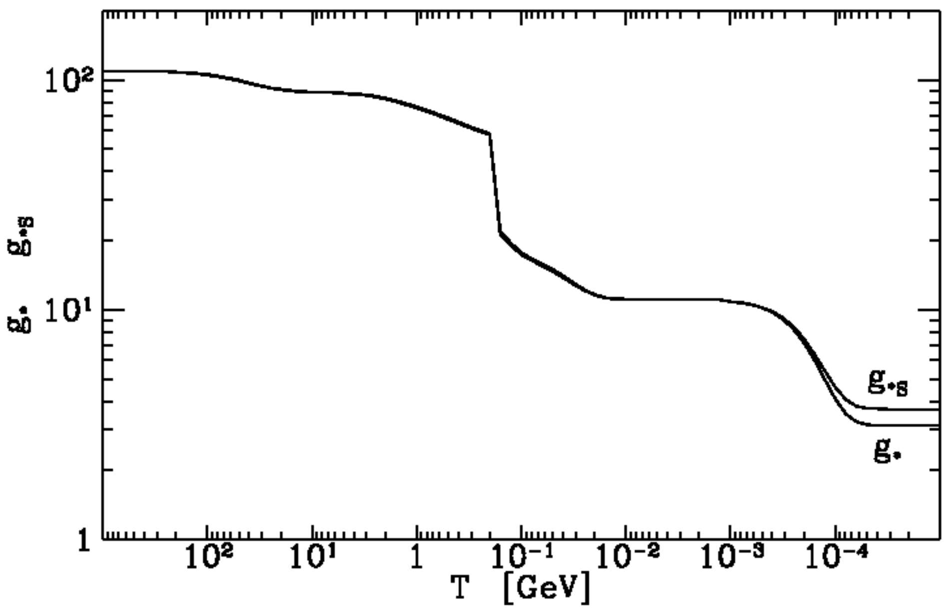
The contribution to the ***total energy density*** ρc^2 (as well as the total pressure p) of the non-relativistic species is negligible (due to the exponential cut), so ρc^2 can be well approximated only by the contribution of relativistic species

$$\rho c^2 \cong \rho_R c^2 = \frac{1}{2} a_B T^4 \underbrace{\left[\sum_{i=bosons,rel} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=fermions,rel} g_i \left(\frac{T_i}{T}\right)^4 \right]}_{g_*(T)} = \frac{1}{2} a_B \, g_*(T) T^4$$

where $g_*(T)$ represents the total, ***effective number of degrees of freedom*** of relativistic) particles.

For $kT<<1\ MeV$ the only relativistic species are photons and the three neutrinos (if m_ν is negligible); since (see the proof below) $T_\nu=(4/11)^{1/3}T_\gamma$, $g_*=2+7/8\cdot 2\cdot 3\cdot (4/11)^{4/3}=3.36$ ($2=\nu+\bar{\nu}$, $3=N_\nu$). For $1\ MeV\leq kT\leq 100\ MeV$ we add e^+ and e^- and $T_\nu=T_\gamma$, $g_*=43/4=10.75$. Above $300\ GeV$ all particles included in the Standard Model are relativistic, and $g_*=474/4=106.75$. At energies higher than $E_{EW} \sim M_W c^2 \sim 100\ GeV$ (Electroweak breacking) g_* depends on the adopted theory (for instance, in the minimal model of *GUT*, *SU(5)*, for $kT > E_{GUT} \sim 10^{16} GeV$, $g_* \sim 160$).

In supersymmetric models, at each particle corresponds a supersymmetric partner, and $g*$ approximately doubles. If some *sparticles* have mass smaller than the Higgs boson, then there may be some changes in the following graph representing the behaviour of $g*$ as a function of temperature for the Standard Model of particle physics.



Time scale: In the Radiation Dominated (*RD*) era the Universe is well approximated by an *EdS* model, so $\rho=3/(32\pi G\, t^2)$, $E \sim 3kT$, $\rho = \rho_R$ and

$$t^2 \approx \frac{3c^2}{32\,\pi\, G\, \rho_R c^2} \approx \frac{45 c^5 \hbar^3}{16\,\pi^3\, G\, g_*} \cdot \frac{3^4}{(3kT)^4}$$

$$t(\mathrm{sec}) \approx \frac{2.2\times10^{-5}}{g_*^{1/2} E_{GeV}^2} \approx \frac{2.4}{g_*^{1/2} \left(kT\right)_{MeV}^2}$$

Thermodynamic equilibrium (TE): The Universe turns out to be in TE for

$$\boldsymbol{1\ MeV \leq kT \leq 10^{-3}\, M_{Pl}\, c^2 \sim 10^{16} GeV\ (\sim E_{GUT})}$$

The upper limit is set by interactions mediated, at very high energy, by ultrarelativistic gauge bosons. The lower limit corresponds to interactions mediated by a massive gauge boson, like W^+, W^- and Z^0 below the scale of electroweak symmetry breaking (~ *100 GeV*). At a mean particle energy of ~ *1 MeV* these interactions are no more effective, are "***frozen out***".

Neutrinos do no interact any more with matter and radiation: they ***decouple*** when the **mean energy** per particle is **about *1 MeV***.

Moreover, the mean free path of the particles is much greater than their average mutual distance $\Rightarrow$ *perfect gas*.

Entropy

In thermodynamic equilibrium, the entropy S in a comoving volume element is preserved during the expansion (entropy can increase if processes like particle decay or phase transitions happen under condition which do not preserve thermodynamic equilibrium).

Entropy S and the first law of thermodynamics are related by (we use $d(pV)=pdV+Vdp$)

$$dQ = T\, dS = dU + dL \longrightarrow T\, dS = d(\rho c^2 V) + pdV = d[(\rho c^2 + p)V] - Vdp$$

If we consider $S=S(V,T)$

$$dS(V,T) = \frac{1}{T} d[\varrho(T) c^2 V] + \frac{p}{T} dV$$

$$dS(V,T) = \frac{V}{T}\frac{d[\varrho(T)c^2]}{dT} dT + \frac{\varrho(T)c^2 + p(T)}{T} dV$$

Since entropy is a function of state, its differential form is exact and the integrability condition $\frac{\partial^2 S}{\partial V \partial T} = \frac{\partial^2 S}{\partial T \partial V}$ gives

$$\frac{\partial}{\partial T}\left[\frac{\varrho(T)c^2 + p(T)}{T}\right] = \frac{\partial}{\partial V}\left[\frac{V}{T}\frac{d[\varrho(T)c^2]}{dT}\right]$$

$$\frac{1}{T^2}\left[\left(\frac{d\,\rho c^2}{dT} + \frac{dp}{dT}\right)\cdot T - (\rho c^2 + p)\right] = \frac{1}{T}\frac{d\,\rho c^2}{dT}$$

$$\frac{dp}{dT} = \frac{\rho c^2 + p}{T} \quad \rightarrow \quad dp = (\rho c^2 + p)\frac{dT}{T}$$

We can use this result in the previous relation

$$T\,dS = d[(\rho c^2 + p)V] - Vdp$$

and we get

$$dS = \frac{1}{T}d[(\rho c^2 + p)V] - V(\rho c^2 + p)\frac{dT}{T^2} = d\left[\frac{(\rho c^2 + p)V}{T} + const.\right].$$

So, up to an additive constant, the entropy S for a comoving volume $V=a^3$ (a is the scale factor) can be written as

$$S = \frac{a^3(\rho c^2 + p)}{T}$$

The **entropy density** s is defined as

$$s \equiv \frac{S}{V} = \frac{\rho c^2 + p}{T}$$

This (due to the exponential cut in number density) is dominated by the contribution of relativistic particles. For each relativistic species $s_i=(\rho_i c^2+1/3\cdot\rho_i c^2)/T=4\rho_i c^2/3T$, and the total contribution becomes, with good approximation

$$s = \frac{2}{3}a_B T^3 \underbrace{\left[\sum_{i=b,rel} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i,f,rel} g_i\left(\frac{T_i}{T}\right)^3\right]}_{g_{*S}(T)} = \frac{2\pi^2 k}{45} g_{*S}(T)\left(\frac{kT}{\hbar c}\right)^3$$

Note that if $T_i \equiv T$ for all relativistic particles, as it is for most of the time in the early Universe, then $\boldsymbol{g_* = g_{*S}}$ (see the figure above).
Also note that s is proportional to n_γ; in fact

$$s = \frac{\pi^4}{45\zeta(3)} k\, g_{*S}(T)\, n_\gamma = 1.8\, k\, g_{*S}(T)\, n_\gamma$$

Today (*$kT \leq 1$ MeV*) $g_{*S}=2+7/8\cdot 2\cdot 3\cdot 4/11=3.909$ and

$$s \cong 7.04 \cdot k\, n_\gamma$$

Above ~ *1 MeV* : $g_* \approx g_{*S}$ (**Note**: g_{*S} depends in general on $T \Rightarrow s$ *an*d n_γ cannot be always considered as proportional!)

Entropy S conservation implies $s \propto a^{-3}$, and also

$$g_{*S} \cdot T^3 \cdot a^3 = \text{constant}$$

while the Universe expands.

The physical size of a comoving volume is $\propto a^3$ and, since $s \propto a^{-3}$, it is also $\propto s^{-1}$. The number N of particles of a species inside a comoving volume (named *comoving number density*), $N \equiv n \cdot a^3$, is also equal (actually, proportional) to n/s, so we also write $\boldsymbol{N_i \equiv n_i/s}$. If particles are neither created nor destroyed, then $\boldsymbol{N_i \equiv n_i/s = const.}$ For *relativistic particles* in *TE* the comoving number density can be written as

$$N_i = F_{bf} \cdot \frac{g_i \zeta(3)}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \cdot \frac{45}{2\pi^2 k\; g_{*S}(T)}\left(\frac{kT}{\hbar c}\right)^{-3} = F_{bf} \cdot \frac{45\zeta(3)}{2\pi^4} \cdot \frac{g_i}{k\; g_{*S}(T)}$$

where F_{bf} is equal to 1 for bosons and to ¾ for fermions.

The baryon number N_B (the difference between baryons b and antibaryons $\bar{b}$) in a comoving volume is

$$N_B = \frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s}$$

As long as the interactions violating baryon number conservation (if they exist!) are very slow, n_B/s is conserved.

However, the ***baryon-photon ratio*** η, a crucial parameter in primordial (or Big Bang) nucleosynthesis,

$$\eta \equiv \frac{n_B}{n_\gamma} = 1.8\, k\; g_{*S}(T) \cdot \frac{n_B}{s}$$

doesn't stay constant since g_{*S} depends on T. But after e^+ add e^- annihilation (at ~ *0.5 MeV*) g_{*S} is constant (=*3.909*), so $\boldsymbol{\eta \approx 7.04\; k\; n_B/s}$ or n_B/s can be indifferently used.

We shall see that primordial nucleosynthesis requires that $\eta \approx 5\times10^{-10}$, so in our Universe there are today about 10^9 photons for each baryon. Also the entropy per baryon, $\boldsymbol{s/n_B=7.04\ k/\eta \approx 7\times10^{10}\ k/\eta_{10}}$, is extremely high ($\eta_{10}\equiv\eta/10^{-10}$)

The fact that $\boldsymbol{S = const.}$ implies

$$\boxed{T \propto g_{*S}^{-1/3} \cdot a^{-1}}$$

If g_{*S} is costant $\boldsymbol{T \propto a^{-1}}$. The $g_{*S}^{-1/3}$ factor enters the game when a species becomes non relativistic, annihilates and disappears (since annihilation is less and less balanced by pair creation): its entropy is transferred to photons and to the other interacting relativistic particles, so T decreases more gently.

If a relativistic particle decuples at time $t=t_D$, when $T=T_D$ and $a=a_D$, it doesn't benefits of the entropy exchange due to the annihilation (at $T<T_D$) of the other species. After decoupling $P\propto 1/a \Rightarrow P=(a_D/a)P_D$ and (if the particle is stable) $n=(a_D/a)^3 n_D$; since $P\propto 1/a$, n will be given by

$$n = \frac{g_i}{2\pi^2\hbar^3}\left(\frac{a_D}{a}\right)^3 \int_0^\infty \frac{P_D^2\, dP_D}{e^{\frac{c}{kT}\cdot\frac{a_D P_D}{a}} \pm 1}$$

which gives the right dependance on a if $T=(a_D/a)T_D$. The distribution function of momenta keeps its shape, but with $T\propto a^{-1}$ instead of $T\propto g_{*S}^{-1/3}a^{-1}$ which holds for particles still coupled. If the particle, for instance a "light" neutrino, becomes eventually non relativistic, the shape of the distribution function of its momentum is preserved, with $T\propto a^{-1}$.

This also explains the reason for *CMB* photons shows a black body spectrum even after the last scattering (at $z_{ls} \approx 1100$), when they decouple from baryons and are no more in thermodynamic equilibrium.

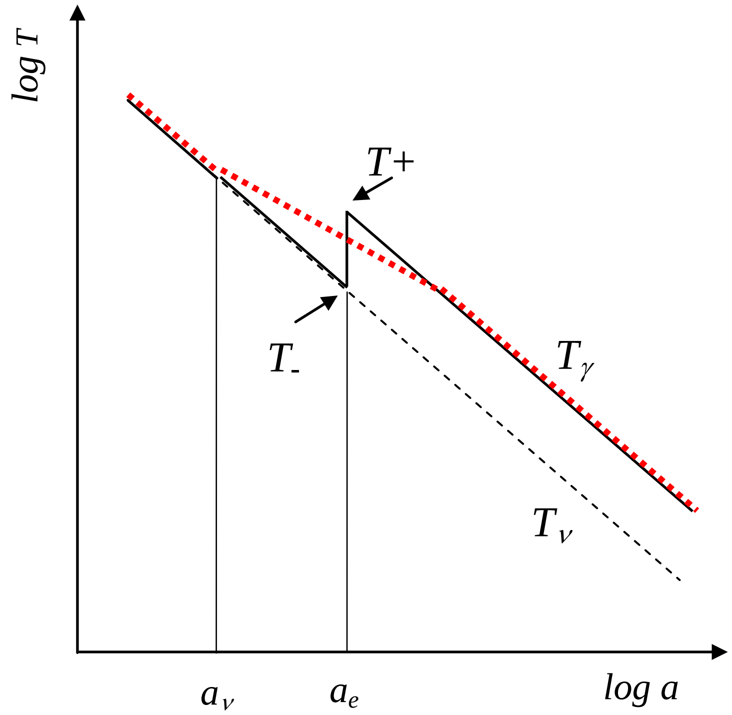
Neutrinos

We have already seen that at $kT\sim E\sim 1Mev$, when $a=a_\nu$, neutrinos (ν) decouple from other species, and so, while before $T_\nu=T_\gamma$, after decoupling $T_\nu=T_\gamma(a_\nu)a_\nu/a$. However, at a slightly lower Energy, at $E\sim 0.5Mev$ $(a=a_e)$, electrons and positrons annihilate and their entropy goes to photons, but not to the decoupled neutrinos. Entropy is conserved ($g_{*S}\, T^3 a^3 = const.$) for still coupled particles (e^+, e^- e γ for $a < a_e$, only γ for $a > a_e$). We denote with a_-, T_- and a_+, T_+ the values just before and immediatleyafter electron-positron; we suppose that annihilation occurs instantaneously and we have[2] ($a_+ \approx a_e \approx a_-$):

[2] We could also add, both on the left hand side and on the right hand side, the contribution of neutrinos, but this contribution is the same immediately before and after annihilation, since neutrinos are decoupled. So we omit their contribution.

$$g_{*S} T^3 a^3 = \overbrace{(\underset{\gamma}{2} + \frac{7}{8}\cdot \underset{e^+ + e^-}{2} \cdot \underset{g_i}{2})\, T_-^{\,3} a_-^{\,3}}^{before} = \overbrace{\underset{\gamma}{2} T_+^{\,3} a_+^{\,3}}^{after}$$

From this relation we get the the ratio (see also the following figure)



$$\frac{T_-}{T_+} = \left(\frac{4}{11}\right)^{1/3} = \frac{T_\nu}{T_\gamma}$$

After a_e both T_ν and T_γ scale as $1/a$, and their ratio stays constant until now. So, if $T_{\gamma 0}=2.73\ K$, $T_{\nu 0}=1.95\ K$.

Actually, the photon temperature does not rise abruptly at $a=a_e$, but decreases more slowly than $1/a$ until the annihilation of e^+ and e^- ends (see the dotted line).

It is now easy to derive the present values of number densities of CMB photons and of cosmological neutrinos.

For today's ***CMB*** the density and the number density are easily derived:

$$\rho_{\gamma 0} = \frac{a_B T_{\gamma 0}^4}{c^2} = 4.67 \cdot 10^{-34} \left(\frac{T_{\gamma 0}}{2.73}\right)^4 g\, cm^{-3}$$

$$\frac{s_0}{k} = \frac{2\pi}{45} g_{*S}(T_{\gamma 0}) \left(\frac{k T_{\gamma 0}}{\hbar c}\right)^3 = 2934 \left(\frac{T_{\gamma 0}}{2.73}\right)^3 cm^{-3}$$

$$n_{\gamma 0} = \frac{2}{\pi^2}\zeta(3)\left(\frac{kT_{\gamma 0}}{\hbar c}\right)^3 = 417\left(\frac{T_{\gamma 0}}{2.73}\right)^3 \ cm^{-3}$$

For each neutrino family, counting ν and $\bar{\nu}$,

$$\frac{n_{\nu+\bar{\nu}}}{n_\gamma} = \frac{3}{4}\cdot\frac{\overset{g_\nu}{1}\cdot\overset{\nu+\bar{\nu}}{2}}{\underset{g_\gamma}{2}}\cdot\left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{3}{11}$$

$$n_{\nu+\bar{\nu},0} = 114\left(\frac{T_{\gamma 0}}{2.73}\right)^3 \ cm^{-3}$$

COSMIC RELICS

The Universe seems to be neutral both from the point of view of electric charge and color charge. So Dark Matter candidates are thought to be indifferent to electromagnetic and strong forces.

It is possible to foresee the cosmological effect produced by weakly interacting massive particles (WIMPs) or, viceversa, to see the constraints posed by cosmological observations on the properties of such particles. Here we assume that these particles interact exactly as neutrinos do, but the term WIMP is also used for much weaker, possible interactions beyond the Standard Model of Particle Physics.

There are two main cases: WIMPs can decouple when they are still relativistic (Hot Dark Matter, HDM, $kT_D \gg m_W c^2$) or when they are non relativistic (Cold Dark Matter, CDM, $kT_D \ll m_W c^2$).

(CR12

$\boxed{\text{HDM}}$ If particles do not decay after decoupling, $n_0 a^3 \propto n/s \propto N \sim$ const.

today ↓ decouple ↓

$$N(T_0) = N(T_D) = \begin{Bmatrix} 3/4 - f \\ 1 - b \end{Bmatrix} \frac{45\,\zeta(3)}{2\pi^4} \frac{g_i}{k g_{*S}(T_D)} = \frac{n_0}{s_0}$$

F_{bf}

remember

$$s = 1.8\,k\,g_{*S}(T)\,n_\gamma \quad ; \quad n_\gamma = \frac{2}{\pi^2}\zeta(3)\left(\frac{kT}{\hbar c}\right)^3$$

$$T_0 \sim 2.728\,\mathrm{K} \qquad g_{*S}(T_0) = 3.909$$

$$\to n_0 = \begin{Bmatrix} 3/4 \\ 1 \end{Bmatrix} \frac{\zeta(3)}{\pi^2} \frac{g_i\, g_{*S}(T_0)}{g_{*S}(T_D)} \cdot \left(\frac{kT_0}{\hbar c}\right)^3 =$$

$$\simeq \begin{Bmatrix} 3/4 \\ 1 \end{Bmatrix} \frac{g_i}{g_{*S}(T_D)} \cdot 813 \left(\frac{T_0}{2.728}\right)^3 \mathrm{cm}^{-3}$$

If fermionic WIMPs:

$$n_0 \sim 610 \frac{g_i}{g_{*S}(T_D)} \mathrm{cm}^{-3}$$

[for neutrinos: $g_i = 2 = 1 \cdot 2$ $(\nu + \bar\nu)$; $g_{*S}(T_D) = 10.75$

$\to n_0 \sim 114\ \mathrm{cm}^{-3}$]

If these WIMPs are non-relativistic today

$$\rho_0 \simeq n_0 m_W \to \Omega_W = \frac{n_0 m_W}{\rho_{0cr}}$$

$$\boxed{\Omega_W h^2 \simeq 0.058 \frac{g_i}{g_{*S}(T_D)} \cdot \left(\frac{m_W}{1\,\mathrm{eV}/c^2}\right)} \quad (*)$$

For 3 $\nu + \bar\nu$ families

$$\boxed{\Omega_\nu h^2 \sim 0.011 \sum_{i=1}^{3} \left(\frac{m_{\nu_i}}{1\,\mathrm{eV}/c^2}\right)}$$

From CMB + Large scale structure | CR | 3

$$\sum \left(\frac{m_{\nu i}}{1\,eV/c^2}\right) \lesssim 0.3 - 0.7$$

- The relation (*) can be applied also to other weakly interacting particles, provided they are relativistic at $T \sim T_D \sim 1$ MeV; for $m_W \gtrsim 1$ MeV/c^2 (*) is no more correct; $\Omega_W h^2$, for $m_W \sim 1$ MeV/c^2, is $\sim 10^4$!
- When HDM particles (ν) decouple, they are relativistic and $\langle E\rangle \sim 3kT_\nu$, $\langle E\rangle \sim \langle p\rangle \cdot c \rightarrow \langle p\rangle \cdot c \sim 3kT_\nu$

Since both $\langle p\rangle$ and $T_\nu \propto \frac{1}{a}$, this relation holds also when particles are no more relativistic and $\langle p\rangle \rightarrow m\langle v\rangle$, so

$$m\langle v\rangle c \sim 3kT_\nu \rightarrow \langle v\rangle \sim \frac{3kT_\nu}{mc} \sim 150 \left(\frac{m}{1eV/c^2}\right)^{-1} km/s$$

This speed is high enough to prevent the formation of structures with escape velocities lower than $\langle v\rangle$, i.e. galaxies. Moreover, if we consider that structures were already formed at $z \sim 6$ (QSOs), $\langle v\rangle$ has to be multiplied by $(1+z) \sim 7$, giving values corresponding to the scale of big galaxy clusters ($> 10^{15} M_\odot$).

In this scenario big structures form and then fragment to produce smaller structures [top-down scenario].

- Note that m_ν depends on T_D via $g_{*S}(T_D)$. If we consider a particle, still relativistic at decoupling, but with interactions (much) weaker than neutrinos, decoupling happens at $T_D \gg 1$ MeV, g_{*S} is larger, m_ν smaller, and higher values for m_W are allowed by cosmology.

CR14

If $T_D > 300$ GeV, $g_{*s} \gtrsim 100$ and

$$\Omega_w h^2 \sim 1.1\times10^{-3}\left(\frac{m_w}{1\,eV/c^2}\right) \sim \frac{m_w}{910\,eV/c^2}$$

Since m_w is larger than that of neutrinos, $\langle v \rangle$ decreases and smaller structures can form. In this case we speak of <u>Warm Dark Matter</u> (WDM)

Possible candidates are are photinos and gravitinos with masses $\lesssim 1$ keV, or sterile neutrinos with similar masses.

WDM alleviates some problems of CDM (see below), on small scales, such as an excess of small structures, with masses $\sim$ globular clusters or dwarf galaxies.

CDM In this case particles decouple when no more relativistic, the number density is cutted exponentially and, for a given Ω_w, masses can be much higher, and $\langle v \rangle \propto \frac{1}{m}$ much smaller: very small structures can form and we have a so called <u>bottom-up scenario</u> in which small structures merge, as time goes on, to form larger and larger structures. This scenario is in in quite good agreement with the observations.

- But in this case the final result depends on the details of the "freezing", at variance with the HDM case.
- Let's see the equation ruling the "freezing". If there no creation/destruction of particles, the proper number density $n \propto 1/a^3$: ($A \equiv$ const.)

$$n = \frac{A}{a^3} \rightarrow \dot{n} = -3\underbrace{\frac{A}{a^3}}_{n}\cdot\frac{1}{a}\cdot\dot{a} = -3Hn$$

CR15

So $\dot{n} + 3Hn = 0$

If we have annihilation and creation (with a source term S), the above eq. becomes

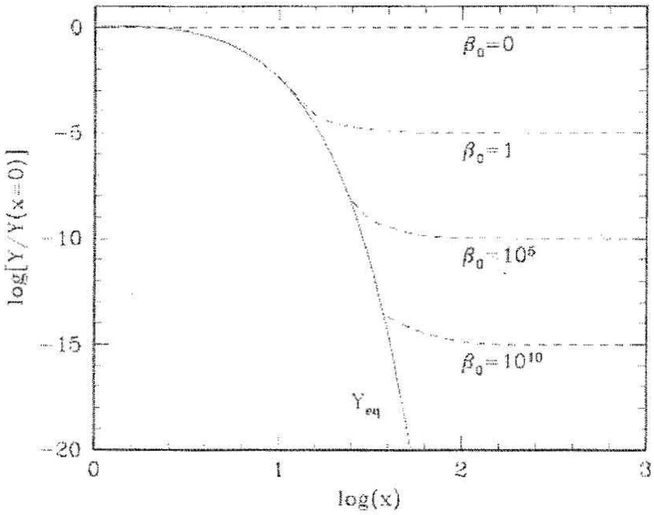
$$\dot{n} + 3Hn = -\langle\sigma v\rangle n^2 + S$$

where σ is the annihilation cross section and v the particle speed (n^2 because ann. is a two-body interaction)

If we were in stationary conditions ($H \equiv 0$), the value of n would be n_{eq}, the thermal equilibrium value at the mean temperature of the universe, so ($\dot{n} = 0, H = 0$)

$$-\langle\sigma v\rangle n_{eq}^2 + S = 0 \rightarrow S = \langle\sigma v\rangle n_{eq}^2$$

This relation can be derived directly from Boltzmann eq. (see, if you want, MoVdBW)



$\beta_0 \equiv \sigma_{ann}$

Fig. 3.5. The solution of Eq. (3.16) assuming a constant annihilation cross-section: $\beta = \beta_0$ (dashed curves). The solid curve shows the equilibrium abundance.

Note that the higher the value of β_0, the lower the final density of relics. The weakest wins!

$\dot{n} + 3Hn = -\langle\sigma v\rangle n^2 + S'$; If $H=0$ $\dot{n}=0$
annihilat. creazione $-\langle\sigma v\rangle n_{eq}^2 + S = 0$

CR | 6

$S = \langle\sigma v\rangle n_{eq}^2$

$\dot{n} + 3Hn = -\langle\sigma v\rangle [n^2 - n_{eq}^2]$

the collision rate is $\Gamma = n\langle\sigma v\rangle$; likewise, the source term for thermal particle creation is $S = \langle\sigma v\rangle n_T^2$; thus, the continuity equation changes to read

$$\dot{n} + 3Hn = -\Gamma n + S = -\Gamma n\left(1 - \frac{n_T^2}{n^2}\right) \tag{1.115}$$

- we now introduce the comoving number density $N := a^3 n$; substituting from $\dot{N} = a^3(3Hn + \dot{n})$ in (1.115) yields

$$\dot{N} = -\Gamma N\left(1 - \frac{N_T^2}{N^2}\right) \tag{1.116}$$

substituting further

$$\frac{d}{dt} = \dot{a}\frac{d}{da} = aH\frac{d}{da} = H\frac{d}{d\ln a} \tag{1.117}$$

yields

$$\boxed{\frac{d\ln N}{d\ln a} = -\frac{\Gamma}{H}\left(1 - \frac{N_T^2}{N^2}\right)} \tag{1.118}$$

- thus, if the comoving number density is thermal, $N = N_T$, it does not change; if N deviates from N_T, it needs to change for re-adjustment to its thermal equlibrium value N_T; this is impossible if $\Gamma \ll H$ because then the rate of change becomes too small; then, the particles freeze out of thermal equilibrium

- for relativistic particles, $n \propto T^3 \propto a^{-3}$, thus $N = a^3 n = \text{const.}$; according to the freeze-out equation (1.118),

$$\frac{d\ln N}{d\ln a} = 0 \Rightarrow N = N_T \tag{1.119}$$

this implies that relativistic particle species retain their thermal-equilibrium density regardless of Γ/H, i.e. even after freeze-out

- for non-relativistic particles, the comoving number density in thermal equilibrium is

$$N_T \propto T^{-3/2} e^{-mc^2/kT} \tag{1.120}$$

$n_i \propto T^{3/2} e^{-\frac{mc^2}{kT}}$

$N_i \propto n_i a^3$ $a \sim \frac{1}{T}$

$N_i \propto \frac{T^{3/2}}{T^3} e^{-\frac{mc^2}{kT}}$

for $kT \lesssim mc^2$, N_T drops exponentially, i.e. very quickly $N_T \ll N$, then

$$\frac{d\ln N}{d\ln a} \approx -\frac{\Gamma}{H} \to 0 \tag{1.121}$$

as the collision rate falls below the expansion rate; the actual comoving number density of particles then remains constant, while its thermal-equilibrium value drops to zero

$\frac{\Gamma}{H} \sim 1$ good approximation for "freezing" ~~per congelamento: buona approssimazione~~

To estimate ~~Per stimare~~ Γ we need ~~occorre~~ $\langle \sigma_A \cdot v \rangle$. (CR17

- ~~Per~~ For $m_\nu < m_{Z^0}$ $\langle \sigma v \rangle \sim \sigma_0 c$

$$\sigma_0 \sim \frac{5}{2\pi} \frac{G_F^2 m_\nu^2}{\hbar^4} = \frac{5}{2\pi} \frac{G_F^2 (m_\nu c^2)^2}{(\hbar c)^4} \propto m_\nu^2$$

The condition ~~La condizione~~ $\Gamma / H \simeq 1$ leads to ~~porta alla for...~~ (~~vedi dettagli in~~ see details on "The Early Universe" by Kolb & Turner p. 129 e segg.) or MVdBW

$$\boxed{\Omega_\nu h^2 \sim 3 \left[1 + \frac{1}{6} \ln\left(\frac{m_\nu c^2}{GeV}\right) \right] \cdot \left(\frac{m_\nu c^2}{GeV}\right)^{-2} \sim 3 \left(\frac{m_\nu c^2}{GeV}\right)^{-2}}$$

$$\Downarrow$$

$0.1 < \Omega_\nu h^2 < 1 \Rightarrow 2 \lesssim m_\nu \lesssim 5$ GeV, $\boxed{m_\nu \sim 1 \text{ GeV}}$

~~Limite di~~ Limit Lee-Weinberg 1 (~~vedi figura pag successive~~ See fig. 3.6 of MVdBW)

- ~~Per~~ For $m_\nu > m_{Z^0}$ $\sigma_0 \propto m_\nu^{-2}$

$$\sigma_0 \sim \frac{5}{2\pi} \frac{G_F^2}{(\hbar c)^4} (m_\nu c^2)^2 \cdot \left(\frac{m_Z c^2}{m_\nu c^2}\right)^4$$

$$\Gamma / H \sim 1 \Rightarrow \Omega_\nu h^2 \approx 0.1 \left[1 - \frac{\ln (m_\nu c^2)_{TeV}}{30} \right] (m_\nu c^2)^2_{TeV}$$

$\boxed{\Omega_\nu h^2 \approx 0.1 (m_\nu c^2)^2_{TeV}} \Rightarrow 0.1 \lesssim \Omega_\nu h^2 \lesssim 1$ implica

$1 \lesssim m_\nu \lesssim 3$ TeV, $\boxed{m_\nu \sim 1 \text{ TeV}}$

We see that CDM candidates, interacting like neutrinos, must ~~be~~ have $m_\nu \sim 1$ GeV or $m_\nu \sim 1$ TeV to be in agreement with cosmological observation, or to be a relevant contributor to dark matter.

Lee-Weinberg limit

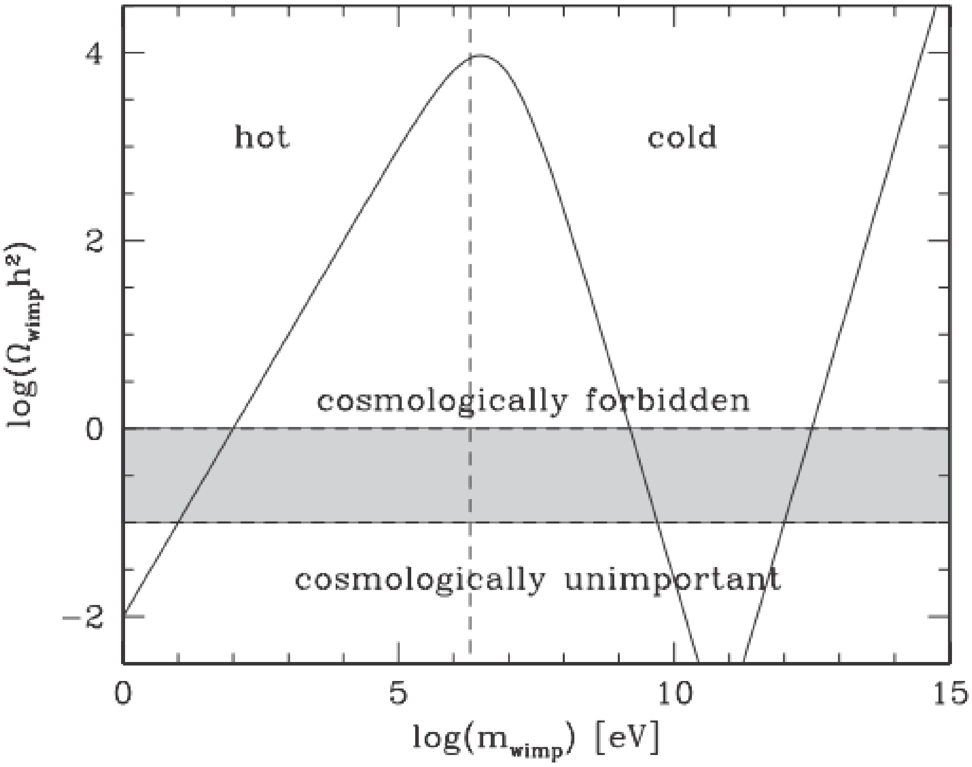


Fig. 3.6. Cosmological constraints on the mass of weakly interactive dark matter particles under the assumption that they interact as a Dirac-type neutrino. The solid curve shows the predicted cosmological density parameter of the WIMPs as a function of WIMP mass, while the shaded area roughly brackets the observed range of the cosmological density parameter. The mass ranges in which the particles make up 'hot' and 'cold' dark matter are indicated.

If DM particles candidates are taken from (CR/8
SUSY, estimates of their density largely depend
on the many free parameters of theories.
If R-parity is conserved, the lightest (LSP)
neutralino (χ) is the most natural DM candidate.
Neutralino states are a mixture of the bino ($\tilde{B}$),
wino ($\tilde{W}_3$) and two higgsinos ($\tilde{H}_0^1$, $\tilde{H}_0^2$). Bino and
Wino are SUSY partners of the gauge bosons B
and W_3 which, during EW symmetry breaking, convert
to photons and Z^0's.

All the relics mentioned above are thermal relics:
there was a time when they were in thermal equilibrium
with the energy bath, until they decoupled and freezed.
But there are also non-thermal relics, particles that
were never in thermal equilibrium, and were produced with negligible velocities. Their evolution
is then similar to CDM.
The axion is one of these relics: it has very low mass,
but its velocity is low, and

can form very small structures (remember that $< v > \propto 1/m$).

Recombination and Last Scattering

When the temperature of the Universe drops below $kT \sim 13.6\ eV$ (the ionization potential of hydrogen in the ground level) protons and electron begin to combine and form neutral hydrogen. This is the epoch of the ***recombination*** (actually, recombination is the name of the radiative process involved; for the Universe "first combination" would be more appropriate). But, due to the very large number of photons for each baryon (about 10^9, as we have seen), hydrogen becomes (almost) neutral at a lower temperature ($kT \sim 0.3\ eV$, $T \sim 3000\ K$)[3]. We neglect recombination of He, which takes place earlier.

There are different mechanisms involved in the making of neutral hydrogen. If recombination takes place in an isolated cloud of ionized hydrogen (HII cloud), two processes are dominant: direct recombination to the ground state, and the capture of an electron to an excited state which then cascades to the ground level. In the first case, a Lyman continuum photon (with energy larger than $13.6\ eV$) is produced, while in the second case one of the recombination photons must have an energy higher than or equal to that of Ly-α. If the cloud is optically thin (optical depth $\tau \ll 1$), all recombination photons can escape and do not contribute to further ionization.

In the case of cosmological recombination, however, recombination photons will be absorbed again because they cannot escape from the Universe. In fact, the direct capture of electrons to the ground state does not contribute to the net recombination, because the resulting photon is energetic enough to ionize another hydrogen atom from its ground state. The normal cascade process is also ineffective, because the Lyman series photons produced can excite hydrogen atoms from their ground states, so that multiple absorptions lead to re-ionization. Therefore, recombination in the early Universe must have proceeded by different means.

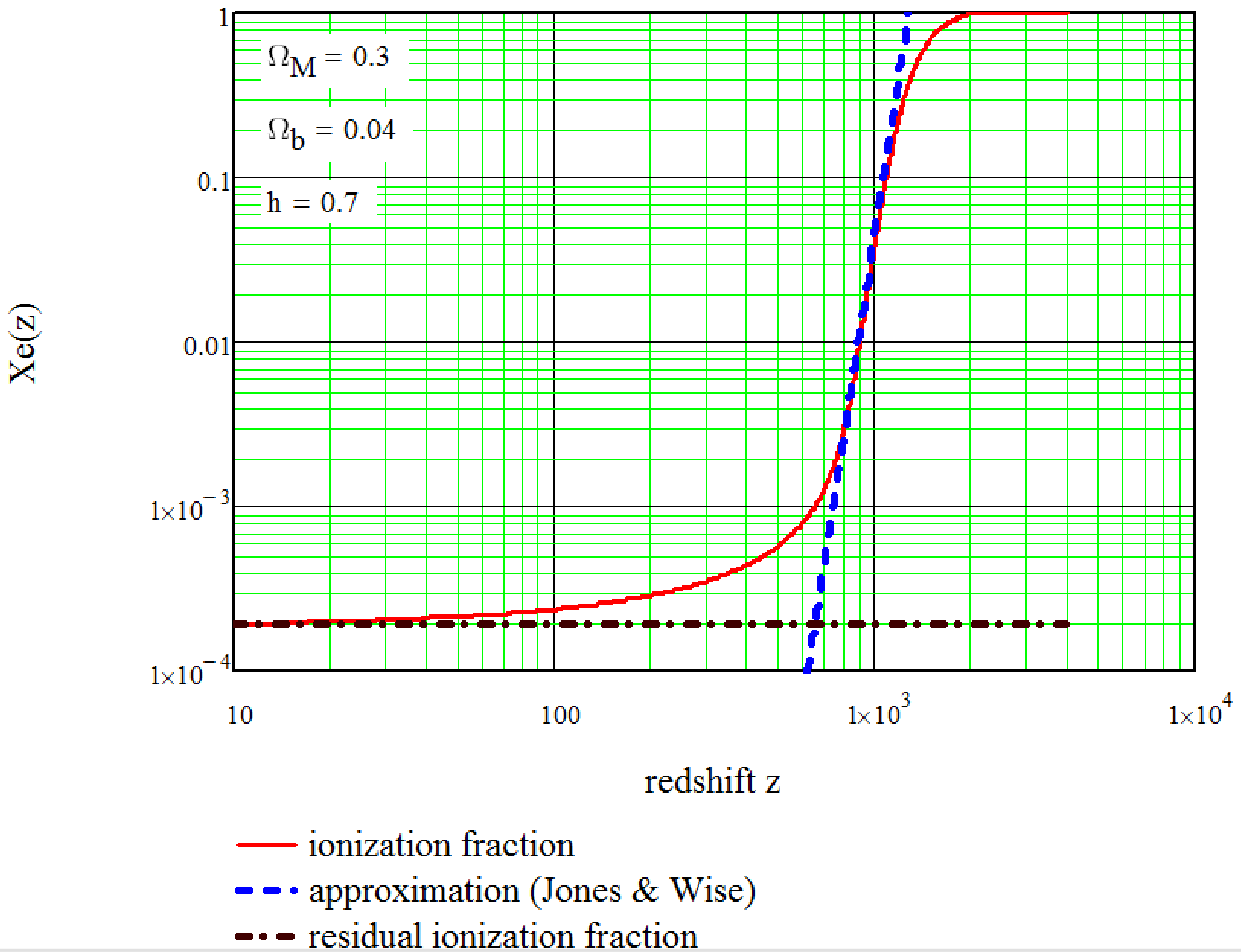
That leaves two main processes for the production of neutral, atomic hydrogen. One is ***two-photon decay*** from the metastable *2s* level to the ground state, at the rate $\Gamma_{2\gamma} \approx 8.23\ s^{-1}$ (in this process two photons must be emitted in order to conserve both energy and angular momentum, and the energies of the two photons may not be able to contribute to ionization). The second is the loss of the Lyman-α resonance photons by the cosmological redshift. Two-photon decay turns out to be the dominant process.
Moreover, since expansion dilutes proton and electrons, at a certain time (redshift) recombination stops, is frozen, and a tiny fraction of ionized hydrogen remains.

We use the following definitions and relations: ionization fraction $X_e \equiv n_p/(n_p+n_H)$, $\eta \equiv n_B/n_\gamma = const. = 2.7\times10^{-8}\Omega_b h^2$, $n_B = n_p+n_H = \rho_{0b}/m_p$, m_p proton mass, $\rho_b = \rho_{0b}(1+z)^3$, $\rho_{0b} = \Omega_b\, \rho_{0cr}$, $T = T_{\gamma 0}(1+z)$. So electron (and proton) density is given by

$$n_e(z) = X_e(z) n_B = X_e(z)\rho_b / m_p = X_e(z)\times 1.13\times10^{-5}\Omega_b h^2 (1+z)^3$$

[3] For order of magnitude estimates, the Kelvin temperature T_K can be linked to energy by: $T_K \sim 10^{13} E_{GeV}$

The following figure shows the evolution of the ionization fraction versus redshift for $\Omega_M=0.3$, $\Omega_b=0.04$ and $h=0.7$.



Conventionally recombination corresponds to $X_e=0.1$. We see in the figure that $X_e \sim 0.1$ at a redshift around *1100*. The figure also shows that recombination is never complete. The recombination process freezes, and a residual ionization remains (at z ~ 10):

$$X_{residual} \approx 10^{-5} \frac{\sqrt{\Omega_M h^2}}{\Omega_b h^2}$$

on the order of 10^{-4} .

The dependence on cosmological parameters is due to the balance between the recombination rate, proportional to n_p (equal to n_e), and the expansion rate H. So

$$\Gamma_{rec} \propto n_e \propto X_e(z)\Omega_b h^2 \qquad H(z) = H_0\sqrt{\Omega_M}(1+z)^{3/2} \text{ (MD EdS)}$$

$$\Gamma_{rec} \sim H \implies X_e(z)\Omega_b h^2 \propto H_0\sqrt{\Omega_M} \implies X_e \propto \frac{H_0\sqrt{\Omega_M}}{\Omega_b h^2} \propto \frac{\sqrt{\Omega_M h^2}}{\Omega_b h^2}$$

An approximation for $X_e(z)$, good for *800 <z <1200* is given by (Jones & Wise, 1985):

$$X_e(z) \cong 2.4\times10^{-3}\frac{\left(\Omega_M h^2\right)^{1/2}}{\Omega_b h^2}\left(\frac{z}{1000}\right)^{12.75}$$

Recombination is also associated to the last scattering of CMB photons, since after recombination the Universe becomes finally transparent.

A useful parameter is the optical depth: since $d\tau = -\, n_e\,\sigma_T\, c\; dt$ (τ grows starting from us, cosmic time increases toward us), where σ_T is the *Thomson scattering* cross section (***σ_T=6.65×10^{-25}cm^2***). When we integrate we have $[dt/dz = -1/(1+z)H(z)]$

$$\int_\tau^0 d\tau' = \int_t^{t_0} n_e\sigma_T c\, dt' = \int_z^0 n_e(z')\sigma_T c\,\frac{-1}{(1+z')H(z')}\,dz'$$

$$\tau(z) = \int_0^z n_e(z')\,\sigma_T\, c\,\frac{1}{(1+z')H_0E(z')}\,dz'$$

When estimating the optical depth τ, the dependence on cosmological parameters disappears since $n_e(z)=X_e(z)\, n_B(z) \sim X_e(z)\,\Omega_B h^2$, and $H_0\, E(z) \sim \Omega_M^{1/2}\, h$, so

$$\tau(z) \cong 0.37\left(\frac{z}{1000}\right)^{14.25}$$

The probability of receiving a photon from the optical depth τ is equal to $e^{-\tau}$. The probability of receiving a photon from the interval between τ and $\tau+d\tau$ corresponds to the probability of receiving it from the interval between z and $z+dz$:

$$e^{-\tau}d\tau = g(z)dz \Rightarrow g(z) = e^{-\tau}\frac{d\tau}{dz}$$

With the above approximation for $\tau(z)$

$$g(z) = 5.26\times10^{-3}\left(\frac{z}{1000}\right)^{13.25}\exp\left[-0.37\left(\frac{z}{1000}\right)^{14.25}\right]$$

which has a maximum for *z=1067*, and conventionally we assume that the ***last***

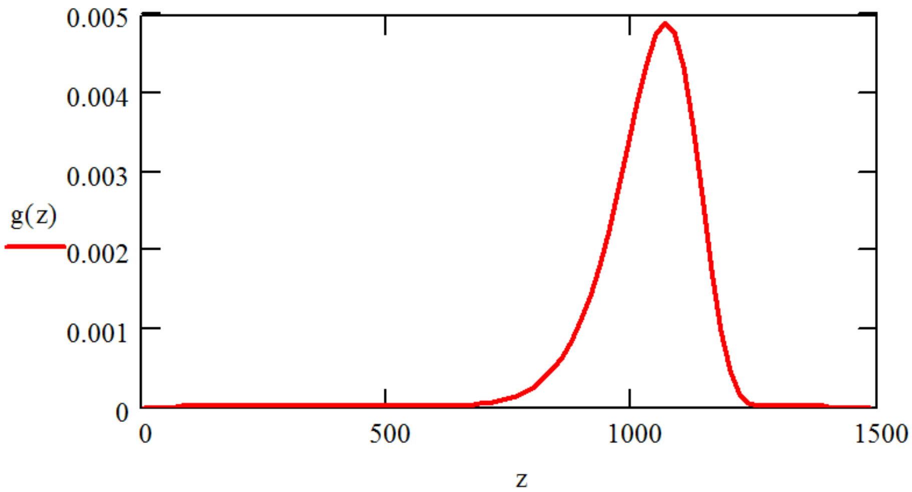
scattering corresponds to this redshift ($z_{ls} \approx 1067$). The following figure shows the probability distribution for the last scattering redshift. The 68% probability is included in a $\Delta z \approx 170$ around the maximum, so the last scattering event is not instantaneous and does not correspond to a single redshift. This means that the last scattered photons have a spread in their temperatures, but this is compensated by the higher redshift suffered by photon which decoupled earlier.

The age of the Universe at the last scattering can be derived, approximately, by using a *MD – EdS* model with $\Omega_M = 0.3$ and $h=0.7$, which gives

$$t(z_{ls}) \approx \frac{2}{3H_0\sqrt{\Omega_M}(1+z_{ls})^{3/2}} \approx 4.8 \times 10^5 \; years$$

while a better approximation gives about $4{\times}10^5$ years.

Probability distribution for the last scattering redshift



BIG BANG NUCLEOSYNTHESIS (BBN)

According to Fig. 15.6, at times earlier than about 1 second the temperature was greater than 10^{10} K, corresponding to an average kinetic energy per particle of more than an MeV. At such energies, nuclear physics processes like

$$n + \nu_e \leftrightarrow p + e^-, \qquad n + e^+ \leftrightarrow p + \bar{\nu}_e, \qquad n \leftrightarrow p + e^- + \bar{\nu}_e \qquad (15.50)$$

and other processes involving leptons and photons such as (Fig. 15.4)

$$\nu + \bar{\nu} \leftrightarrow e^+ + e^- \leftrightarrow \gamma + e^+ + e^- \qquad (15.51)$$

can all be in thermal equilibrium. For the nucleons, $kT \ll mc^2$ and (neglecting the irrelevant chemical potential) (15.36) gives the energy density

$$\rho c^2 = mc^2 \frac{g}{\hbar^3}\left(\frac{mkT}{2\pi}\right)^{3/2} e^{-mc^2/kT}, \qquad (15.52)$$

the last being the usual Boltzmann suppression factor. Hence, the ratio of the number of neutrons to protons will be

$$r \equiv \frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-\frac{(m_n - m_p)c^2}{kT}} \cong e^{-\frac{(m_n - m_p)c^2}{kT}}$$

where m_n and m_p are neutron and proton masses, and $(m_n\text{-}m_p)c^2=1.293$ *MeV*. The rate of the interactions exchanging n into p and vice versa is (G_F = Fermi weak coupling constant) :

$$\Gamma_{n\leftrightarrow p} \cong 2(kT)^5_{MeV}\ s^{-1} \propto G_F^2 T^5$$

Compare this with $H=1/2t$ (*EdS* in RD era), where

$$t(\sec) \approx 2.4\, g_*^{-1/2}\,(kT)^{-2}_{MeV}$$

($g_* \approx 10$). $\Gamma_{n\leftrightarrow p} \approx H$ for $kT_{D\nu} \approx 0.7\ MeV$, $t_{D\nu} \approx 1.5\ sec$. The neutron to proton ratio freezes at

$$r_0=n_{n,0}/n_{p,0} \cong exp(-1.293/0.7) \cong 0.16.$$

Only neutron β *decay* is possible, with $\tau_n=885.7\pm0.8\ sec$ (about 15 minutes).

. The neutrons will then start to decay, $n \rightarrow p + e^- + \bar{\nu}_e$, but the lifetime for this is long ($\approx$15 min) compared to the age of the universe at this point. Alternatively, they can combine with protons through very fast processes such as

$$p + n \leftrightarrow {}^2H + \gamma, \qquad {}^2H + n \rightarrow {}^3H + \gamma, \qquad {}^3H + p \rightarrow {}^4He + \gamma,$$
$$ {}^2H + p \rightarrow {}^3He + \gamma, \qquad {}^3He + n \rightarrow {}^4He + \gamma. \qquad (15.54)$$

The key process is the formation of deuterium ^2H, which has a binding energy B_D = 2.23MeV. Because of the relatively large number of photons with respect to baryons, the high energy tail of the distribution of photons immediately dissociates the deuterium which is formed, and this until the number of dissociating photons n_γ^{diss} becomes comparable with that of baryons, n_B . We will have:

$$\frac{n_\gamma^{diss}}{n_B} = \frac{n_\gamma^{diss}}{n_\gamma} \cdot \frac{n_\gamma}{n_B} = \frac{1}{\eta} \cdot \frac{n_\gamma^{diss}}{n_\gamma}$$

with

$$n_\gamma = \frac{2}{\pi^2}\zeta(3)\left(\frac{kT}{\hbar c}\right)^3$$

The density of the dissociating photons is obtained by putting $P = E/c$ in the relation that gives the density of photons, placing B_D as the lower limit in the integration:

$$n_\gamma^{diss} = \frac{2}{2\pi^2(\hbar c)^3}\int_{B_D}^{\infty}\frac{E^2 dE}{e^{E/kT}-1} \approx \frac{1}{\pi}\left(\frac{kT}{\hbar c}\right)^3\left(\frac{B_D}{kT}\right)^2 e^{-B_D/kT},$$

a good approximation since $E/kT > B_D/kT \gg 1$. For $1<\eta_{10}<10$, $n_\gamma^{diss}/n_B \approx 1$ if $kT \approx 0.1\ MeV$, $\boldsymbol{T \approx 10^9 K}$. ($\eta_{10} = \eta / 10^{-10}$)

At this time the deuterium is no longer destroyed by photons and quick reactions occur leading to the formation ^{4}He: this is the era of BBN. The universe has an age of about ($g_* = 3.36$ at $kT = 0.1\ MeV$)

$$t_{BBN}(\mathrm{sec}) \approx 2.4\, g_*^{-1/2}\,(0.1)_{MeV}^{-2} \approx 150\, s$$

That is about <u>**three minutes**</u>.

Between the freezing, $t_{D\nu} \approx 1.5\ sec$, and t_{BBN} neutrons decay to protons and, from $r_0 \approx 0.16$, we arrive to

$$r_{BBN} \cong \frac{n_{n,0}e^{-t_{BBN}/\tau_n}}{n_{p,0} + n_{n,0}\left(1-e^{-t_{BBN}/\tau_n}\right)} \cong 0.13$$

After the bottleneck of deuterium, all neutrons that did not decay end up embedded in the nuclei of ^{4}He. Since it takes two neutrons for each ^{4}He nucleus and this has atomic weight 4, the abundance in mass Y_{BBN}, of ^4He is

$$Y_{BBN} \equiv \frac{mass\,of\ ^4He}{mass\,of\ ^4He + mass\,of\ free\ protons} = \frac{4\cdot n_n/2}{4\cdot n_n/2 + 1\cdot(n_p - n_n)}$$

$$Y_{BBN} = \frac{2\,r_{BBN}}{1+r_{BBN}} \approx 0.23$$

The detailed calculation, much more complicate, provides similar values, in agreement with the experimental data that suggest Y_{obs} around 0.24-0.25.

As shown in the following figure, the predicted abundance of ^{4}He does not vary much with the baryon-to-photon ratio $\boldsymbol{\eta}$, because τ_n is long (compared to the age of the universe) and neutrons decay slowly. However, Y_{BBN} depends strongly on $T_{D\nu}$, which depends on H, which in turn depends on g_* at a temperature of about *1 MeV*:

$$g_* = 2 + \frac{7}{8}(4 + 2\cdot N_\nu)$$

where N_ν is the number of neutrino species. The higher the value of N_ν the higher is $T_{D\nu}$ and so the greater are r_0 and Y_{BBN} ($\Delta N_\nu = 1 \Rightarrow \Delta Y_{BBN} \cong 0.013$, see the lines in the figure). The observational limits on Y_{BBN} give $N_\nu = 3 \pm 1$. In the 80ies, until *LEP* at *CERN* measured the decay (width) of Z^0 and obtained $N_\nu = 2.994 \pm 0.012$, the best estimate of N_ν was given by BBN. We notice that BBN and *LEP* are sensitive to different kinds of particles: BBN is sensitive to particles that were relativistic at $kT \sim 1\ MeV$; the width of Z^0 is sensitive to neutrinos with masses $m_\nu < M_{Z^0}/2$. So they measure different things.[4]

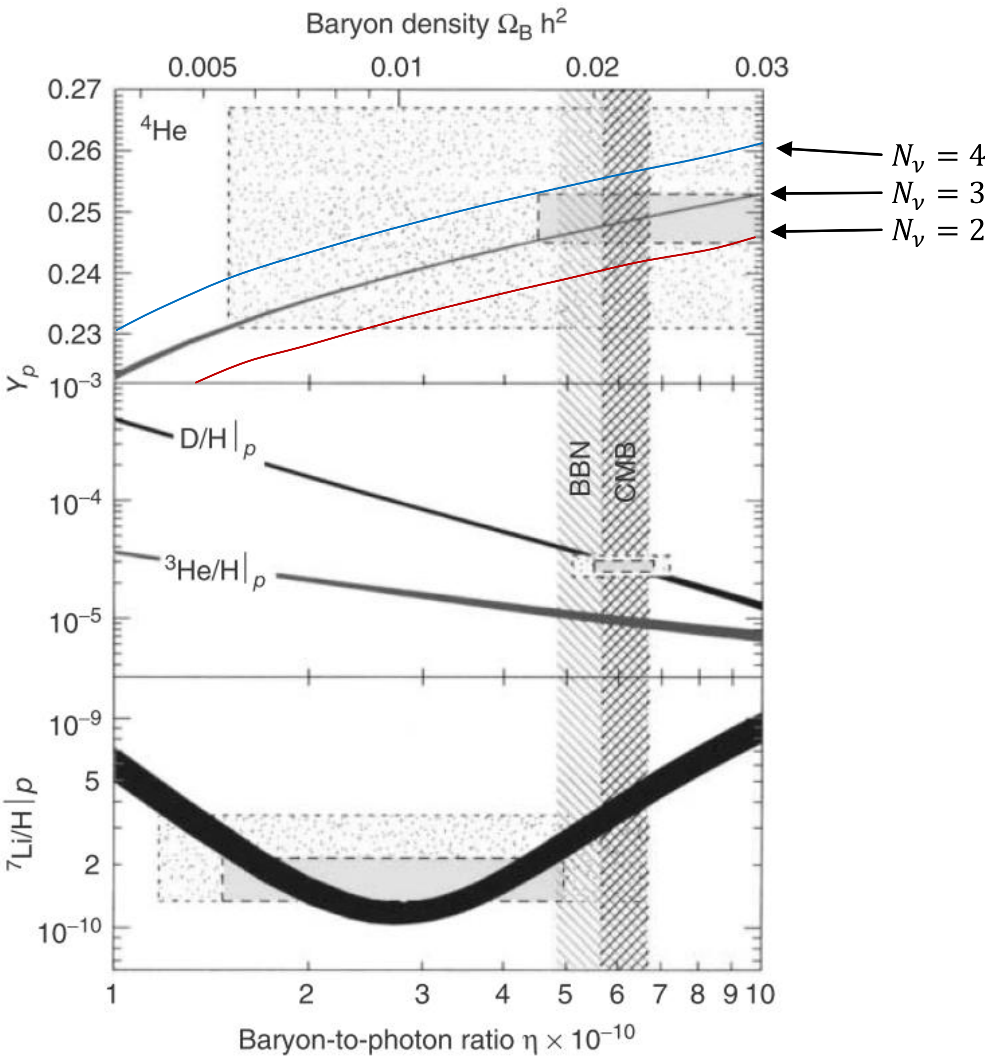


Figure 8.7 Baryon components of the universe. The unit of abscissa is $\eta_{10} \equiv n_B/n_\gamma \times 10^{10}$. Values determined from observation of light elements agree well with those determined from the nucleosynthesis [733, 739] and CMB (cosmic microwave background radiation) spectrum by WMAP [705, 740]. (Reproduced with permission of [7].)

[4] If you are interested in the possibilities offered by BBN to explore physics beyond the Standard Model, look at the Particle Data Group site (*http://pdg.lbl.gov/*), and in particular the review on BBN (*http://pdg.lbl.gov/2014/reviews/rpp2014-rev-bbang-nucleosynthesis.pdf*).

Although the bulk of the neutrons end up in ^{4}He, it is predicted that some ^{2}H, ^{3}H, and ^{3}He will remain, and, despite the absence of stable nuclides with atomic mass number $A = 5$ or 8, and the large Coulomb barrier between higher-Z nuclei, some heavier elements will be formed by processes such as

$$^3\mathrm{He} + {^4\mathrm{He}} \rightarrow {^7\mathrm{Be}} + \gamma, \qquad {^4\mathrm{He}} + {^3\mathrm{H}} \rightarrow {^7\mathrm{Li}} + \gamma, \tag{15.56}$$

but the rates depend more critically on the baryon density, as Fig. 15.7 illustrates. Of course later, when matter condensed into stars, its temperature again became sufficiently high for nuclear reactions to start up

So, for the production of Carbon, Nitrogen Oxigen and so on, we have to wait for the formation and evolution of stars.

Concordance, Dark Matter, and the CMB

We now use the observed light element abundances to test the theory. We first consider standard BBN, which is based on Standard Model physics alone, so $N_\nu = 3$ and the only free parameter is the baryon-to-photon ratio η. Thus, any abundance measurement determines η, while additional measurements overconstrain the theory and thereby provide a consistency check. Also observations of the CMB constrain the value of η.

First we note that the overlap in the η ranges spanned by the larger boxes (which include systematic errors) in the Figure above indicates overall concordance. More quantitatively, when we account for theoretical uncertainties, as well as the statistical and systematic errors in observations, there is acceptable agreement among the abundances when

$$5 \leq \eta_{10} \leq 6.5 \ (95\% \ CL).$$

However, the agreement is much less satisfactory if we use only the quoted statistical errors in the observations. In particular, as seen in the Figure, D and ^{4}He are consistent with each other, but favor a value of η which is higher than that indicated by the ^{7}Li abundance determined in stars. Actually, there is a possible problem with Lithium, which maybe requires new physics.

Even so, the overall concordance is remarkable: using well-established microphysics we have extrapolated back to an age of ∼ 1 s to correctly predict light element abundances spanning 9 orders of magnitude. This is a major success for the standard cosmology, and inspires confidence in extrapolation back to still earlier times. This concordance provides a measure of the baryon content

$$0.019 \leq \Omega_b h^2 \leq 0.024 \ (95\% \ CL),$$

a result that plays a key role in our understanding of the matter budget of the Universe.

Primordial Baryosynthesis

The above picture still leaves us, however, with the problem of where the net baryon number and lepton number of the universe have come from, i.e., why there are more quarks than antiquarks, and more electrons than positrons. We know that the solar system is made of matter not antimatter. The very small proportion of antimatter in the cosmic radiation ($\approx 10^{-4}$), the failure to observe the X-rays that would result if matter–antimatter annihilation occurred at all commonly in the collisions of stars, gas clouds, or galaxies, and the lack of any very convincing mechanism for separating matter and antimatter on a cosmic scale, all suggest that the universe is made just of matter. If in the early universe the numbers of quarks and antiquarks had been equal, as Fig. 15.4 suggests, their final annihilation once $kT < 1$ GeV is estimated to yield only

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \approx 10^{-19}, \qquad \text{rather than} \qquad \eta \approx 10^{-9} \gg \frac{n_{\bar{B}}}{n_\gamma}, \qquad (15.57)$$

which is actually observed.

7.6 Criteria for a baryon asymmetry

If we assume unitarity (all probabilities of interactions add up to 1) and CPT is a good symmetry, then a nonzero baryon asymmetry can be generated if the following conditions hold†, (Sakharov, 1967; Kuzmin, 1970).

i) baryon number is not conserved
ii) C and CP are not conserved
iii) there is departure from thermal equilibrium

Condition (i) is necessary if we are to pass from a state with baryon number (B), zero to $B \neq 0$; however, it is not a sufficient condition. The C operator changes n_q—$n_{\bar{q}}$ so if C is conserved we must have $n_q = n_{\bar{q}}$ in the system and

†C : charge conjugation, C (particle) = antiparticle; P parity reversal, P (right hand) = left hand; T - time-reversal.

hence $B=0$. Since the P operator leaves both n_q and $n_{\bar{q}}$ unchanged the CP operator also requires $n_q = n_{\bar{q}}$ and hence $B=0$. Hence the condition (ii) is necessary. Finally, if thermal equilibrium obtains then T is a good symmetry and so CPT symmetry would imply CP symmetry and $B=0$ by (ii): therefore, we require condition (iii). [For more detail see especially Kolb and Wolfram (1980)].

The asymmetry could be linked to the breaking of GUTs or of the electro-weak interaction. The question is open and, very likely, requires new physics.