Test of Mathematics

January 27th, 2015

Name:.....................................**Surname:**................................

Matriculation number:........................

1. Determine the domain of the function

$$f(x) = arcsin(1 - \sqrt{x}).$$

2. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} arctang\, 2x & \text{if } x \leq 0 \\ \frac{1}{x} & \text{if } x > 0 \end{cases}.$$

   Determine the inverse function $x = f^{-1}(y)$.

3. Determine the following limit:

$$\lim_{x \to 0^+} \frac{1}{x} \sqrt{\frac{x - sin\, x}{x}}.$$

4. Study the following function and draw its graph (just consider the first derivative and determine the unique point x such that $f(x) = 0$):

$$f(x) = log(log\, x - 1).$$

5. Determine the following indefinite integral:

$$\int \frac{cos(log\, x)}{x}\, dx.$$

6. Determine the derivatives $f'_x(x, y)$ and $f'_y(x, y)$ of the following real-valued function of two real variables:

$$z = f(x, y) = \sqrt{x} cos(xy).$$