

Esercizio 3

October 29, 2016

Trovo il polinomio di Taylor della funzione e^{x+y^2} nel punto $(1, 1)$.

$$f(1, 1) = e^2$$

$$\text{grad}f = (e^{x+y^2}, 2ye^{x+y^2})$$

$$\text{grad}f(1, 1) = (e^2, 2e^2)$$

$$Hf = \begin{pmatrix} e^{x+y^2} & 2ye^{x+y^2} \\ 2ye^{x+y^2} & 2e^{x+y^2} + 4y^2e^{x+y^2} \end{pmatrix}$$

$$Hf(1, 1) = \begin{pmatrix} e^2 & 2e^2 \\ 2e^2 & 6e^2 \end{pmatrix}$$

$$f(x, y) = e^2 + e^2(x-1) + 2e^2(y-1) + \frac{1}{2} \begin{pmatrix} e^2(x-1) + 2e^2(y-1) \\ 2e^2(x-1) + 6e^2(y-1) \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} + r =$$

$$= e^2 + e^2(x+2y-3) + \frac{1}{2}e^2(x+2y-3)(x-1) + \frac{1}{2}e^2(2x+6y-8)(y-1) + r$$

con:

$$\lim_{(x,y) \rightarrow (1,1)} \frac{r}{\sqrt{(x-1)^2 + (y-1)^2}} = 0$$