

From 5.8 → 5.12 b

$$\frac{\partial \bar{v}_0}{\partial t} + \boxed{\varepsilon \frac{\partial \bar{v}_1}{\partial t}} + (\bar{v}_0 \bar{\nabla}) \bar{v}_0 + \boxed{\varepsilon (\bar{v}_1 \bar{\nabla}) \bar{v}_0} + \boxed{\varepsilon (\bar{v}_0 \bar{\nabla}) \bar{v}_1} + \varepsilon^2 (\bar{v}_1 \bar{\nabla}) \bar{v}_1 =$$

$$= \frac{-\bar{\nabla} p_0}{\rho_0 + \varepsilon \rho_1} - \frac{\varepsilon \bar{\nabla} p_1}{\rho_0 + \varepsilon \rho_1} - \cancel{\bar{\nabla} \phi_0} - \boxed{\varepsilon \bar{\nabla} \phi_1}$$

$$\left\{ \begin{aligned} -\bar{\nabla} p_0 \\ (\rho_0^2 - \varepsilon^2 \rho_1^2) \end{aligned} \right. (\rho_0 - \varepsilon \rho_1) = \cancel{\frac{-\bar{\nabla} p_0}{\rho_0}} + \boxed{\frac{\varepsilon \rho_1 \bar{\nabla} p_0}{\rho_0^2}}$$

$$\boxed{} = - \frac{\varepsilon \bar{\nabla} p_1}{(\rho_0^2 - \varepsilon^2 \rho_1^2)} (\rho_0 - \varepsilon \rho_1) = \boxed{\frac{-\varepsilon \bar{\nabla} p_1}{\rho_0}} + \cancel{\frac{\varepsilon^2 \rho_1 \bar{\nabla} p_1}{\rho_0^2 - \varepsilon^2 \rho_1^2}}$$

From eq. at the equilib.

II order

remnant parts / $\varepsilon \Rightarrow$ eq. 5-12b

From definition of $h(\bar{x}, t)$ 5-14

$$h(\bar{x}, t) = \int_0^{p(\bar{x}, t)} \frac{dp(p')}{\rho(p')}$$

$$h = h_0 + \varepsilon h_1$$

$$h_0 + \varepsilon h_1 = \int_0^{\rho_0} \frac{dp(p')}{\rho'} + \int_0^{\rho_0 + \varepsilon \rho_1} \frac{dp(p')}{\rho'}$$

$$\varepsilon h_1 = \left(\frac{dp}{dp} \right)_0 \frac{1}{\rho_0} \varepsilon \rho_1$$

$$h_1 = \left(\frac{dp}{dp} \right)_0 \frac{\rho_1}{\rho_0} = v_s^2 \frac{\rho_1}{\rho_0} = \frac{p_1}{\rho_0}$$

5-12d

$$\begin{aligned} \bar{\nabla} h_1 &= \bar{\nabla} \left(\frac{p_1}{\rho_0} \right) = \\ &= \frac{\rho_0 \bar{\nabla} p_1 - p_1 \bar{\nabla} \rho_0}{\rho_0^2} = \\ &= \frac{\bar{\nabla} p_1}{\rho_0} - \frac{\left(\frac{dp}{dp} \right)_0 p_1 \bar{\nabla} \rho_0}{\rho_0^2} = \\ &= \frac{\bar{\nabla} p_1}{\rho_0} - \frac{\bar{\nabla} \rho_0}{\bar{\nabla} \rho_0} \frac{p_1 \bar{\nabla} \rho_0}{\rho_0^2} \end{aligned}$$

$$\bar{\nabla} h_1 = \frac{-p_1}{\rho_0^2} \bar{\nabla} \rho_0 + \frac{1}{\rho_0} \bar{\nabla} p_1 \rightarrow 5-12b$$

From 5-19 \rightarrow 5-18 \rightarrow 5-20

$$p_1 = c e^{i(\bar{k}\bar{x} - \omega t)} \quad i(\bar{k}\bar{x} - \omega t) = \text{cloud}$$

$$\frac{\partial p_1}{\partial t} = c e^{\text{cloud}} (-\omega i)$$

$$\frac{\partial^2 p_1}{\partial t^2} = c e^{\text{cloud}} (-\omega^2)$$

$$\frac{\partial^2 p_1}{\partial x_1^2} = \frac{\partial}{\partial x_1} \left(c e^{\text{cloud}} (i k_1) \right) = c e^{\text{cloud}} (-1 k_1^2)$$

5-18

$$\cancel{c e^{\text{cloud}}} / (-\omega^2) - v_s^2 \cancel{c e^{\text{cloud}}} / [-1(k_1^2 + k_2^2 + k_3^2)] - 4\pi G \rho_0 \cancel{c e^{\text{cloud}}} = 0$$

$$-\omega^2 + k^2 v_s^2 - 4\pi G \rho_0 = 0 \quad \rightarrow \text{5-20}$$