

$$f(x, y) = \sin(x-y) \cos x$$

$$\begin{aligned} \nabla f(x, y) &= (\cos(x-y) \cos x + \sin(x-y) (-\sin x), -\cos(x-y) \cos x) = \\ &= (\cos(x-y) \cos x - \sin(x-y) \sin x, -\cos(x-y) \cos x) \end{aligned}$$

Determino i punti stazionari risolvendo il sistema:

$$\begin{cases} \cos(x-y) \cos x - \sin(x-y) \sin x = 0 \\ -\cos(x-y) \cos x = 0 \end{cases}$$

$$\begin{cases} \dots \dots \dots \begin{cases} \pm \sin(\frac{\pi}{2} + k\pi - y) = 0 \\ \cos x = 0 \end{cases} \begin{cases} \frac{\pi}{2} + k\pi - y = h\pi \\ x = \frac{\pi}{2} + k\pi \end{cases} \begin{cases} y = \frac{\pi}{2} + (k-h)\pi \\ x = \frac{\pi}{2} + k\pi \end{cases} \end{cases}$$

$$\begin{cases} \dots \dots \dots \begin{cases} \pm \sin x = 0 \\ \cos(x-y) = 0 \end{cases} \begin{cases} x = h\pi \\ x - y = \frac{\pi}{2} + k\pi \end{cases} \begin{cases} y = h\pi - \frac{\pi}{2} + k\pi = -\frac{\pi}{2} (h+k)\pi \end{cases} \end{cases}$$

I punti stazionari sono "periodici", allora considero i seguenti punti:

$$\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \left(\frac{\pi}{2}, -\frac{\pi}{2}\right) \left(-\frac{\pi}{2}, -\frac{\pi}{2}\right) \left(0, \frac{\pi}{2}\right) \left(0, -\frac{\pi}{2}\right) \left(\pi, \frac{\pi}{2}\right) \left(\pi, -\frac{\pi}{2}\right)$$

Calcolo la matrice hessiana

$$\frac{\partial}{\partial x} (\cos(x-y) \cos x - \sin(x-y) \sin x) = -\sin(x-y) \cos x + \cos(x-y) (-\sin x) = -[\cos(x-y) \sin x + \sin(x-y) \cos x]$$

$$\begin{aligned} &= -\sin(x-y) \cos x - \cos(x-y) \sin x - \cos(x-y) \sin x - \sin(x-y) \cos x = \\ &= -2 \sin(x-y) \cos x - 2 \cos(x-y) \sin x \end{aligned}$$

$$\frac{\partial}{\partial y} (\cos(x-y) \cos x - \sin(x-y) \sin x) = +\cos x \sin(x-y) + \sin x \cos(x-y)$$

$$\frac{\partial}{\partial x} [-\cos(x-y) \cos x] = \sin(x-y) \cos x - \cos(x-y) (-\sin x) = \sin(x-y) \cos x + \cos(x-y) \sin x$$

$$\frac{\partial}{\partial y} [-\cos(x-y) \cos x] = \cos x (-\sin(x-y)) = -\cos x \sin(x-y)$$

$$Hf\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \quad \det(Hf\left(\frac{\pi}{2}, \frac{\pi}{2}\right)) = -1 \quad \begin{array}{l} \text{il determinante \u00e8 negativo, allora} \\ \text{i due autovalori hanno segno opposto} \end{array}$$

$\Rightarrow \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ punto di sella

$$Hf\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{la matrice hessiana \u00e8 la stessa del punto } \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\Rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ punto di sella