

Coefficiente di dispersione: $S \triangleq -\frac{\lambda}{c} \frac{\partial^2 n_e}{\partial \lambda^2}$

Dispersione $D = S \cdot L \cdot \Delta \lambda_s$

$$V = a \cdot k \cdot NA \quad NA = m_2 \sqrt{2\Delta}$$

$$\Delta = \frac{m_1^2 - m_2^2}{2m_2^2} \quad b = \frac{\left(\frac{\beta}{k}\right)^2 - m_1^2}{m_2^2 - m_3^2}$$

$(\Delta \ll 1)$

Essendo $\beta \approx m_2 k (1 + \Delta \cdot b)$

$$\tilde{e} \quad m_e = \frac{\beta}{k} \approx m_2 (1 + \Delta \cdot b) = m_2 \left(1 + \frac{m_1 - m_2}{m_2} \cdot b\right)$$

Ipotesi: Δ indipendente da λ (cut-off)

$$\frac{\partial^2 n_e}{\partial \lambda^2} = \frac{\partial^2 m_2}{\partial \lambda^2} + \frac{\partial}{\partial \lambda} \left(\frac{\partial}{\partial \lambda} m_2 \Delta \cdot b \right)$$

$$= \frac{\partial^2 m_2}{\partial \lambda^2} + \frac{\partial}{\partial \lambda} \left(\frac{\partial}{\partial \lambda} \frac{V b \lambda \sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} \right)$$

(essendo $V \approx \frac{a \cdot 2\pi \cdot m_2 \cdot \sqrt{2\Delta}}{\lambda} \rightarrow m_2 \sqrt{\Delta} = \frac{V \cdot \lambda}{2\pi \cdot a \cdot \sqrt{2}}$)

$$\left(\frac{\partial}{\partial \lambda} \left(\frac{\partial}{\partial \lambda} \frac{V b \lambda \sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} \right) \right) = \frac{\partial}{\partial \lambda} \left(\frac{V b \sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} + \frac{\lambda \sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} \frac{\partial V b}{\partial V} \frac{\partial V}{\partial \lambda} \right)$$

$$= \frac{\sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} \frac{\partial}{\partial V} \left(V b + \left(-\frac{V}{\lambda}\right) \cdot \lambda \frac{\partial V b}{\partial V} \right) \cdot \left(-\frac{V}{\lambda}\right)$$

$(V = a \cdot k \cdot NA = a \cdot \frac{2\pi}{\lambda} \cdot NA; \text{ trascurando le dip. di } NA \text{ da } \lambda \text{ e } \frac{\partial V}{\partial \lambda} = -\frac{V}{\lambda})$

$$= \frac{\sqrt{\Delta}}{2\pi \cdot a \cdot \sqrt{2}} \cdot \left(-\frac{V}{\lambda}\right) \cdot \left(-\frac{\partial^2 V b}{\partial V^2}\right)$$

$$= \frac{m_2 \Delta}{\lambda^2} \cdot V \cdot \frac{\partial^2 V b}{\partial V^2}$$