INFLATION

We have already mentioned two problems affecting the Hot Big Bang model: the ***flatness problem*** and the ***horizon problem***. To them one can add the ***magnetic monopoles problem*** (monopoles are zero-dimensional topological defects, that are produced at the time of the phase transition corresponding to the breaking of GUTs; theirnumber density, coupled with their very high mass, would produce a value of Ω clearly unacceptable).

The paradigm of ***inflation***, which solves these problems, has been proposed by *Alan Guth* in 1981. It assumes that there has been an accelerated expansion phase between the times t_i and t_f (with $t_{Pl} < t_i < t_f << t_{eq}$), produced by an equation of state that mimics that of a cosmological constant:

$$t_i < t < t_f \qquad a(t) \approx a(t_i) e^{H(t-t_i)}$$

(H ~ constant). The scale factor grows as in a de Sitter model (which has $\Lambda \neq 0$ and density of matter negligible), instead of growing as $a(t) \sim t^{1/2}$, like an EdS model in the RD era.

The exponential growth, if sufficiently prolonged, produces a growth of the particle horizon d_H sufficient to solve the horizon problem; Ω converges towards unity (as in models dominated by the cosmological constant), resolving the flatness problem (remember also that the curvature of the spatial section scale as $a(t) \sim t^{-2}$, and the exponential growth of *a(t)* force this curvature towards zero). The problem of monopoles is resolved through a strong dilutionof their number density.

If inflation occurs around the time of the breaking of grand unification (GUT), the above problems are solved provided

$$\ln\left(\frac{a_f}{a_i}\right) \equiv \mathcal{N} \geq 60$$

where $\mathcal{N}$ is named number of ***e-foldings***.

Lagrangian formulation of Field Equations

As we have seen, to have a phase of inflation is necessary that the universe possesses, for a certain time interval, an equation of state of the type $p = -\rho c^2$.

This can be achieved in a natural way by means of a scalar field present in the early stages of the early universe (a scalar field has also the property of being isotropic). To understand the mechanism it is necessary to introduce some concepts used in Quantum Field Theory.

In Classical Mechanics the equations of motion of a dynamical system can be derived from a ***Lagrangian function L***

$$L(q_i,\dot{q}_i) = T(\dot{q}_i) - V(q_i)$$

where q_i are the generalized coordinates, *T* is the kinetic energy and *V* is the potential energy. The action *S*, involved in the motion of the system from one configuration at time t_1 to another at the time t_2, is given by

$$S = \int_{t_1}^{t_2} L dt$$

and, according to the ***principle of least action***, the evolution of the system between the two configurations is that which corresponds to the minimum value of *S*. This condition leads to the *Euler-Lagrange equations*:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

These relations describe the motion of particles, that is, of localized objects. A field instead occupies a certain region of space, and the Field Theory wants to calculate one (or more) functions of position and time: $\phi = \phi\,(x, y, z, t)$ (eg .: temperature, electric potential, the three components of the magnetic field in a room). While, in the mechanics of particles, the Lagrangian ***L*** is a function of the coordinates q_i and of their derivatives, Field Theory works with a ***Lagrangian density*** $\boldsymbol{\mathcal{L}}$ which is a function of the field ϕ and of its derivatives with respect to *x, y, z*, and *t*. To keep the relativistic covariance of physics more apparent, we use space-time coordinates $x_0 = ct$ and $x_1, x_2, x_3 = x, y, z$, so that the Lagrangian is the volume integral of $\boldsymbol{\mathcal{L}}$

$$L = \int \boldsymbol{\mathcal{L}}\, d^3x$$

and the action is

$$S = \frac{1}{c}\int \boldsymbol{\mathcal{L}}\, d^4x$$

(the factor *1/c*, inessential, serves to keep the dimensions of the action).

In relativistic field theory q_i is replaced by the field ϕ, and the index *i* is replaced by space-time coordinates x^α. Since each time derivative can be associated to a similar term involving a gradient, we use all the covariant derivatives $\partial\phi/\partial x^\alpha = \partial_\alpha \phi$ and Euler-Lagrange equations become

$$\partial_\alpha\left[\frac{\partial(\boldsymbol{\mathcal{L}})}{\partial(\partial_\alpha \phi)}\right] - \frac{\partial(\boldsymbol{\mathcal{L}})}{\partial \phi} = 0$$

Actually this writing is correct in a Euclidean space and in orthogonal coordinates; to take account of a more general choice of coordinates (e.g. co-moving spatial coordinates) the volume element d^4x is replaced with $\sqrt{-g}\, d^4x$ where g is the determinant of the metric $g_{\alpha\beta}$. So the Euler-Lagrange equation becomes

$$\partial_\alpha \left[\frac{\partial\left(\sqrt{-g}\,\mathcal{L}\right)}{\partial(\partial_\alpha \phi)} \right] - \frac{\partial\left(\sqrt{-g}\,\mathcal{L}\right)}{\partial \phi} = 0$$

In a flat, static Minkowski space, the metric is $g_{\alpha\beta} = \eta_{\alpha\beta} = \eta^{\alpha\beta} = diag(1,-1,-1,-1)$. Then

$$\partial_\alpha = \frac{\partial}{\partial x^\alpha} \equiv \left(\frac{1}{c}\frac{\partial}{\partial t}; \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \left(\partial_0; \overline{\nabla}\right)$$

$$\partial^\alpha = \eta^{\alpha\nu}\partial_\nu \equiv \left(\frac{1}{c}\frac{\partial}{\partial t}; -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z} \right) = \left(\partial^0 (\equiv \partial_0); -\overline{\nabla}\right)$$

$$\partial_\alpha \partial^\alpha = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \equiv \Box^2$$

If we use co.moving coordinates ($r = a\,x$) in a flat, expanding space: $g_{\alpha\beta} = diag(1, -a^2, -a^2, -a^2)$ and $\sqrt{-g} = a^3$. We have then ($\overline{\nabla}_x$ is the gradient referred to the co-moving coordinate x)

$$\partial_\alpha = \left(\partial_0; \overline{\nabla}_x\right) \qquad \partial^\alpha = \left(\partial^0; -\frac{1}{a^2}\overline{\nabla}_x\right) \qquad \partial_\alpha \partial^\alpha = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{a^2}\nabla_x^2$$

Let’s consider, for instance, the following Lagrangian (density):

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \phi^2 = \frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - \frac{1}{2}\mu^2\phi^2$$

where ϕ is a real, single scalar field. In this case, *i.e.* Minkowski space, ($\sqrt{-g} = 1$),

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha \phi)} = \partial^\alpha \phi \qquad \frac{\partial \mathcal{L}}{\partial \phi} = -\left(\frac{mc}{\hbar}\right)^2 \phi = -\mu^2 \phi$$

and hence Euler-Lagrange formula requires

$$\partial_\alpha \partial^\alpha \phi + \mu^2 \phi = \left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \mu^2 \right]\phi = \left[\Box^2 + \mu^2\right]\phi = 0$$

which is the ***Klein-Gordon equation***, describing (in Quantum Filed Theory) a particle of spin 0 and mass *m*.

In analogy with *L = T - V*, in the Lagrangian written above the first term, $\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)$, is named kinetic energy term, while the second term, in this case quadratic in ϕ (the term corresponding to the mass), is the potential energy term. For a scalar field we will write the Lagrangian in the general form

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha\phi)(\partial^\alpha\phi) - V(\phi)$$

where $V(\phi)$ is a suitable potential ($V(\phi)=1/2\ \mu\ \phi^2$ in *Klein-Gordon* case). If $\mathcal{L}$, written as above, depends on *x* only through ϕ and its derivatives $\partial_\alpha\phi$, the following quantity (***energy-momentum tensor***) is preserved (i.e., has four-divergence equal to zero):

$$T^{\alpha\beta} \equiv \partial^\alpha\phi\, \partial^\beta\phi - g^{\alpha\beta}\mathcal{L}$$

In the case of a **perfect fluid** we have seen that the energy-momentum tensor has the form

$$T^{\alpha\beta} = (p + \rho c^2)u^\alpha u^\beta - p\, g^{\alpha\beta}$$

where *P* is the pressure, ρc^2 the Energy density and u^α is the four-velocity ($u^\alpha \equiv dx^\alpha/ds$); in the co-moving reference frame $u^\alpha=(1,0,0,0)$. In a flat space, by using co-moving coordiantes, the comparison of the two relations gives:

$$\begin{cases} \rho c^2 \equiv T^{00} = \frac{1}{2}\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}\frac{1}{a^2}\left(\overline{\nabla}_x\phi\right)^2 + V(\phi) \\ p \equiv a^2\left(\frac{T^{11}+T^{22}+T^{33}}{3}\right) = \frac{1}{2}\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{6}\frac{1}{a^2}\left(\overline{\nabla}_x\phi\right)^2 - V(\phi) \end{cases}$$

(see the following scanned page for the proof).

In the case in which the field is spatially homogeneous (from which $\overline{\nabla}_x\phi = 0$; even if $\overline{\nabla}_x\phi$ is different from zero the term containing $\overline{\nabla}_x\phi$ becomes rapidly negligible due to the a^{-2} factor [1]) and the term $1/2c^2\ (\partial\phi/\partial t)^2$ is negligible compared to the potential $V(\phi)$, we have

$$\rho c^2 = V(\phi)$$
$$p = -V(\phi) = -\rho c^2$$

that is, an equation of state that mimics that which corresponds to the cosmological constant!

[1] Actually there are small fluctuations on the scale of the Hubble radius, which are the "seeds" of the large-scale structure of the universe

$$T^{\alpha\beta} = \partial^\alpha\phi\,\partial^\beta\phi - g^{\alpha\beta}\mathcal{L} \quad \Big| \quad \mathcal{L} = \frac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi - V(\phi)$$

and

$$T^{\alpha\beta}_{fe} = (p+\rho c^2)u^\alpha u^\beta - p\,g^{\alpha\beta} \qquad \boxed{r \equiv a\cdot x} \text{ comoving coord.} \qquad \partial^0 = \partial_0$$

$$g_{\alpha\beta} = (1, -a^2, -a^2, -a^2)\ ;\ g^{\alpha\beta} = \left(1, -\frac{1}{a^2}, -\frac{1}{a^2}, -\frac{1}{a^2}\right);\ \partial^i = -\frac{1}{a^2}\partial_i$$

$$T^{00} = \partial^0\phi\,\partial^0\phi - \underbrace{g^{00}}_{\equiv 1}\left[\frac{1}{2}\left(\partial_0\phi\partial^0\phi + \partial_1\phi\partial^1\phi + \partial_2\phi\partial^2\phi + \partial_3\phi\partial^3\phi\right) - V(\phi)\right] =$$

$$= \left(\frac{1}{c}\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left\{\left(\frac{1}{c}\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{a^2}\underbrace{\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right]}_{\overline{\nabla}_x\phi\cdot\overline{\nabla}_x\phi}\right\} + V(\phi)$$

$$\boxed{T^{00} = \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2a^2}(\overline{\nabla}_x\phi)^2 + V(\phi) = \rho c^2 = T^{00}_{fe}}$$

$$T^{11}_{fe} = -p\cdot -\frac{1}{a^2} = \frac{p}{a^2} = T^{22}_{fe} = T^{33}_{fe}$$

$$T^{11} = \partial^1\phi\,\partial^1\phi - \left(-\frac{1}{a^2}\right)\left[\qquad\qquad\right] =$$

$$= \frac{1}{a^4}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{a^2}\left[\frac{1}{2}\left(\frac{1}{c}\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\frac{1}{a^2}(\overline{\nabla}_x\phi)^2\right] - \frac{1}{a^2}V(\phi)$$

$$+\ T^{22} = \frac{1}{a^4}\left(\frac{\partial\phi}{\partial y}\right)^2 + \quad '' \quad ''$$

$$+\ T^{33} = \frac{1}{a^4}\left(\frac{\partial\phi}{\partial z}\right)^2 + \quad '' \quad ''$$

$$T^{11}+T^{22}+T^{33} = \frac{1}{a^4}(\overline{\nabla}_x\phi)^2 + \frac{3}{2a^2}\left[\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{a^2}(\overline{\nabla}_x\phi)^2\right] - \frac{3}{a^2}V(\phi) = \frac{3p}{a^2} \quad \Big/ \frac{a^2}{3}$$

$$p = \frac{1}{a^2}\cdot\frac{1}{3}(\overline{\nabla}\phi)^2 + \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2a^2}(\overline{\nabla}\phi)^2 - V(\phi) \qquad \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}$$

$$\boxed{p = \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{6a^2}(\overline{\nabla}_x\phi)^2 - V(\phi)}$$

If $\forall\alpha : \partial_\alpha \phi$ is negligible with respect to $V(\phi)$

$$T^{\alpha\beta} \approx -p\, g^{\alpha\beta} \approx \rho c^2 g^{\alpha\beta} \approx V(\phi) g^{\alpha\beta} \equiv \frac{\Lambda_{eff} c^4}{8\pi G} g^{\alpha\beta}$$

and the term of potential energy corresponds to an effective cosmological constant.

Phase transitions and Symmetry Breaking

In the history of the early universe one or more phase transitions have occurred. At high energies, according to the unified theory of the electroweak interaction, the weak and electromagnetic interactions were manifestations of a single force. Then, due to the progressive cooling produced by cosmic expansion, at a certain time (around a critical temperature T_{EW}~$10^{15}K$, E_{EW}~$10^2 GeV$) the universe has undergone a phase transition, after which the two interactions separated.

The Grand Unified Theories (GUTs), which attempt to unify electromagnetism and weak and strong interactions, in turn, require a phase transition in the universe at critical temperature T_{GUT}~10^{28}-$10^{29}K$, E_{GUT}~10^{15}-10^{16} GeV, above which there was symmetry between the three interactions.

Let’s consider an analogy with the magnetization of a ferromagnetic material. Above the Curie temperature T_C the magnetic moments linked to the spins of atoms are randomly oriented and rapidly fluctuating, there is rotational symmetry around each point of the material and the expectation value of (the mean value) of the spin is null ($\langle \mathbf{S} \rangle = 0$). However, falling the temperature below T_C, alignemet of spins becomes energetically more favorable, and there is a phase transition to a magnetized state, with $\langle \mathbf{S} \rangle \neq 0$ in a certain direction $\hat{\imath}$. The original symmetry is lost, broken, because the different domains that begin to form, independently of each other, have spins with different directions. In the end, when the whole mass has turn into domains, defects form at the borders of the different regions.

In a similar way, while above T_{GUT} there was symmetry between the three interactions, below T_{GUT} it is broken. Going back to the case of the ferromagnetic material, the way in which the rotational symmetry is broken in the different portions of the mass can be measured by the growth of the spin $\mathbf{S}$ and the orientation of the different domains. Similarly, the way in which the symmetry between the three interactions breaks down can be characterized by the acquiring of non-null values of parameters named ***Higgs fields***; this phenomenon is called ***spontaneous symmetry breaking*** (*SSB*). The symmetry is present when the Higgs fields have zero expectation value; it is spontaneously broken when at least one of the boson fields acquires an expectation value other than zero. As in the case of ferromagnetic domains, defects remain at the boundaries of the different regions in which the symmetry is broken in different ways, assuming different sets of values for the Higgs fields. These defects are called ***topological defects***, and may be two-dimensional (domain walls), uni-dimensional (cosmic strings) and zero-dimensional (magnetic monopoles). During the phase transition that leads to the breaking of the symmetry a period of exponential expansion may also occur: the inflation. Let's see how.

For simplicity we consider a single Higgs field, the scalar field ϕ. We consider again a Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \phi)(\partial^\alpha \phi) - V(\phi)$$

The equation of motion, a generalization of *Klein-Gordon* (*KG*) equation, becomes

$$\Box^2 \phi + \frac{\partial V}{\partial \phi} = 0$$

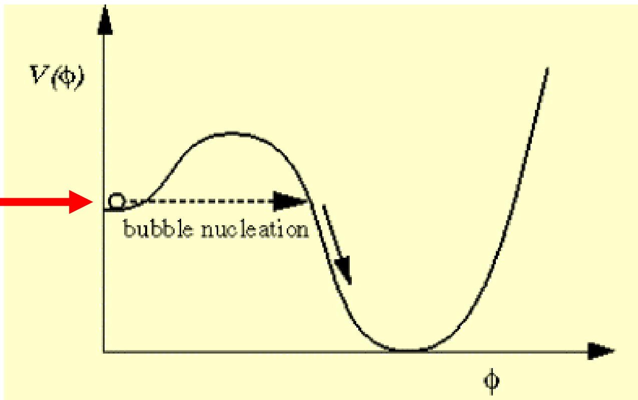
Free particle states are the solution of this equation with only a quadratic term in ϕ in the potential $V(\phi)$, like in *KG* case; the coefficient of this term specifies the mass m of the particle: $V(\phi)=1/2\ \mu\ \phi^2$, $\mu = m\,c/\hbar$. The "***vacuum***" state, which by definition is the state in which there are no particles, occurs when $\partial V/\partial\phi=0$; in the *KG* case this occurs at $\phi=0$.

Higher-order terms in $V(\phi)$ correspond to the interactions between these particles. The equation written above admits the solution $\phi=constant$ at any value of ϕ for which $\partial V/\partial\phi=0$. The vacuum (no particles) state will therefore be one of those in which the expectation value of ϕ assumes one of these constant values. There are several possibilities:

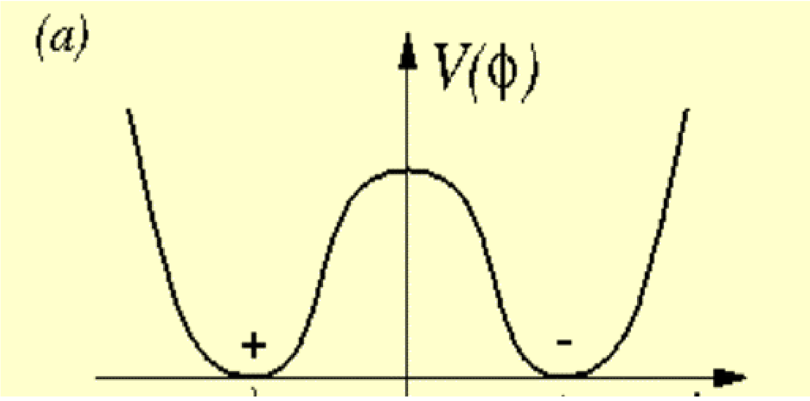
- It may be that the $\partial V/\partial\phi=0$ has only one solution. In order for the energy to be bounded below, this should correspond to a minimum of $V(\phi)$ and also corresponds to the unique vacuum of the theory
- On the other hand there may be multiple solutions of $\partial V/\partial\phi=0$. The maxima of the potential are unstable, but all the minima are possible vacua of the theory. If there is more than a minimum, the lowest would be the ultimate vacuum, the "***true vacuum***" of the universe.
- However, the universe may be, at a certain moment, in a local minimum with a higher value of the potential; it would be in a "***false vacuum***", with the possibility, for instance by tunnel effect, to move to the *true vacuum*.

Potential shape for "*old inflation*" (see below).

The Inflation phase, for this potential, corresponds to the trapping of the field in the well at $\phi=0$.



- In some cases there may be several such minima that have the same value of the potential, and the vacuum is degenerate.



This Figure corresponds to a potential of the form

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 + V(0)$$

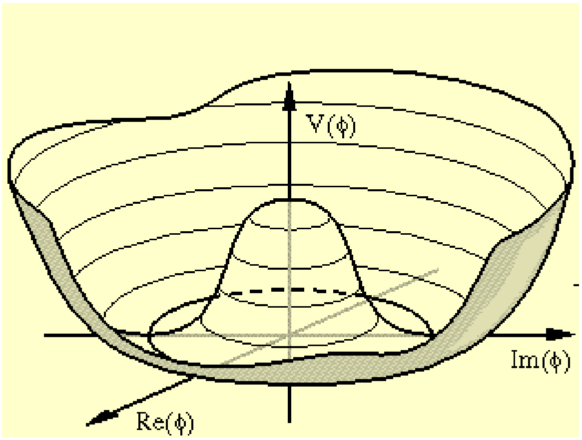
with μ and λ real constants ($\lambda > 0$ if the potential is bounded from below). The first term on the r.h.s. looks like a mass term and the second like an interaction, but the sign of the mass term is wrong, the mass should be imaginary! However, $\phi=0$ is a maximum for the potential, and we have two, degenerate, minima corresponding to possible "vacua" or groud states for

$$\phi = \pm\mu/\sqrt{\lambda} \equiv \pm v$$

Peturbation theory involves an expansion of $\mathcal{L}$ in ϕ around a minimum of the potential. We arbitrarily choose one of the two minima, for instance $+v$, and define a new field $\eta \equiv \phi - v$. We write the potential as a function of the new field η and now the Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \eta)(\partial^\alpha \eta) - [\mu^2\eta^2 + \text{terms cubic and higher in } \eta + \text{const.}]$$

which possess the right sign of the mass term and complicated interactions. If we chose the other minimum the mass term remains the same (only the η^3 term changes his sign).



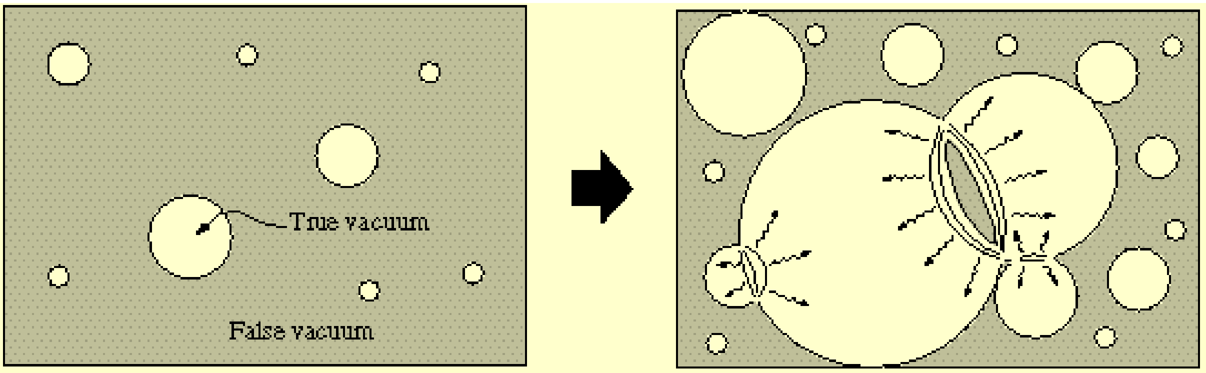
This is an example with only two possible values for the true vacuum, but more general potentials can lead to an infinite number of possible values in which the true vacuum may end. Here we see a two-dimensional (complex) case for the potential [we substitute $\phi\phi^*$ to ϕ^2 and $(\phi\phi^*)^2$ to ϕ^4, where ϕ^* is the complex conjugate of ϕ]. True possible vacua correspond to points belonging to the circle of the minimum values of $V(\phi)$.

The random choice of one of the minima generates a ***spontaneous symmetry breaking*** (*SSB*), similar to the formation of a domain with a particular orientation of the spins of its atoms in a portion of a ferromagnetic material that cools below T_C.

The potential written above has this form, with a negative coefficient for ϕ^2, at a temperature $T = 0$. But in the early universe, when the temperature is very high, to take into account this effect, corrections to $V(\phi)$ produce an effective potential with additional terms proportional to $\phi^2 T^2$. In this way the coefficient of ϕ^2 is positive at a temperature high enough, the minimum of the potential is at $\phi = 0$, and the symmetry is unbroken.

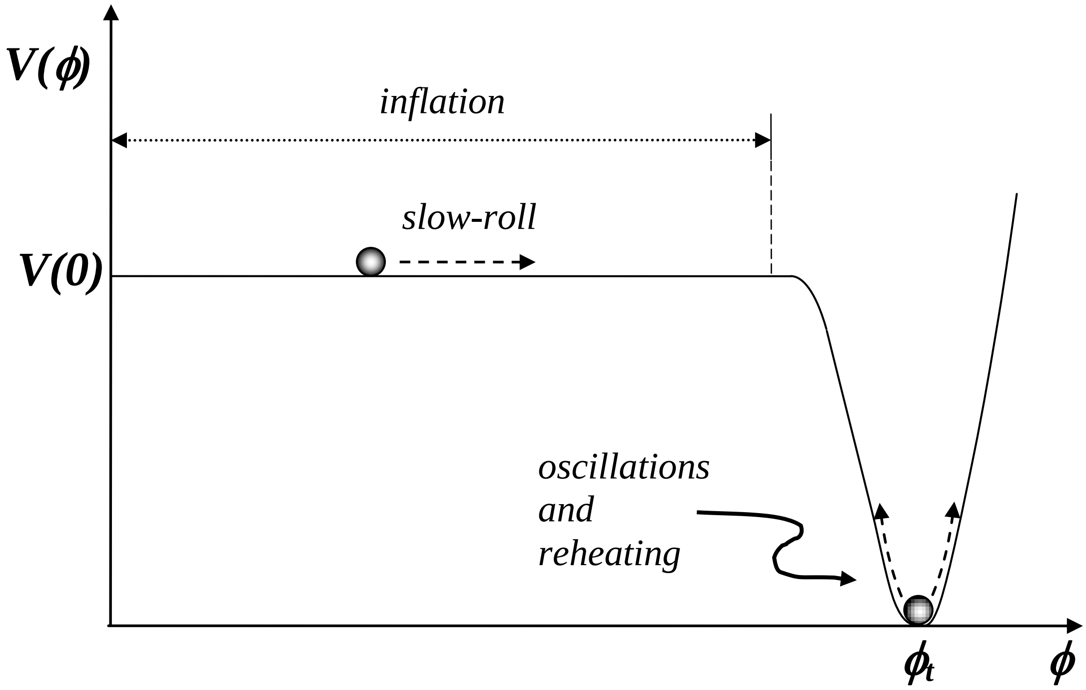
During the cooling of an expanding universe, according to details that depend on the particular shape of $V(\phi)$, the spontaneous symmetry breaking will take place:

- Through a ***phase transition of first order***, in which the field, initially in $\phi = 0$, crosses, by tunnel effect, a potential barrier within which it remains trapped for a certain time; inflation, with $\partial\phi/\partial t = 0$, occurs during this trapping phase. This is the model initially proposed by Guth, named ***old inflation***, which presents, however, some problems. In fact, a phase transition of first order occurs through the formation of bubbles of the new phase in the middle of the old phase; these bubbles expand, collide and coalesce until the new phase completely replaces the old one.



  But in the model of Guth, to have a phase inflationary sufficiently long, the probability of forming bubbles is low and, since the false vacuum expands exponentially, the bubbles can not coalesce and the transition to the true vacuum does not occur.

- Through a ***phase transition of the second order***, in which the field evolves smoothly from one phase to another. This is the model of ***new inflation***, proposed by *Linde, Albrecht and Steinhardt* in 1982, in which the field evolves very slowly (***slow-roll***) from the condition of false vacuum at $\phi = 0$ to the true vacuum. Again, if the evolution from $\phi = 0$ takes place slowly and $V(\phi) \simeq V(0)$ for a long enough time before falling into the true vacuum, we have a phase of inflation (we will see, later, what are the conditions for this to happen).



Orders of magnitude for Inflation

The expansion in the Early Universe, if we neglect the curvature (which, however, tends rapidly towards zero due to the enormous growth of the scale factor), will be given by the equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

and the energy density is given by

$$\rho c^2 = \rho_R c^2 + \rho_\Lambda c^2$$

with

$$\rho_R c^2 = \frac{\pi^2}{30} \cdot \frac{g_*(T)(kT)^4}{(\hbar c)^3}$$

$$\rho_\Lambda c^2 = V(\phi = 0)$$

Where $V(\phi)$ corresponds to the energy density of the field ϕ that, at high temperature, has its minimum in $\phi = 0$.
Until ρc^2 is dominated by $\rho_R c^2$, the universe behaves as in the model of EdS dominated by radiation, with $a(t) \sim t^{1/2}$. But while $\rho_R c^2$ scale as $1/a^4$, $\rho_\Lambda c^2$ remains constant. At a certain time t_i, $\rho_R c^2 \sim \rho_\Lambda c^2$, and, from that moment, the expansion becomes dominated by an "effective" cosmological constant Λ_{eff}, as in the exponentially expanding de Sitter model:

$$a(t) = a_i \exp\left[\sqrt{\frac{8\pi G}{3c^2}V(\phi = 0)}\left(t - t_i\right)\right]$$

$$H = \cos t = \sqrt{\frac{8\pi G}{3c^2}V(\phi = 0)} \equiv \sqrt{\frac{\Lambda_{eff}c^2}{3}}$$

where a_i *is the scale factor at* t_i. At the equality time, at $T=T_\Lambda$,

$$\rho_\Lambda c^2 = \rho_R c^2 = \frac{\pi^2}{30} \cdot \frac{g_*(T_\Lambda)\left(kT_\Lambda\right)^4}{\left(\hbar c\right)^3}$$

$$\approx 1.1\times 10^{40}\left(kT_\Lambda\right)^4_{GeV} \text{ erg/cm}^3$$

$$\approx 1.1\times 10^{100}\left(kT_{\Lambda 15}\right)^4 \text{ erg/cm}^3$$

where $kT_{\Lambda 15}$ is the energy scale in units of 10^{15}*GeV*. To this energy density corresponds an "effective" cosmological constant

$$\Lambda_{GUT} = \frac{8\pi G}{c^4}\rho_\Lambda c^2 = 2.3\times 10^{-8}(kT_\Lambda)^4_{GeV}\ cm^{-2}$$

$$\approx 2.3\times 10^{52}(kT_{\Lambda 15})^4\ cm^{-2}$$

If we compare this value (Λ_{GUT}) for $kT_{\Lambda 15} = 1$, with that of the cosmological constant today ($\Lambda_0 \simeq 10^{-56}$ cm^{-2}) we get a huge ratio (fine-tuning?):

$$\frac{\Lambda_{GUT}}{\Lambda_0} \approx 10^{108}$$

The Hubble constant, during the phase in which the system is trapped in the false vacuum and the expansion occurs exponentially, is

$$H = \sqrt{\frac{\Lambda}{3}} \cdot c = 2.6\times 10^6(kT_\Lambda)^2_{GeV} = 2.6\times 10^{36}(kT_{\Lambda 15})^2\ s^{-1}$$

If we take $kT_{\Lambda 15} = 1$, and we want that $Ht_f \geq 60$ to solve the horizon and flatness problems, then we have that

$$t_f \geq \frac{60}{H} \approx 2\times 10^{-35}\ s$$

as the epoch of the end of inflation, while the start, using a model of EdS dominated by radiation, is given by

$$t_i \approx \frac{1}{2H} \approx 2\times 10^{-37}\ s$$

These are order of magnitude estimates and depend on the value of kT_Λ adopted.

Dynamics of the Inflaton

Let us derive the evolution equation of ***inflaton***, i.e. the scalar field ϕ, starting from the Lagrangian density:

$$S_\phi = \frac{1}{c}\int \sqrt{-g}\ \mathcal{L}\, d^4x$$

For an expanding universe, spatially flat, in orthogonal coordinates, $\sqrt{-g} = a^3$ and the Euler-Lagrange equations are applied to the quantity $a^3\mathcal{L}$:

$$a^3\mathcal{L} = \frac{a^3}{2}\partial_\mu\phi\partial^\mu\phi - a^3 V(\phi)$$

If the inflaton is solely dependent on time, and not on the spatial coordinates, only ∂_0 will be different from zero and

$$a^3\mathcal{L} = \frac{a^3}{2}\cdot\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - a^3 V(\phi)$$

and the Euler-Lagrange equations give:

$$\frac{\partial\left(\mathcal{L}a^3\right)}{\partial\left(\partial_\mu\phi\right)} = \frac{\partial\left(\mathcal{L}a^3\right)}{\partial\left(\dfrac{\partial\phi}{c\,\partial t}\right)} = c\cdot\frac{a^3}{2c^2}\cdot 2\frac{\partial\phi}{\partial t} = \frac{a^3\dot{\phi}}{c}$$

$$\partial_\mu\left(\frac{\partial\left(\mathcal{L}a^3\right)}{\partial\left(\partial_\mu\phi\right)}\right) = \frac{1}{c}\frac{\partial}{\partial t}\left[\frac{a^3\dot{\phi}}{c}\right] = \frac{1}{c^2}\left[a^3\ddot{\phi} + \dot{\phi}\cdot 3a^2\dot{a}\right]$$

$$\frac{\partial\left(\mathcal{L}a^3\right)}{\partial\phi} = -a^3\frac{dV}{d\phi}$$

Putting together and simplifying we get ($\dot{a}/a = H$)

$$\ddot{\phi} + 3H\dot{\phi} + c^2 \frac{dV}{d\phi} = 0$$

which represents the evolution of the inflaton.
This equation, if we refer to the typical potential of *new inflation*, has two different regimes, one called "***slow roll***" and one during which rapid oscillations around the minimum develop. Let's look at them in more detail.
a) ***Slow-roll regime***: is this phase there is a slow "rolling" of the non-accelerated field ϕ which corresponds to the ***phase of inflation***. In this regime the term $\ddot{\phi}$ is negligible and the equation of motion reduces to

$$3H\dot{\phi} = -c^2 \frac{dV}{d\phi}$$

that is, the “friction” due to the expansion is dynamically balanced by the acceleration due to the slope of the potential.

By using the derivative of the above relation, recalling that H is essentially constant during inflation, and naming V'' the $d^2V/d\phi^2$, the condition $|\ddot{\phi}| \ll |3H\dot{\phi}|$ gives

$$\left|\frac{c^2 V''\dot{\phi}}{3H}\right| << \left|3H\dot{\phi}\right| \rightarrow \left|V''\right| << \frac{9H^2}{c^2} \approx \frac{9}{c^2}\cdot\frac{8\pi G}{3}\cdot\frac{V}{c^2} \approx \frac{24\pi G V}{c^4}$$

$$\eta(\phi) \equiv \frac{c^4}{24\pi G}\left|\frac{V''}{V}\right| << 1$$

Another crucial condition to have $p = -\rho c^2$ is that $\dot{\phi}^2/2c^2 \ll V(\phi)$, which leads to

$$\left(\frac{c^2 V'}{3H}\right)^2 << 2c^2 V \rightarrow c^2 V'^2 << 2V \cdot 9H^2 \approx 2V \cdot 9 \cdot \frac{8\pi G V}{3c^2}$$

$$\varepsilon(\phi) \equiv \frac{c^4}{48\pi G}\left(\frac{V'}{V}\right)^2 << 1$$

The two constraints on the potential, $\eta \ll 1, \varepsilon \ll 1$ are the *slow-roll conditions*.

b) ***Fast oscillations***: At the end of the inflation phase, the potential "falls" in the true vacuum and the inflaton oscillates rapidly around the minimum. If nothing more happened, we would have oscillations undergoing redshift as time goes on, in a universe that has already cooled dramatically during the inflationary adiabatic expansion. In order that the thermal history of the universe evolves as suggested by the evidence (e.g. BBN) it is required that the energy of the false vacuum is converted into matter and radiation with a certain efficiency. This process is called ***reheating***. We have already noted that inflation rapidly diluted magnetic monopoles because the energy density of the scalar field remains constant, while the density of monopoles decreases as $1/a^3$ (this does not mean that they disappeared completely, one day they will return within the horizon) . However, in order not to be recreated by reheating, it is necessary that this does not bring again the temperature of the universe to values able to remake them.

The number of e-foldings: It is immediate to calculate the number $\mathcal{N}$ of *e-foldings*. We start from

$$H = \frac{\dot{a}}{a} = \frac{da}{a\,dt} \to \frac{da}{a} = H\,dt \to \int_{a_i}^{a_f} d\ln(a) = \int_{t_i}^{t_f} H\,dt$$

$$\mathcal{N} \equiv \ln\left(\frac{a_f}{a_i}\right) = \int_{t_i}^{t_f} \frac{H^2}{H}\,dt = -\int_{t_i}^{t_f} \frac{8\pi G}{3} \cdot \frac{V(\phi)}{c^2} \cdot \frac{3\dot{\phi}}{c^2 V'(\phi)}\,dt$$

$$\mathcal{N} = -\frac{8\pi G}{c^4} \int_{\phi_i}^{\phi_f} \cdot \frac{V(\phi)}{V'(\phi)}\,d\phi \,.$$

Other models of inflation

In the model of inflation that we have described, a spontaneous breaking of the symmetry occurs, but it is possible to achieve inflation even without SSB, as in the case of the so-called ***chaotic inflation*** proposed by Linde (1983), in which the potential $V(\phi)$ is simply

$$V(\phi) = \lambda\phi^4$$

and the potential has a minimum at $\phi = 0$. The phase of inflation takes place if, within the horizon, the field, due to quantum fluctuations, assumes a value different from zero in a region of the universe and then returns toward the minimum. This is more likely to happen at the end of Planck era, rather than at the time of the breaking of great unification.

To solve the problems of the standard model it is not necessary a stage with exponential expansion of the scale factor; it is sufficient that

$$a(t) \propto t^p \qquad p > 1$$

(***power-law inflation***) The required potential has the shape

$$V(\phi) \propto e^{\alpha\,\phi}$$

In the above discussions we have assumed that space is flat, homogeneous and isotropic. What happens if it is not the case? It can be seen (see for example chap. 8, paragraph 6, in "The Early Universe" Kolb and Turner) that, unless the initial space curvature is so high to force the universe to recollapse before inflation, this phase produces, for a wide class of models, huge regions uniform and flat, which exceed in size the current Hubble radius, and then solve the problems of the standard model. Inhomogeneity and/or anisotropy are, however, only delayed and will eventually reappear.

Cosmological constant and Dark Energy

How can we interpret today's cosmological constant? In Einstein's equations (and those of Friedmann), if we remove all matter and radiation, the cosmological constant is the only source of the field: Λ corresponds to the ***density of the vacuum***.
But, according to Quantum Field Theory, the vacuum is not the nothingness of metaphysics, but the ground state of minimum energy, with no particles, of the field itself. We have seen that the cosmological constant behaves as a perfect fluid with $\rho_\Lambda c^2=\varepsilon_\Lambda=\Lambda c^4/8\pi G$ and $p_\Lambda=-\varepsilon_\Lambda=-\rho_\Lambda c^2$, and the energy-momentum tensor is diagonal

$$T^{\alpha\beta} \equiv \begin{pmatrix} \varepsilon_\Lambda & 0 & 0 & 0 \\ 0 & p_\Lambda & 0 & 0 \\ 0 & 0 & p_\Lambda & 0 \\ 0 & 0 & 0 & p_\Lambda \end{pmatrix}$$

Moreover, it must be expected that the values of ε_Λ and p_Λ, that define the state of vacuum, are the same in any, not accelerated, reference frame, so they have to be relativistic invariants.

If, for example, we make a Lorentz transformation with velocity $v=\beta c$ [$\gamma^2=1/(1-\beta^2)$] along the axis x^1, $T^{\alpha\beta}$ changes according to the rule

$$T'^{\alpha\beta} = \Lambda^\alpha_\gamma \Lambda^\beta_\delta T^{\gamma\delta}$$

where

$$\Lambda^\alpha_\gamma \equiv \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and $T^{\gamma\delta}$ is diagonal, as above. We get (see the following scanned page for the proof):

$$T'^{00} = \varepsilon'_\Lambda = \frac{\varepsilon_\Lambda + \beta^2 p_\Lambda}{1-\beta^2}$$

$$T'^{01} = \gamma^2 \beta (\varepsilon_\Lambda + p_\Lambda) = T'^{10}$$

$$T'^{11} = \frac{\beta^2 \varepsilon_\Lambda + p_\Lambda}{1-\beta^2}$$

$$T'^{22} = T'^{33} = p_\Lambda$$

$$\Lambda^{\alpha}{}_{\beta} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \gamma^2 = \frac{1}{1-\beta^2} \qquad \rho_\Lambda c^2 = \varepsilon_\Lambda$$

$$T^{\alpha\beta} \begin{pmatrix} \rho_\Lambda c^2 \; (=\varepsilon_\Lambda) & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

Se $\alpha=\beta$ $T^{\alpha\beta} \neq 0$

$$T'^{00} = \Lambda^0_\alpha \Lambda^0_\beta T^{\alpha\beta} = \Lambda^0_\alpha \Lambda^0_\alpha T^{\alpha\alpha} = \gamma^2 \rho_\Lambda c^2 + \gamma^2\beta^2 p = \gamma^2[\varepsilon_\Lambda + \beta^2 p]$$

$$\boxed{\varepsilon'_\Lambda = \frac{\varepsilon_\Lambda + \beta^2 p}{1-\beta^2}}$$

$$T'^{01} = \Lambda^0_\alpha \Lambda^1_\beta T^{\alpha\beta} = \Lambda^0_\alpha \Lambda^1_\alpha T^{\alpha\alpha} = \gamma^2\beta\,\varepsilon_\Lambda + \gamma^2\beta\, p = \boxed{\gamma^2\beta(\varepsilon_\Lambda + p)} = T'^{10}$$

$$T'^{11} = \Lambda^1_\alpha \Lambda^1_\beta T^{\alpha\beta} = \Lambda^1_\alpha \Lambda^1_\alpha T^{\alpha\alpha} = \gamma^2\beta^2\varepsilon_\Lambda + \gamma^2 p = \gamma^2[\beta^2\varepsilon_\Lambda + p] = \boxed{\frac{\beta^2\varepsilon_\Lambda + p}{1-\beta^2}}$$

$$T'^{02} = \Lambda^0_\alpha \Lambda^2_\alpha T^{\alpha\alpha} = 0 \ldots$$

$$T'^{22} = \Lambda^2_\alpha \Lambda^2_\alpha T^{\alpha\alpha} = p = T'^{33}$$

Se $p = -\varepsilon_\Lambda$ $\qquad \varepsilon'_\Lambda = \dfrac{\varepsilon_\Lambda[1-\beta^2)}{1-\beta^2} = \varepsilon_\Lambda$

$T'^{01} = T'^{10} = 0 \qquad T'^{11} = \dfrac{p[1-\beta^2)}{1-\beta^2} = p$

and all the other terms of $T'^{\alpha\beta}$ are null. In order that $\varepsilon'_\Lambda = \varepsilon_\Lambda$, $p'_\Lambda = p_\Lambda$ and $T'^{\alpha\beta}$ is diagonal $p_\Lambda = -\varepsilon_\Lambda = -\rho_\Lambda c^2$ is required. Thus we see that the "false" vacuum of inflation, and the "true" void share a similar equation of state.

We can also consider the vacuum as a "substance" with given ε_Λ and p_Λ, in the sense that the relation $dL = -dV = -pdV$ (since $dQ = 0$) is satisfied. Indeed $dU = d(\varepsilon_\Lambda V) = \varepsilon_\Lambda dV = -p_\Lambda dV$ *if* $p_\Lambda = -\varepsilon_\Lambda$.

But can we estimate the expected value for ε_Λ? (Note that in the following pages $c=1$).

5.3 The Λ problem

We now return to the contribution of quantum fluctuations to the vacuum energy density. First consider a quantum mechanical harmonic oscillator. Its energy eigenvalues are given by

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) \qquad n = 0, 1, \ldots . \tag{5.29}$$

The vacuum ($n = 0$) therefore has a finite amount of energy (zero point energy). A relativistic field can be considered as a sum of harmonic oscillators of all possible frequencies ω. In the simple case of a scalar field with mass m, the vacuum energy is the sum of all contributions:

$$E_0 = \sum_j \frac{1}{2}\hbar\omega_j . \tag{5.30}$$

This summation can be carried out by putting the system in a box of size L^3 and then considering the limit as L tends to infinity (see e.g. [Car92]). Assuming periodic boundary conditions, equation (5.30) becomes (using again $\hbar = 1$)

$$E_0 = \frac{1}{2}L^3 \int \frac{d^3k}{(2\pi)^3}\omega_k \tag{5.31}$$

where $k = 2\pi/\lambda$ corresponds to the wave vector. Now, using the relation

$$\omega_k^2 = k^2 + m^2 \tag{5.32}$$

and a maximum cut-off frequency $k_{\mathrm{max}} \gg m$, the integration can be carried out, resulting in

$$\rho_V = \lim_{L\to\infty} \frac{E_0}{L^3} = \int_0^{k_{\mathrm{max}}} \frac{4\pi k^2}{(2\pi)^3} \mathrm{d}k \frac{1}{2}\sqrt{k^2+m^2} = \frac{k_{\mathrm{max}}^4}{16\pi^2}. \tag{5.33}$$

Assuming the validity of the general theory of relativity up to the Planck scale ($l_{Pl} \simeq (8\pi G)^{-\frac{1}{2}}$), $l_{Pl} = k_{\mathrm{max}}$ results in a value which lies 121(!) orders of magnitude above the experimental value. This would lead to a vacuum energy of [Car92]

$$\rho_V = 10^{74}\ \mathrm{GeV}^4 \approx 10^{92}\ \mathrm{g\ cm}^{-3}. \tag{5.34}$$

It is rare for an estimate to be quite so incorrect. In addition, a value for k_{max} at approximately the electroweak scale of about 200 GeV, leads to a discrepancy of 54 orders of magnitude. Even with a k_{max} of order Λ_{QCD}, the prediction is still 42 orders of magnitude away from observation.

This is the so-called ***cosmological constant problem*** from the point of view of Field Theory. There is a second problem linked to the coincidence between matter-energy density and cosmological constant: why do they are comparable today?
Many attempts have been done to find a solution to these problems.
The discovery of SUSY led to the hope that, since bosons and fermions (of identical mass) contribute equally but with opposite sign to the vacuum expectation value, the cosmological constant should be zero. But SUSY is today broken, so Λ could be zero only in the early universe. Attempts has been done to produce a (almost) vanishing cosmological constant also with broken SUSY.
Anthropic explanations have been proposed. In several cosmological theories the observed big bang is just one member of an ensemble. The ensemble may consist of different expanding regions at different times and locations in the same spacetime, or of different terms in the wave function of the universe. If the vacuum energy density ρ_V varies among the different members of this ensemble, then the value observed by any species of astronomers will be conditioned by the necessity that this value of ρ_V should be suitable for the evolution of intelligent life.
The anthropic bound on a positive vacuum energy density is set by the requirement that ρ_V should not be so large as to prevent the formation of galaxies (the accelerated expansion stops the growing of the amplitude of density fluctuations). A negative value for the cosmological constant, as we have seen, acts as an additional self-gravity and forces the recollapse of the universe; if this recollapse happens too early, no intelligent life can develop.

Dynamical models of Dark Energy

Many ideas have been proposed to solve the problem of Dark Energy (if you are interested in this subject you can refer to the book "DARK ENERGY, Theory and Observations" by Luca Amendola and Shinji Tsujikawa, 2010, Cambridge University Press).

There are basically two approaches for the construction of dark energy models. The first approach is based on "modified matter models" in which the energy-momentum tensor $T_{\mu\nu}$ on the r.h.s. of the Einstein equations contains an exotic matter source with a negative pressure. The second approach is based on "modified gravity models" in which the Einstein tensor $G_{\mu\nu}$ on the l.h.s. of the Einstein equations is modified.

Here we mention the so-called ***quintessence***[2] model as one of the representative modified matter models.

Quintessence is a canonical scalar, uniform field Q with a potential $V(Q)$ responsible for the late-time cosmic acceleration. Unlike the cosmological constant, the equation of state of quintessence dynamically changes with time: $p_Q = w_Q \rho_Q c^2$ with

$$\rho_Q c^2 = \frac{\dot{Q}^2}{2c^2} + V(Q)$$

$$p_Q = \frac{\dot{Q}^2}{2c^2} - V(Q)$$

$$w_Q = \frac{\frac{\dot{Q}^2}{2c^2} - V(Q)}{\frac{\dot{Q}^2}{2c^2} + V(Q)}$$

where w_Q can be in the range from -1 to +1. Here we can use relations similar to those used when working on inflation. We assume a flat, $k=0$, universe. The evolution of the field and the dynamics of the universe are given by the already known relations

$$\ddot{Q} + 3\frac{\dot{a}}{a}\dot{Q} + c^2\frac{dV(Q)}{dQ} = 0$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_Q + \rho)$$

$$\rho_Q c^2 = \frac{\dot{Q}^2}{2c^2} + V(Q)$$

Peebles and Ratra proposed a potential like

[2] According to ancient Greek science, the quintessence (from the Latin "fifth element") denotes a fifth cosmic element after earth, fire, water, and air.

$$V(\phi) \simeq \frac{\varkappa}{Q^{\alpha}}$$

and assumed that at high redshift the density of the field is "subdominant" with respect to that of matter/radiation, in order to preserve the BBN. We assume that the scale factor grows as

$$a(t) \propto t^q$$

The equation of the field is

$$\frac{dV(Q)}{dQ} = \frac{d}{dQ}\left[\frac{\varkappa}{Q^{\alpha}}\right] = -\varkappa\alpha Q^{-\alpha-1}$$

$$\ddot{Q} + 3\frac{\dot{a}}{a}\dot{Q} - \frac{c^2\varkappa\alpha}{Q^{\alpha+1}} = 0$$

$$a \sim t^q \Rightarrow \frac{\dot{a}}{a} = \frac{q\,t^{q-1}}{t^q} = \frac{q}{t}$$

$$\ddot{Q} + 3\frac{q}{t}\dot{Q} = \frac{c^2\varkappa\alpha}{Q^{\alpha+1}}$$

the solution is

$$Q = At^p \rightarrow \begin{cases} \dot{Q} = pAt^{p-1} \\ \ddot{Q} = Ap(p-1)t^{p-2} \end{cases}$$

$$Ap(p-1)t^{p-2} + 3\frac{q}{t}pAt^{p-1} = \frac{c^2\varkappa\alpha}{A^{\alpha+1}(t^p)^{\alpha+1}}$$

$$A\left[p(p-1) + 3qp\right]t^{p-2} = \frac{c^2\varkappa\alpha}{A^{\alpha+1}}t^{-p(\alpha+1)}$$

$$\Rightarrow p-2 = -p(\alpha+1) \rightarrow p + p(\alpha+1) = 2 \rightarrow p(\alpha+2) = 2$$

$$p = \frac{2}{\alpha+2} \qquad Q \sim t^{\frac{2}{\alpha+2}} \leftarrow$$

← doesn't depend on q !!!

$$\rho_Q c^2 = \frac{1}{2c^2}\dot{Q}^2 + V(Q) = \frac{1}{2c^2}A^2 p^2 t^{2(p-1)} + \frac{\kappa}{A^\alpha t^{\alpha p}} =$$

$$= \left[\frac{A^2 p^2}{2c^2} + \frac{\kappa}{A^\alpha}\right] t^{2p-2} \sim t^{-\frac{2\alpha}{\alpha+2}}$$

$$p = \frac{2}{\alpha+2}$$

$$\alpha + 2 = \frac{2}{p}$$

$$\alpha = \frac{2}{p} - 2 = \frac{2-2p}{p}$$

$$\alpha = \frac{2(1-p)}{p}$$

$$\alpha p = 2(1-p)$$

Since, in EdS, $\rho \propto 1/t^2$, both for matter and radiation

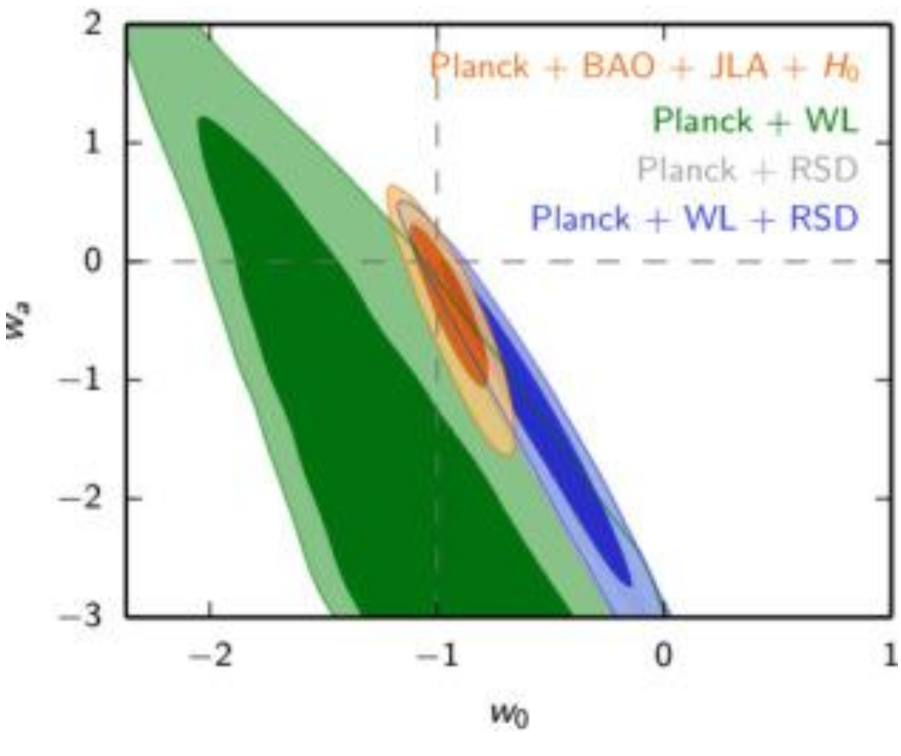
$$\frac{\rho_Q}{\rho} \propto \frac{t^{2p-2}}{1/t^2} \sim t^{2p} \sim t^{\frac{4}{\alpha+2}}$$

α*=*0, *V(Q)=const.*, corresponds to the cosmological constant, $\rho_Q \propto t^0$.
For ***α*>*0*** the scalar field becomes dominant with respect to matter, even if it was negligible at high redshift.

In any case, the first thing to understand is if the vacuum energy density is constant or varies over time. To do that, people uses all the available sets of cosmological observations to fit, for instance, a linear dependence on scale of the equation of state

$$w(a) = w_0 + w_a\,(1 - a/a_0)$$

The results are not conclusive, and a cosmological constant is still consistent with the data (plot taken from Planck satellite 2015 results).



SHORT COSMIC HISTORY

Era	t (sec)	E	T (K)[3]	Events
Planck	10^{-44}	10^{19}GeV	10^{32}	Quantum Gravity
GUT	10^{-38} 10^{-36}	10^{16}GeV 10^{15}GeV	10^{29} 10^{28}	GUT's SSB Inflation? Baryogenesis?
Electroweak	10^{-10}	10^{2}GeV	10^{15}	Electroweak SSB
Adronic	10^{-4}	200 MeV	10^{12}	Quark-adrons transition
Leptonic	0.7	1 MeV	10^{10}	Decoupling of ν_e
	5	0.5 MeV	5×10^{9}	Annichilation e^+e^-
BBN	2-3 min	0.1 MeV	10^{9}	BBN: ^{4}He, ^{3}He, D, ^{7}Li
Radiation-Matter Equality	6×10^{4} yr	2 – 3 eV	$(2-3)\times10^{4}$	Matter-dominated era begins
Recombination	4×10^{5} yr	0.3 eV	3000	The universo becomes neutral and transparent
Void	10 Gyr	10^{-3} eV	3.6	Void-dominated era begins
Today	13.7 Gyr	3×10^{-4} eV	2.73	

yr	sidereal year (1900)	3.1558149984×10^{7} sec
ly	light year	9.4605×10^{17} cm
a.u.	astronomical unit	1.495985×10^{13} cm
pc	parsec	3.0856×10^{18} cm
H_0	Hubble constant	3.241×10^{-18} h sec^{-1}
$1/H_0$	Hubble time	3.086×10^{17} h^{-1} sec
$M_\odot$	solar mass	1.989×10^{33} g
$R_\odot$	solar radius	6.9598×10^{7} cm
$L_\odot$	solar luminosity	3.90×10^{33} erg sec
$M_\oplus$	Earth mass	5.977×10^{27} g
$R_\oplus$	equatorial Earth radius	6.37817×10^{3} km

[3] We use : $T_K \sim 10^{13} E_{GeV}$