Precipitation Formation, and RADAR Equation

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Abstract
Precipitation, by definition, is defined as an ensemble of water particles (solid or liquid) which falls down from a cloud up to the ground. According to this definition, water particles have to be enough large to overcome the air drag as well as the air upward motions. As shown in previous lectures (Kelvin equation) the surface tension of a pure liquid water droplet is so high that the formation of droplets can take place only for extremely high values, currently not observed in nature. For this reason homogeneous nucleation can not be considered as the main mechanism for precipitation formation. When formed, liquid water droplets currently observed in nature are too small to reach the ground in useful times, for this reason even when formed we need a growing mechanism to obtain, starting from the available liquid water droplets, precipitation droplets or rain. Currently two mechanisms for the growing of droplets have been proposed: the coalescence mechanism (or Langmuir mechanism) and the liquid-ice mechanism (Wegener, Bergeron, Findeisen mechanism). Apart from the formation of rain, snow and hail themselves, precipitation are fundamental for the process of cloud electrification and for the lightning formation as well as for the existence and equilibrium of electrosphere.

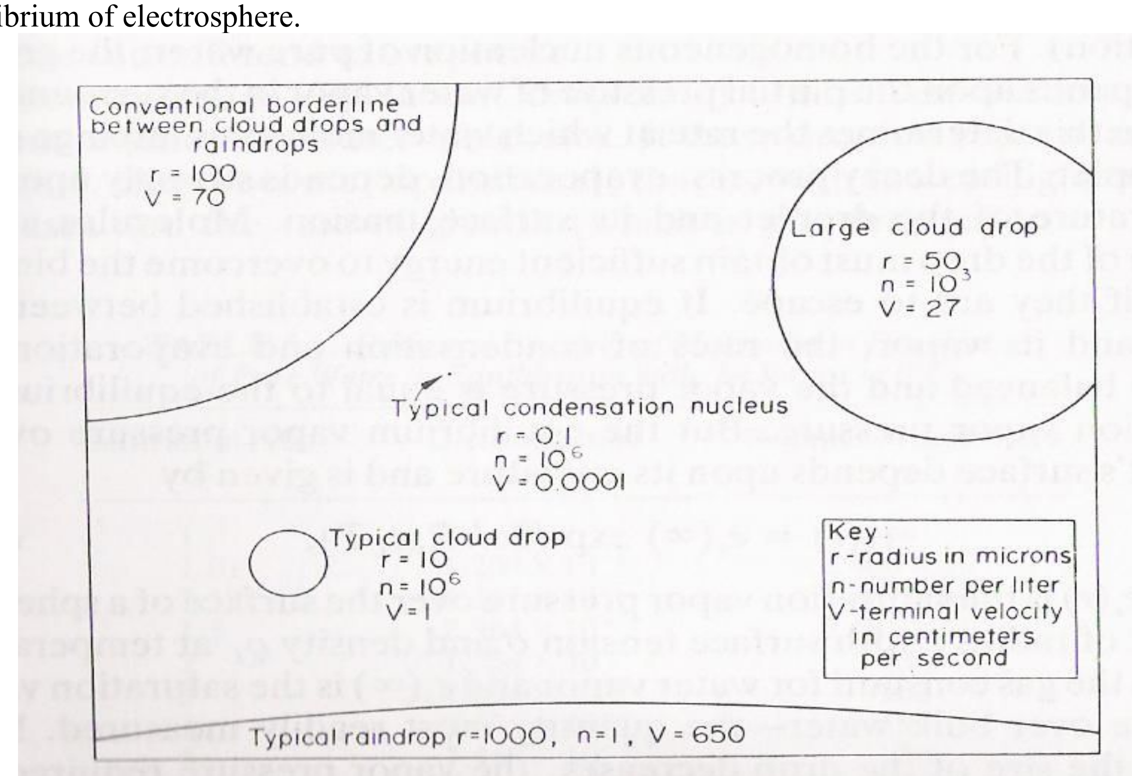

Figure. Relative dimensions and fall speeds of the main atmospheric actors

The most correct way to measure precipitation is counting and measuring their shapes and dimensions (i.e., by way of a disdrometer). Unfortunately, the measuring of precipitation shape and dimension is not an easy task and in general it is possible only on a small portion of surface. Moreover what is really important for the great majority of practical purposes is the total amount of precipitation (mass or volume). For this reason the most widespread instruments used for the measuring of precipitation are rain gages. Even rain gages, however, can sample precipitation only on a relatively small area and, most important, integrating the measurements they sample precipitation on a volume whose shape depends from the time of integration and from the wind field. For this reason currently weather RADARs are used for the measuring of precipitations.

The role of aerosols

Kelvin equation demonstrates that, because of the high surface tension the formation of a liquid water droplet can take place at low supersaturation values (e.g., vapor pressure higher than saturation vapor pressure by just a few percent) only if the droplet is large. Small droplets can form only for extremely high values of supersaturation, currently not observed in nature. For this reason a new ingredient has to enter into the condensation formation. The introduction of aerosols change the the situation because condensation is not taking place homogeneously but on the surface of a pre-existing particle, that is the aerosol particle, overcoming the troubles represented by surface tension. In this new scenario condensation may occur when the equilibrium pressure (i.e., saturation) is reached for the specific type of particle which constitutes the aerosol. This equilibrium pressure can be, and usually it is, lower than the saturation water pressure over bulk liquid water then potentially explaining the currently observed formation of liquid droplets even for small supersaturations or even for sub-saturation over water. The introduction of aerosols gives us the conceptual instruments to explain not only the condensation at the saturation vapor pressure but even the formation of haze, that is the atmospheric turbidity which is observed in several situations. With the introduction of aerosols haze can be explained as due to the presence of particles whose equilibrium water pressure is lower than that of water vapor over bulk water. Since most of the aerosols are part of the condensation mechanism, then of the precipitation formation, they are continuously withdrawn from atmosphere. This is the reason why there should be some sources of aerosols that compensate the sink represented by condensation/precipitation. In any case not all the aerosols present in atmosphere may favor condensation, those that show this behavior are called *condensation nuclei* and are evidenced by the acronym CN. They are measured bringing a fixed volume of water vapor at a supersaturation which is several hundred percents higher than that for bulk water and counting the number of water droplets formed in the volume. The number of CN is extremely high and in general we can assume that the number spans from a few to several thousands per cubic centimeter.

Not all the CN are of meteorological interest because, as often said, the observed supersaturation in atmosphere is that for bulk water or just of the order of a few percents (e.g., 102% or 101%) above that. For this reason a subset of CN is defined to identify those which start to nucleate at ordinary saturation. These, directly involved in the formation of the observed clouds are called *cloud condensation nuclei* and are evidenced by the acronym CCN. The distribution of CCN in atmosphere is function of the place and time of the year but, in general, a distinction is done between *continental air masses* and *maritime air masses* according to the number of CCN contained in it. In particular the continental clouds are characterized by the presence of a few thousands of CCN per cubic centimeter while the maritime clouds are characterized by the presence of a few hundreds of CCN per cubic centimeter. There is even an intermediate class, which is called maritime polluted air masses, which is characterized by an intermediate number of CCN. These intermediate air masses are observed over South Africa and Friuli Venezia Giulia. The observed difference between air masses seems counter-intuitive but it can be explained realizing that the great majority of CCN and CN comes from the air-ground interactions with processes that are so far poorly understood. The distinction between maritime and continental air masses, then clouds, is not merely academical because the radiative properties of clouds are essentially related to the number of the water droplets which constitute them.

In other words continental clouds have an high albedo than maritime ones, then they might have a non negligible impact on the whole Earth's energy balance. In any case, under the precipitation point of view, the number of CCN is so high that at the observed saturation vapor pressures they can not grow simply by condensation of vapor, then they can form but they should not fall down to the ground in a useful time (see paragraphs below).
The same considerations done for the condensation can be done for solidification. In fact the formation of ice requires the overcoming of a surface tension similar to that for condensation of vapor. For this reason the formation of the ice phase starting from the liquid one can be possible only because a small percentage of CCN have even the property to help the overcoming of surface tensions. The CCN which have these properties are called *ice nuclei* and are distinguished with the acronym IN. In any case the number of IN which can trigger solidification exactly at 0 °C is negligible at standard atmospheric conditions, and those usually present in atmosphere start to become active only below -10 °C. Moreover their number is extremely low when compared with that of CCN. In fact normally there is 1 or 0.1 IN every liter of air between -12 °C and -15 °C. This low number of IN is the reason for the extremely frequent presence of supercooled water droplets, that is liquid water below °C in clouds. IN are usually even distinguished according to the physical mechanism which brings to the formation of ice. These mechanisms are contact (the nucleus hits a pre-existing droplet), immersion (the nucleus acts as a CCN and then, being immersed on the droplet, it freezes it), and deposition (the nucleus collects directly water vapor and grows as a crystal).

The growing mechanisms

The terminal fall speed of a spherical droplet can be easily determined assuming the equilibrium between the air drag acting on it and the gravity acceleration. The value of the terminal velocity is function of the droplet diameter and density, of the air density and of the air drag, which is function of the droplet shape.

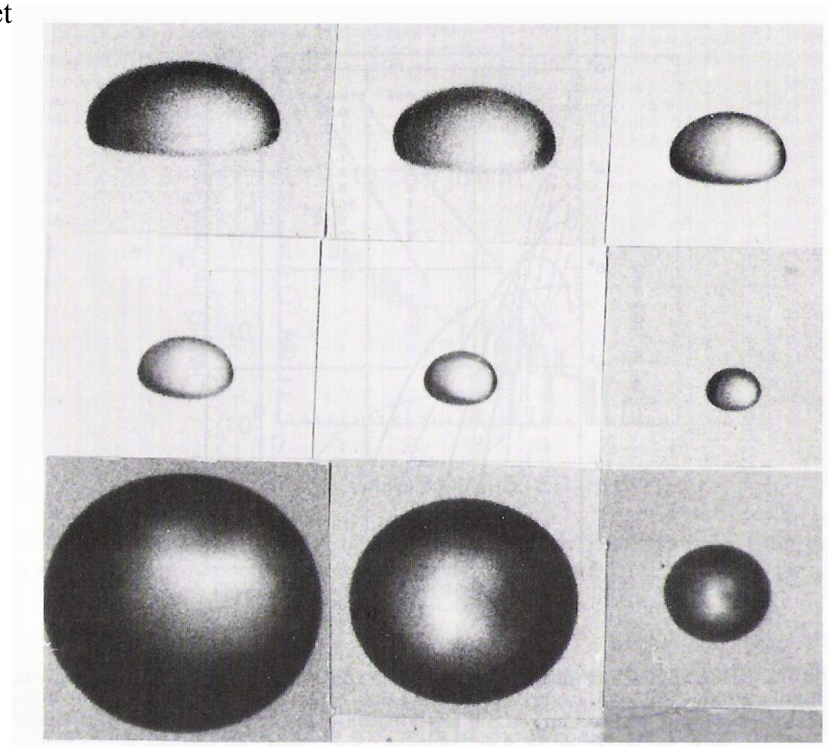

Figure. Shape of rain drops as a function of their dimension. Vertical scale of the lowest panel is 1mm, vertical scale of the upper and middle panel is 1 cm.

$$F_D = F_g$$

that is

$$\frac{\pi R^2}{2} V^2 D \rho_a = \frac{4}{3} \pi R^3 g (\rho_p - \rho_a)$$

where V is the particle terminal velocity, ρ_p indicates the particle density, ρ_a the air density and D is the drag coefficient of the flow.

Using these variables the terminal velocities of cloud droplets are of the order of a few centimeters per second (see figure above). With these velocities cloud droplets cannot reach the ground in useful time, even because the cooling needed to maintain saturation is due to the adiabatic expansion maintained by vertical motions. For this reason we need to introduce a growing mechanism to produce droplets whose dimensions, then terminal velocities, are enough high for the reaching of the ground. The two mechanisms so far introduced and which seems to be those responsible for the precipitation formation are *ice-liquid mechanism* (even called Wegener-Bergeron-Findeisen mechanism) and the *coalescence mechanism* (even called Langmuir mechanism) . The ice-liquid mechanism makes use of the difference between saturation (equilibrium) vapor pressure over ice and liquid phase. In particular when an ice crystal is in the same environment in which liquid water droplets are in equilibrium with vapor because of the continuous activation of new condensation nuclei it is slightly far from equilibrium because of the small difference between equilibrium pressure over ice (slightly lower) and liquid water. For this reason even if condensation nuclei continue to maintain saturation, the ice crystal continues to grow up to the time in which its dimensions are sufficient to warrant a significant terminal velocity. Are the ice crystals that fall down to the ground, melting producing rain, provided that temperature is enough high in the lowest levels near to the ground. This mechanism is enough efficient, but it can take place only in mixed phase clouds and for clouds which have a part at temperatures well below zero. For warm clouds (i.e., those which are completely at temperatures above 0 °C) the coalescence mechanism has to be introduced. According to this mechanism not all the cloud droplets have the same dimension, in particular there will be a distribution of diameters. Fall speed of larger droplets are larger than fall speeds of smaller ones, then because of the differential velocity the probability of collisions will not be null. Because of these collisions, larger droplets will grow quickly than smaller ones, then increasing their fall speed, than increasing their probability of further collisions. This mechanism is enough sufficient, but mainly in maritime clouds (why?) and in clouds characterized by significant (a few meters per second) vertical velocities (why?).

Radar Equation

RADARs work simply emitting a radiation toward a fixed solid angle (in reality RADAR lobes have an extremely complex shape) and measuring the amount of radiation back-scattered by the target. Generally every RADAR is characterized by a gain, defined as the ratio between the power emitted aper unit area along its main lobe and the power per unit mass emitted by a isotropic antenna (i.e, in all the directions), in formulas

$$g = \frac{(\textit{power along beam axis})}{(\textit{power of isotropic antenna})}$$

With the introduction of gain g the power received by a target with scatter section σ which is at a distance r from the antenna which emits a power P is given by

$$p_\sigma = \frac{P\, g\, \sigma}{4\pi\, r^2}$$

if the target emits isotropically (i.e., in the same way toward all the directions), the power received back by the RADAR antenna is

$$p_r = \frac{P\, g\, \sigma\, A_a}{(4\pi\, r^2)^2}$$

where A_a is the effective RADAR antenna area. If the target, assumed as spherical, is single (i.e., just one target on the RADAR lobe) the back scatter section σ is a non monotonic function of its radius. The analytical solution obtained keeping into account the wave propagation of radiation was obtained by James Rayleigh for small wave lengths (when compared to the radius), by Mie for long wave lengths (when compared to the radius) and is equal to the optical scatter section (the sphere silouette) for very long (when compared to the radius) wave lengths. The result is shown the below figure.

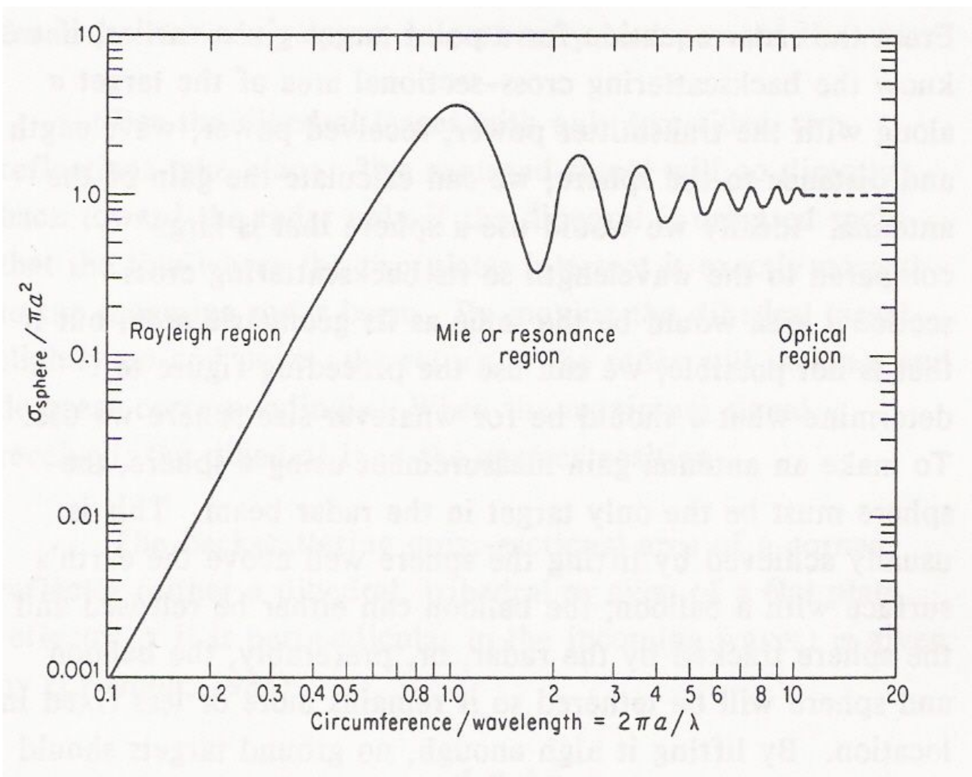

Figure. Normalized (i.e., divided by the optical back scatter section) back scatter section as a function of the normalized (i.e., divided by the radiation wave length) circumference of a single target assumed as spherical.

Meteorological targets are composed by several single particles, then the total back scatter section for a give volume V sampled by the RADAR is given by the relationship

$$\sigma_t = V \sum_{volume} \sigma_i$$

Where the sum is carried out over a unit volume. This is of course possible only assuming that the volume V sampled by the RADAR is homogeneous. Using the solution found by Rayleigh, but which works only when the wave length λ of radiation used is small compared to the radius of the target, we can write

$$\sigma_t = V \sum_{volume} K_i \frac{D_i^6}{\lambda^4}$$

where K_i is a constant function of the target's complex index of refraction $k_i = m + i\, n$ where *m* and *n* are respectively the target refraction and absorption indexes and *i* is the imaginary unit. If we assume that $K_i = K$ is the same for all the targets and since the RADAR wave length is constant, we can write

$$\sigma_t = \frac{VK}{\lambda^4} \sum_{volume} D_i^6$$

At this point we define the quantity

$$z = \sum_{volume} D_i^6$$

which is called *reflectivity* and with it the back-scatter section of the diffused target becomes

$$\sigma_t = \frac{VKz}{\lambda^4}$$

this back scatter section can be inserted into the equation which gives the power received back by the RADAR antenna obtaining

$$p_r = \frac{P\, g\, A_a\, \sigma_t}{(4\pi\, r^2)^2} = \frac{P\, g\, A_a}{(4\pi\, r^2)^2} \frac{V\, K\, z}{\lambda^4}$$

This relationship is called RADAR equation because it creates a relationship between a (diffused) target, the power emitted and the power received. Being the latter two known, we can receive information on *z*, that is on the diffused target. The values of z for meteorological targets span from $z = 1 \times 10^{-3}\, mm^6\, m^{-3}$ for fog up to $z = 5 \times 10^7\, mm^6\, m^{-3}$ for heavy rain. For this reason usually z is measured in a logarithmic range referring them to a diffused target whose reflectivity is $z = 1\, mm^6\, m^{-3}$. In this case we will have

$$Z = 10 \times \log_{10}\left(\frac{z}{1\, mm^6\, m^{-3}}\right)$$

whose units are named *dBZ.*