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ABSTRACT

Central banks affect the resources available to fiscal authorities through the impact of their policies on the public debt, as well as through their income, their mix of assets, their liabilities, and their own solvency. This paper inspects the ability of the central bank to alleviate the fiscal burden by influencing different terms in the government resource constraint. It discusses five channels: (i) how inflation can (and cannot) lower the real burden of the public debt, (ii) how seignorage is generated and subject to what constraints, (iii) whether central bank liabilities should count as public debt, (iv) how central bank assets create income risk, and whether or not this threatens its solvency, and (v) how the central bank balance sheet can be used for fiscal redistributions. Overall, it concludes that the scope for the central bank to lower the fiscal burden is limited.

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1 Introduction

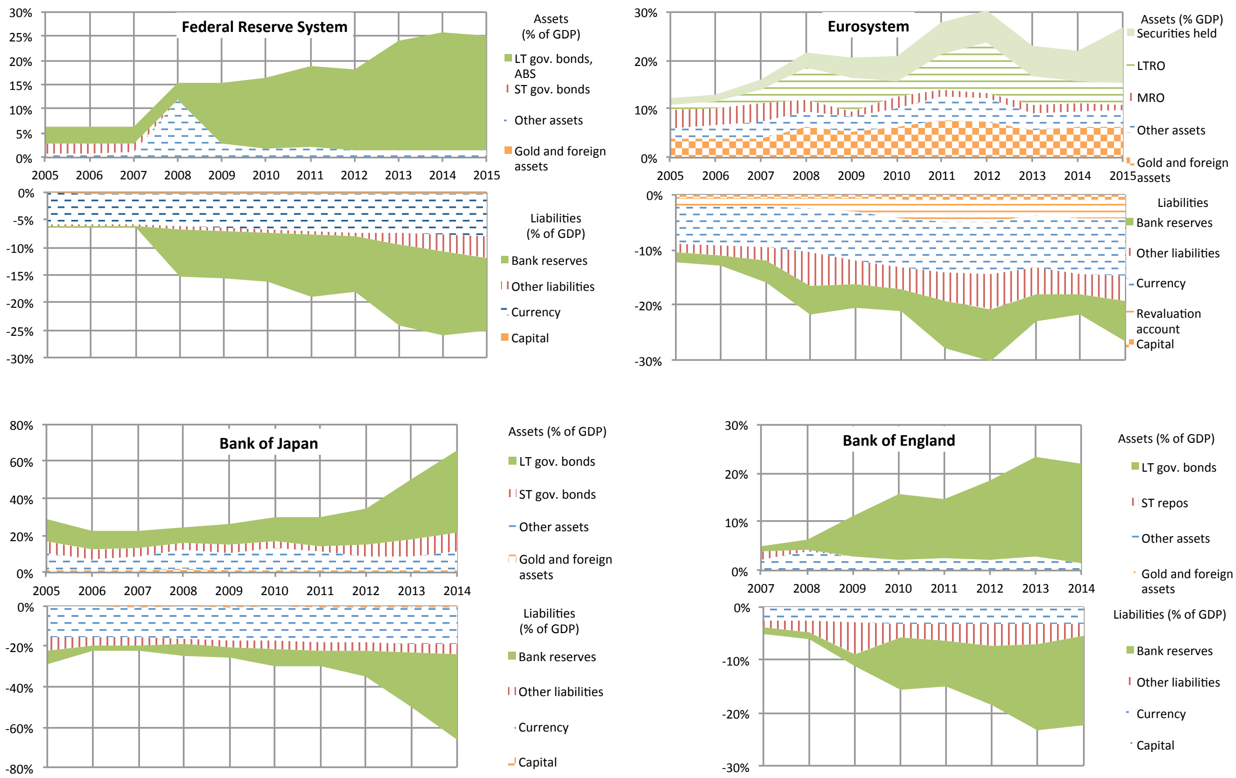
Crises spur the imagination of academics. The U.S. financial crisis followed by the Eurocrisis have led to original proposals on how fiscal stimulus can be delivered, how safe assets can be created in a monetary union without joint liabilities, and how macro prudential regulation can overcome problems in the financial sector, among many others. At the same time, the balance sheets of the four major central banks (ECB, Federal Reserve, Bank of England and Bank of Japan) look quite different than they did just 10 years ago. As figure 1 shows, these central banks have increased the size of their balance sheets through direct purchases of government securities (ECB), issued liabilities in the form of bank reserves that pay market interest rates (Federal Reserve), raised the maturity of bond holdings through special purpose vehicles (Bank of England), and bought large amounts of private securities (Bank of Japan). It is therefore not a surprise that some of the most daring ideas coming from research on macroeconomic policy involve the central bank in some way.

Common to many discussions surrounding the central bank is its supposed ability to alleviate the fiscal burden faced by a country. On both sides of the Atlantic, public debt is at its highest level since World War II; on the right side, fiscal crises in Greece, Ireland and Portugal have already left their marks. With its mystical ability to print money and its frequent purchases of government bonds, it is tempting to look at the central bank as a source of solace and respite.

This perception of the central bank as a relevant fiscal agent takes many forms and common statements. Some say that a country with its own central bank can never have a sovereign debt crisis because it can print money to pay any bills. Others argue that if a country raised its inflation target, it could wipe out the real value of its public debt and alleviate its fiscal burden. Others still make the point that central banks can never go legally insolvent, so when fiscal authorities are on the verge of sovereign insolvency, it is time for the central bank to step up. Finally, in the context of the Eurosystem, some proposed that the ECB could buy and then forgive the debt of the periphery countries, becoming a vehicle for fiscal transfers within the currency union.

Each of these suggestions involves the central bank as a fiscal actor. Of course, any student of monetary economics understands that it is the interaction of monetary and fiscal policy that determines equilibrium outcomes for inflation or real activity. There is a vast literature discussing the fiscal implications of central bank actions, and the required coordination between monetary and fiscal authorities. However, all the suggestions in the previous paragraph were more specific than this general interaction. They involved the central bank

Figure 1: Balance sheets of four major central banks 2005-15



Notes: Vertical axis scale is in % of domestic GDP. Reproduced from Reis (2016a)

creating real resources, and somehow transferring them to some other authorities. They involved the central bank using its balance sheet to have a direct effect on the balance sheet of the fiscal authority.

While a survey of central banks one decade ago could barely mention these fiscal roles for the central bank (Blinder, 2004), they play a central role in more recent surveys (Reis, 2013) and yet still leave much to be discussed. This is the topic of this paper. The closest papers to it in motivation are Stella & Lonnberg (2008) and Archer & Moser-Boehm (2013), but the approach and goal here are quite different. This paper works through the resource constraint facing the central bank with the goal of understanding whether the central bank can generate fiscal revenues, and in doing so relax (or tighten) the resource constraint facing the fiscal authorities.

In spite of this narrow focus, there is still significant ground to cover. Section 2 starts with the fiscal intertemporal resource constraint so that we can precisely define what is the fiscal burden. This section catalogues five different possible channels through which a central bank could feasibly lower this burden. They are each analyzed in turn the succeeding sections. I also provide a minimal definition of a central bank, setting the basics for each succeeding section to progressively expand the actions taken by the central bank, and consider new channels for real transfers to the fiscal authorities. Throughout, these are spelled out using resource constraints.

Section 3 starts by considering the central bank's ability to take deposits and make loans to banks. This gives it control over nominal interest rates and inflation, and also gives rise to the first channel through which it can lower the fiscal burden: inflating away the debt. While the idea is simple, this section discusses the difficulties with estimating the size of this channel, with implementing it effectively, and with making it quantitatively relevant.

Section 4 considers the central bank's provision of liquidity services to the economy, for which it collects a payment commonly known as seignorage. This source of relaxation of fiscal burdens is common during hyperinflations. When inflation is below two digits though, the associated revenues are relatively small, and they are not too sensitive to inflation.

Next, I introduce voluntary interest-paying reserves that banks can hold at the central bank. These central bank liabilities are quite special assets in the economy, and they play a central role in the remainder of this paper. Section 5 starts by describing the unique properties of reserves and by answering one question: should these central bank liabilities be counted as part of the public debt? Taking the other side, this section studies to what extent issuing reserves provides a third channel for the alleviation of fiscal burdens.

Reserves lead to a definition of the meaning of central bank insolvency, together with its tight link to central bank independence. They point to the fourth channel that this paper studies: the sources of income risk facing the central bank and the possibility that these lead to either sources of revenue to the government or further fiscal burdens in the form of central bank insolvency. Sections 6 to 9 study four sources of income risk, and four associated dangers to solvency. Section 6 focuses on the risk from holding foreign currency. This leads to a discussion of the role of central bank income and net worth, and clarifies why across the world there are special rules involving central bank holdings of foreign reserves. Section 7 considers purchases of short-term private bonds by the central bank and their associated default risk. The rules that determine the central bank's dividend to the fiscal authorities and the ability to provision play a key role in determining whether the fiscal authorities must support (or not) the central bank. Sections 8 and 9 turn to government bonds. First, I focus on sovereign default and on the transfers between different economic agents that it entails. Second, I discuss quantitative easing: the purchase of long-term government bonds in exchange for bank reserves. I show that QE does not lead on average to fiscal transfers, but it can put to the test the fiscal capacity of the central bank because of duration risk.

In section 10, the analysis switches from the resources that the central bank can distribute to the redistributions across fiscal entities that it can trigger. Within a currency union, the central bank can be used to relax the fiscal constraints of some regions, while increasing the fiscal burden of others. This is the fifth and final channel considered in this paper. Using the Eurosystem as an example, I describe the many vehicles that could be used to undertake these redistributions, and how the Eurosystem's institutions prevent them.

Finally, section 11 concludes by revisiting some commonly heard statements about what central banks can achieve fiscally. I show that, while they are partly correct, they are mostly misleading. This provides a final application of why the lessons in this paper can be useful in economic policy discussions.

2 The fiscal burden and the central bank

In discussing fiscal resources and central banks, it is important to stay away from two common fallacies.

The first is money illusion. Printing banknotes may generate nominal resources, but because this invariably comes with higher inflation, it creates fewer real resources. People are only willing to lend to the government expecting a real return, and almost all government spending programs require providing or paying for real services or goods. Fiscal burdens are real, and it is an illusion to think that "printing money" makes them magically disappear. To safeguard against this illusion, I will write constraints in real terms; whenever a variable is in nominal units, it will be written using capitals, to make this clear.

The second fallacy is the free lunch. If central banks could magically create something out of nothing without bounds, then the whole of society could use monetary policy to solve any scarcity problem. Free lunches don't usually exist, at least not at the wide macroeconomic scale where there are finite resources. Even if central banks and monetary policy work towards raising welfare, it is dangerous and almost always wrong to believe that they can create real resources out of nothing. When one identifies what seems like a free lunch, it is crucial to make sure that all resource constraints have been spelled out. I will do so by insisting on looking at resource constraints throughout.

At the same time, the analysis will be very general by discussing what central banks can do given those constraints, rather than what they ought to do. There will be many lessons even though the objectives of the central bank, or its actual choices, are never stated. The goal is to figure out what is possible, leaving for later a discussion of what is desirable.

2.1 The government's fiscal burden

Time is discrete, and we will take the perspective of an arbitrary date t. The fiscal burden of the government is equal to the commitments that it has to honor to its creditors:

$$\Phi_t \equiv \frac{\sum_{j=0}^{\infty} Q_t^j B_t^j}{p_t}. \tag{1}$$

On the right-hand side are the nominal promises outstanding at date t for a payment in j periods-time: B_t^j. Note that this expression assumes that all government bonds have a zero coupon—a weak assumption since if they are not, they can be rewritten in their zero-coupon equivalent form. Bonds have a price today Q_t^j, where $Q_t^0 = \delta_t$ is the repayment rate on the bonds, such that $\delta_t = 1$ if there is no sovereign default. Since we care about the real fiscal burden, we divide by the price level p_t. Governments also issue real debt, and this could be included, but it would not change the analysis in any interesting way.

The reason why this is a burden is that people would not be willing to lend to a government that never intends to repay. The government cannot run a Ponzi scheme on its bonds, and this constraint can be written as a resource constraint facing the fiscal authorities:

$$\Phi_t \leq \mathbb{E}_t \left[\sum_{j=0}^{\infty} m_{t,t+j}(f_{t+j} + d_{t+j}) \right]. \tag{2}$$

On the right-hand side are three new variables, m, f, and d.

Starting with the latter, d_t are real transfers from the central bank to the fiscal authority. We will discuss where they come from throughout the paper, but for now keep in mind that they could well be negative. In that case the fiscal authority might be said to be recapitalizing the central bank.

Moving on to f_t, these are the fiscal surpluses, or the difference between spending and revenues. One might argue that some forms of committed spending, like social security payments, should be moved to the left so they are included in the fiscal burden. This could make a significant difference for quantitative assessments, but it is hard to gauge how easy it is to revert those commitments, and what fraction of them is nominal versus real. I will treat them as exogenous and real, and as such they can be included in f_t.

Finally, $m_{t,t+j}$ is the stochastic discount factor that converts units from future real resources to present-day values. An important assumption is that there are no arbitrage opportunities facing the government, including the central bank. If they existed, then it is hard to escape the free-lunch fallacy: why wouldn't the government exploit them with no end and use the free funds to provide valuable government services? There may well be many financial-market imperfections together with arbitrage opportunities across private individual securities that investors can exploit. The assumption here is instead that in the large and liquid market for government liabilities of different maturities, there are no such opportunities that the government can systematically exploit in a quantitatively significant way.

Mathematically, this translates into assuming that the following condition holds:

$$\mathbb{E}_t \left[\frac{m_{t,t+j} \delta_{t+j} p_t}{Q_t^j p_{t+j}} \right] = 1, \tag{3}$$

so that the expectation of the product of the stochastic discount factor with the return on any government security is always equal to one. It is important to note, because it is often forgotten, that the price of government bonds is driven both by default and future inflation. If the bond is expected to default and pay little (small δ_{t+j}), or if inflation is expected to be high until the bond comes due (high p_{t+j}/p_t), both of these depress the price of the bond in the same way or, equivalently, raise the yields on the public debt. From the perspective of the bondholder, whether the country uses inflation to prevent default, or is stuck with low inflation and is forced to default, the payoff is reduced.

Using this condition, we can write the fiscal burden in an alternative useful way:

$$\Phi_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} m_{t,t+j} \left(\frac{\delta_{t+j} B_t^j}{p_{t+j}} \right) \right]. \tag{4}$$

The fiscal burden is the expected present discounted value of the payments promised by the government, taking into account the possibility of sovereign default.

2.2 What is a central bank?

Central banks have a long and rich history and they perform many functions in modern advanced economies (Reis, 2013). Each section of this paper will gradually expand the policies controlled by the central bank so that, by the end, we end up with a satisfactory model caricature of its real-life counterpart. To start, consider the minimal version of what is a central bank.

All central banks in advanced countries perform a crucial task, and some (the Federal Reserve) were created from the start to satisfy this function. Banks issue checks and debit cards that are used to make payments by their customers to other banks' customers. This is only possible because periodically (say at the end of the day) the banks that end up as net creditors can settle their claims on the other banks that end up as net debtors. Making these settlements requires a clearing house that the banks can join, and some form of payment within the clearing house, in the sense of a claim on the house itself that can be used to settle the interbank claims. In the 19$^{\text{th}}$ century United States, many private clearing houses tried to perform this role, but most failed because of the double moral hazard problem that each bank wants to accumulate a large debt to the house and then default, and the house managers want to secretly issue the house's means of payment to enrich themselves. The central bank is a public institution that has a strong and restrictive legal framework governing its operations, and broad powers to regulate, supervise and close down the banks that take deposits and issue these means of payments, and this allows the central bank to ensure that banks almost always satisfy their claims to other banks.

A central bank can then be defined as the sole institution in a country with the power to issue an interbank claim, called bank reserves at the central bank, that is used to settle interbank claims. Since the central bank is the monopoly issuer of these reserves, it can freely choose which interest to pay on these reserves. Insofar as the balances on these reserves are zero, then this comes with no fiscal transfer on aggregate from the banking sector to the government sector. But, the central bank is special among government agencies by being able to issue this type of nominal government liability, independently of the fiscal authority. Moreover, this liability is quite special because it is the unit of account in the economy: its units define what the price level p_t is. From another perspective, the central bank can pay the interest on reserves with more reserves, and this fully satisfies its commitment to the banks, since reserves are the unit for nominal transactions in the economy.

Its ability to issue reserves, choose their interest rate, and in doing so affect their real value which is by definition the inverse of the price level, are the minimal powers of a central bank.

2.3 The channels from the central bank to the fiscal burden

Combining equations (2) and (4) gives the determinants of the fiscal burden in terms of an inequality:

$$\Phi_t = \mathbb{E}_t\left[\sum_{j=0}^{\infty} m_{t,t+j}\left(\frac{\delta_{t+j} B_t^j}{p_{t+j}}\right)\right] \leq \mathbb{E}_t\left[\sum_{j=0}^{\infty} m_{t,t+j}(f_{t+j} + d_{t+j})\right]. \tag{5}$$

At this very general level, we can see the five channels through which the central bank can affect the fiscal burden.

First, the central bank can affect the real value of reserves, and in doing so exert significant control over the price level p_{t+j}. For a fixed level of nominal commitments by the fiscal authority, higher prices lower the real burden of honoring those commitment. In short, the central bank can try to inflate away the debt. Section 3 is devoted to this channel.

Second, the central bank makes real transfers to the government d_{t+j}. Every savings that the central bank achieves in its operations is an extra source of revenue to the fiscal authority. More importantly, if in its activities the central bank can generate revenue to disburse that is not fully offset by lowering the revenues of the fiscal authority, so that $f_t + d_t$ rise, this will lower the fiscal burden. The unique revenues of the central bank come from selling banknotes, which society finds useful to undertake payments: these are studied in section 4.

Third, some of the nominal commitments of the government may have their source in the central bank. That is, Φ_t may include the liabilities issued by the central bank, reserves, or perhaps even banknotes. Section 5 spells out the resource constraint of the central bank and connects it to the fiscal burden to understand what is part of Φ_t and what is not.

The fourth channel through which central banks affect the fiscal burden woks as well through d_{t+j}, but in the opposite direction. If the central bank takes on income risk in its asset holdings, then it may incur losses which must be met by the fiscal authority through a negative d_{t+j}. In the extreme, if the central bank is insolvent, then a large flow of funds may be necessary to recapitalize it. Sections 6 to 9 describe the different sources of income for central banks as a result of holding different assets, and how these affect the flow of dividends to the fiscal authority and the solvency of the central bank.

The fifth and final channel studied in this paper is across fiscal authorities. For a given total fiscal burden for an economic union, the central bank may be able to redistribute it across regions. Section 10 discusses how the central bank can perform these redistributions using all of the previous channels.

There are three further, indirect, channels through which the central bank can lighten the fiscal burden, which this paper will not cover. They occur through three equilibrium variables that the central bank is far from precisely controlling, but which it probably somewhat affects: $m_{t,t+j}$, f_{t+j}, and δ_{t+j}. If there is monetary non-neutrality, then the central bank's actions will affect the real interest rate. If, moreover, the fiscal authority adjusts its plans in response to the central bank's actions, then fiscal revenues are also affected. Finally, if private investors perceive that the attractiveness of sovereign default depends on inflation and on the size of reserves, then the central bank will also have some effect on the recovery rates on public debt. Each of these effects may be important. But they really pertain to discussions of the Phillips curve, of monetary-fiscal interactions, and of equilibrium sovereign default, respectively. These are much studied topics that have been well covered in many other places (e.g., Mankiw & Reis, 2010; Leeper & Leith, 2016; Aguiar & Amador, 2014), and which would require specifying the actions of many more agents than just the central bank. They are left aside in this article.

3 Inflating the debt

The central bank can choose the nominal interest rate that remunerates bank reserves. Because there is no counterparty nominal risk in lending to the central bank—who will always repay and can always do so—this interest rate should put a floor, or lower bound, on other nominal interest rates in the economy. At the same time, in its role as the clearinghouse, the central bank could also announce that it will make loans to banks (which are negative individual reserves) at an announced interest rate. This in turn puts a ceiling on interest rates, since being able to borrow at any time from the central bank, no bank would pay more to borrow from anyone else. Both floor and ceiling combined imply that the central bank is able to pin down, by the sheer force of its announcements, the market nominal interest rate on 1-period investments, which I denote by i_t.

Outside of the fictional model description in the previous paragraph, in reality, all advanced country central banks are able to control the overnight nominal interest rate in their economies. They may not do so through the simple borrowing and lending of reserves as in the previous paragraph, but via direct purchases or repurchase agreements. Moreover, the institutional details of interbank markets in each country may lead to deviations from the floor and/or ceiling described above. Further, the central bank may allow for a gap between its deposit and lending rates, or what is called a "corridor system", while using the quantity of reserves outstanding to pin down a desired rate. In spite of all this diversity, decades of experience have confirmed that through different implementation methods the central bank has an almost perfect control of the overnight safe nominal interest rate in advanced financial systems.

3.1 The central bank's control over inflation

The no-arbitrage condition applied to reserves implies that:

$$\mathbb{E}_t\left[\frac{m_{t,t+1}(1+i_t)p_t}{p_{t+1}}\right]=1. \tag{6}$$

Because the central bank chooses i_t and, under the classical dichotomy, $m_{t,t+1}$ is not affected by these choices, this Fisher relation provides an equation with which to solve for inflation. At the same time, it is clear that this is only one equation for more than one unknown: one has to pin down both the initial price level p_t as well as the future price level p_{t+1} at each of the many possible future states of the world. Therefore, simply choosing one exogenous interest rate will leave the price level indeterminate.

These problems are relatively well understood and the topic of determining the price level has produced a large literature as well as recent surveys (Reis, 2015b), filled with ways in which the central bank can pick the price level to control inflation. The most popular strategy is perhaps the Taylor (1993) rule, while the simplest and most robust one is perhaps the payment on reserves rule of Hall & Reis (2016). In the latter case, the central bank makes its payment of interest at date $t+1$ be indexed to the price level, so that it promises a real payment x_t to the banks holding the reserves. In that case, since $1+i_t=(1+x_t)p_{t+1}$, then plugging this into the no-arbitrage condition, and rearranging, one gets a unique global solution to the price level:

$$p_t=\frac{1}{(1+x_t)\,\mathbb{E}_t(m_{t,t+1})}, \tag{7}$$

as a function of the policy choice x_t and the stochastic discount factor $m_{t,t+1}$.

Whether through this route, or some other, and with larger or smaller errors coming from fluctuations in the stochastic discount factor, the central bank can control inflation. In picking a path for $\{p_{t+j}\}_{j=0}^{\infty}$, it can then affect the real value of nominal bonds, and so the fiscal burden of the government.

While this control of the price level is achieved via the power to choose the remuneration of reserves, it does not require a large central bank balance sheet or risks to the dividends it pays. Given its total issuance of reserves and credits on aggregate to the whole banking sector, v_t, the law of motion for reserves net of credit by the central bank is:

$$v_t = (1 + r_t)v_{t-1} + d_t, \tag{8}$$

where $1 + r_t = (1 + i_{t-1})p_{t-1}/p_t$ is the ex post real return paid on the reserves. The central bank can issue more reserves to pay for the transfers it sends to the fiscal authority and for its past commitments to remunerate reserves. For the control of the price level, it is not necessary to have net reserves outstanding; arbitrage suffices. The size of the balance sheet of the central bank has no direct link to inflation (Reis, 2016a). The central bank can choose $v_t = 0$ at all dates, and still the price level will be under control simply by the promised interest rate on reserves that could be issued on demand. Under this scenario, dividends to the fiscal authority are always zero, and the central bank is always solvent. Its effect on the fiscal burden works solely through inflation.

3.2 Surprises and persistence

By how much can the central bank lower the fiscal burden depends not just on the amount, but also on the type of inflation that it engineers, and how it interacts with the maturity of the outstanding debt. To see this clearly, consider two special and extreme cases.

First, imagine that there is a single J-period government bond outstanding that always repays its holders in full. In that case:

$$\Phi_t = B_t^J \, \mathbb{E}_t \left(\frac{m_{t,t+J}}{p_{t+J}} \right). \tag{9}$$

Imagine also that because of the delays in setting policy or in its transmission to the real economy, the central bank's interest policy only has an effect on inflation in T periods' time. In that case, if $T \leq J$ then the central bank can still affect p_{t+J} and lower the fiscal burden by as much as it wants to. But, if $T > J$, then the central bank can set inflation to whatever it wants yet this will be neutral with respect to the fiscal burden. The more general lesson is that only if the central bank engineers a surprise increase in the price level before most of the debt is due will it be able to inflate away a significant amount of it.

Hilscher et al. (2014) show that this extreme case is not so far-fetched. They construct the distribution of privately-held debt in the United States for 2012 and find that it has dramatically lowered in duration relative to just a decade before. The average duration is only 3.7 years. In turn, using market expectations of inflation, they also find that the probability that the price level will jump significantly in the space of just one or two years is quite low. Inflation is inertial, and the Federal Reserve would find it difficult to suddenly and unexpectedly shift policy so much so as to change the price level by much in only a few years. As a result, the $T > J$ case above is not so far as an approximation to the U.S. situation in 2012. Hilscher et al. (2014) conclude that the probability that the United States could inflate away its debt by even as little as 5% of GDP is well below 1%.

For the second case, consider a different set of special assumptions: (i) the stochastic discount factor across one period is a constant $m \in (0, 1)$ so that $m_{t,t+j} = m^j$; (ii) there are only two types of bonds outstanding entering period t, a one-period bond that pays B^s right away, and a consol that pays B^c every period in perpetuity; (iii) the central bank can choose p_t freely right away, but after that the price level evolves according to $p_{t+j} = \pi^j p_t$. With these restrictions, the fiscal burden is:

$$\Phi_t = \frac{1}{p_t}\left[B_t^s + \frac{B_t^c}{1 - m/\pi}\right]. \tag{10}$$

The more persistent is inflation, the lower the real value of the debt will be. Moreover, more persistence of inflation implies that the fiscal burden falls by more per unit of higher prices today. Intuitively, higher persistence implies that the price level will be higher in the future, eating away at the real value of the consol. This formula also makes clear how this effect depends on the interaction with the maturity of the government's debt. The larger is the ratio B^c/B^s, the larger the effect that higher inflation persistence has on inflating the debt.

To conclude, being able to inflate more quickly and with more persistence increases the reduction in the fiscal burden that can be achieved. But both of these properties interact with the maturity of the debt. With more complicated maturities than the two special cases in this section, figuring out this interaction is not easy, and it can make a large difference for the calculations.

3.3 An options-based approach

In back of the envelope calculations, it is common to assume that inflation can be increased instantly and permanently and that the distribution of the maturity of government debt is exponential. However, Hilscher et al. (2014) show that these seemingly innocent assumptions dramatically overstate the effect of higher inflation on the public debt. Moreover, while so far we have assumed for analytical transparency that higher inflation does not affect real interest rates, this Phillips curve effect can be substantial in the data. Is there a relatively simple way to provide reliable estimates given the large error in these and other approximations?

The answer is in the formula for the debt burden. It is a result in financial economics that if $f(\pi_{t,t+j})$ is the probability density for inflation $\pi_{t,t+j} \equiv p_{t+j}/p_t$, and $r^f_{t,t+j}$ denotes the risk-free rate between dates t and $t+j$ defined as $(1+r^f_{t,t+j})^{-1} = \mathbb{E}_t(m_{t,t+j})$, then we can define the risk-adjusted measure for inflation as: $f^Q(\pi_{t,t+j}) \equiv (1+r^f_{t,t+j})f(\pi_{t,t+j})m_{t,t+j}(\pi_{t,t+j})$. The fiscal burden without default is then equal to:

$$\Phi_t = \sum_{j=0}^{\infty}(1+r^f_{t,t+j})^{-1}\left(\frac{B^j_t}{p_t}\right)\mathbb{E}^Q\left(\frac{1}{\pi_{t,t+j}}\right). \tag{11}$$

where $\mathbb{E}^Q$ is the expectations operator under the risk-adjusted measure.

This result shows that the harmonic mean of inflation under the risk-adjusted measure at different horizons is the sufficient statistic from inflation that multiplies the maturity distribution of the debt to give the fiscal burden. This formula can either be used probabilistically to provide "value-at-risk" probabilities for whether inflation will lower the real value of the debt by a certain threshold, or to easily calculate counterfactuals of the effects of higher inflation. Up until recently, this formula could not be implemented because there were no good estimates of the $f^Q(.)$ probability density. But, Hilscher et al. (2014) use the active recent markets on inflation options to obtain these marginal densities for inflation, and propose an econometric estimator to convert these into joint distributions for inflation.

4 Dividends from seignorage

A part of financial regulation in most advanced economies consists of forcing banks to hold a required amount of reserves at the central bank, usually against their deposits. These required reserves often pay zero interest, so they are distinct from the voluntary reserves v_t that were discussed already. We denote them by H_t instead (recalling that capitals denote variables in nominal units). Still taking the case $v_t = 0$, without loss of generality, the resource constraint of the central bank becomes:

$$p_t d_t = H_t - H_{t-1}. \tag{12}$$

By forcing banks to hold more of these required reserves, the central bank generates resources that it can then rebate to the fiscal authorities. Ultimately, this regulation is a form of taxation of the banks, and as such it is subject to the usual tradeoffs: on the one hand it generates fiscal revenues, on the other hand, it also introduces distortions on the optimal size of banks and on the extent of deposit financing.

Closely related is the ability of the central bank to issue banknotes. By definition, they pay zero interest. This section studies this central bank product in more detail.

4.1 The central bank as a seller of payment services

Central banks all over the world issue banknotes. Currency is the sum of banknotes or coins, and while the latter are sometimes issued by the Treasury this is usually under strict rules on the ratio of coins to banknotes so that the central bank effectively has almost perfect control over total currency. Reserves and currency the dual bedrock of the monetary system. They come with the promise that they can be exchanged for reserves and vice versa, at any time, on simple request, at a fixed exchange rate of one to one (which is why currency and reserves can be interchangeably referred to as the unit of account).

For some reason, people find currency useful to make payments. There is an enormous literature on why this is so (e.g., Nosal & Rocheteau, 2011). For our purposes, it only matters that people happen to demand these services, and the central bank is their monopoly supplier. Because they are durable, the monopolist sales are the first difference of the stock of currency; because of the one-to-one exchange rate with reserves, their real price is $1/p_t$. Therefore with only a slight abuse of notation, we can denote the sum of required reserves and currency by H_t, so the revenue from providing these payment services is:

$$s_t = \frac{H_t - H_{t-1}}{p_t}. \tag{13}$$

This is typically called seignorage.

In the most standard model of monetary economics, due to Sidrauski (1967), the central bank rebates this revenue to households every period. An alternative story, due to Friedman (1969) imagines the central bank dropping the banknotes from a helicopter, or effectively giving away the seignorage revenue by distributing currency for free. In either of these two cases, the fiscal effect is neutral. The central bank pays zero dividends to the fiscal authorities, and the analysis of the previous section applies, so the central bank can only affect fiscal burdens via debasement of the nominal debt. But this is not how central banks supply currency.

4.2 Fiscalist inflation

Closer to reality, central banks collect the seignorage revenue and pay it off to the fiscal authority every period, so the resource constraint of the central bank is simply:

$$d_t = s_t. \tag{14}$$

In this case, the central bank rebates the revenue from its monopoly supply of a publicly-provided good to the fiscal authorities. From a different perspective, the central bank is a collector of the inflation tax on the holders of currency. Either way, the central bank is not all that different from the parking tickets office, or the issuer of permits for boats: it collects a revenue and uses it to provide direct fiscal transfers that lower the fiscal burden of the government.

It is tempting to say that by printing more banknotes the central bank can just increase this revenue. While this is often taught in introductory economics classes, it is quite far removed from the reality of central banking. No modern central bank chooses H_t exogenously. Rather, they choose the interest on reserves and in doing so give rise to some equilibrium inflation. Then they simply stand ready to accommodate whatever demand for currency there is. The amount of banknotes in circulation is not chosen by the central bank, as they satisfy *any* demand that there is for this means of payment.

How is the amount of currency in circulation then endogenously determined? Because currency pays no interest, and households and firms could instead hold their savings in a bond that pays the nominal interest rate, the opportunity cost of holding money is approximately equal to the short-term interest rate i_t. Almost all models of the demand for currency then give rise to a declining function of this opportunity cost: $L(.) : \mathbf{R}^+ \to \mathbf{R}^+$. This gives the demand:

$$\frac{H_t}{p_t y_t} = L(1 + i_t). \tag{15}$$

$L(.)$ is sometimes called the inverse velocity of currency. The central bank therefore chooses an interest-rate, which pins down inflation, and this determines the demand for currency. Printing currency is not an independent option from changing interest rates and inflation.

Sargent & Wallace (1981) clarified this point in a forceful way. Imagine the central bank was solely focussed on generating a fiscal revenue for the fiscal authorities, so d_t/y_t was imposed on it exogenously. Then, this would pin down an inflation path, since combining equations (13)-(15) given an exogenous d_t/y_t, one gets a stochastic difference equation for inflation:

$$\frac{d_t}{y_t} = L\left(\frac{1}{\mathbb{E}_t\left[\frac{m_{t,t+1}p_t}{p_{t+1}}\right]}\right) - L\left(\frac{1}{\mathbb{E}_{t-1}\left[\frac{m_{t-1,t}p_{t-1}}{p_t}\right]}\right)\left(\frac{y_{t-1}p_{t-1}}{y_t p_t}\right). \tag{16}$$

Inflation would therefore be pinned down by the fiscal demands placed on the central bank.

The general lesson is that "printing money" to generate revenue is not an independent choice from setting interest rates and determining inflation. The central bank is the monopoly supplier of a durable good; it cannot just "sell more" to get more revenue, rather it must change the price for its good. In the case of the central banks' output, banknotes, the only way to raise its revenue, seignorage, is to let inflation rise.

4.3 How large is it?

Because printing banknotes can be done quickly, seignorage is an appealing source of revenue for the government, and it is an effective way for the central bank to lower the fiscal burden, albeit at a cost of higher inflation. A long literature has discussed the use of this source of revenue (Cagan, 1956; Fischer et al., 2002). A common conclusion is that it is relatively small.

To understand why it is so, start by considering seignorage in a steady state where $1 + g = y_t/y_{t-1}$, $1 + \pi = p_t/p_{t-1}$ and $m = \mathbb{E}_t(m_{t,t+1})$. Seignorage then is:

$$\frac{s}{y} = L\left(\frac{1+\pi}{m}\right)\left(1 - \frac{1}{(1+g)(1+\pi)}\right). \tag{17}$$

Inverse velocity for non-interest paying monetary base in the United States between 1960 and 2015 was 6.11% on average, while average nominal income growth rate was 6.32%. Seignorage has therefore been approximately 0.36% of GDP. By comparison, in the data after taking away the costs of running its operations, the Federal Reserve's average remittances to the U.S. Treasury have been 0.31% of GDP.

Increasing π would raise the last term in brackets on the right-hand side, but at the same time it would lower inverse-velocity. This is the typical taxation effect: higher taxes directly raise tax revenue, but lower it by reducing the tax base. A standard specification of the demand for currency writes velocity as an iso-elastic function $L(1 + i_t) \equiv L_0[i_t/(1 + i_t)]^{-\eta}$. The L_0 is a constant, while Lucas (2000) suggests that $\eta = 0.5$ is roughly consistent with the U.S. data. Under this particular specification, increasing inflation from its 1960-2015 average of 3.31% by one percentage point would raise seignorage by a mere 0.02% of GDP. Even raising inflation by a whole 10% would increase seignorage by only 0.17% of GDP.

It is not easy to estimate the peak of this Laffer curve for banknotes. Historically, Fischer et al. (2002) suggest from looking at a cross section of countries that the peak happens at 174% inflation with seignorage of about 6% of GDP. They also document that even in deep fiscal cises, when fiscal balances run at a 10% deficit, usually inflation is only in the order of 20%. Outside of steady state, Hilscher et al. (2016) estimate the present value of seignorage for the United States through a variety of different perspectives using market inflation expectations. Even under their most generous estimates, the present value of seignorage is never above 30% of GDP, and a more accurate estimate is slightly above 20%. The amount of banknotes in circulation is simply not that large for steady-state seignorage to be all that fiscally significant.

Another source of seignorage revenue comes from requiring some reserves to be held by banks. Measuring the limits on this implicit tax is harder. Directly, this tax on collecting deposits should reduce savings in the banking sector, therefore affecting the tax base. Perhaps more importantly though is the indirect effect. As banks face a higher marginal cost of financing, this should lead to fewer projects being financed by bank credit, which in turn should lower economic activity, the amount of wealth, and thus the amount held as deposits. Estimating the full general-equilibrium Laffer curve is therefore not easy. Insofar as a sudden increase in bank reserves can precipitate a financial crisis, trying this policy option can be very costly (Hoggarth et al., 2002).

To conclude, seignorage may be useful to smooth small business-cycle fluctuations in government revenue while keeping tax rates fixed (Chari et al., 1991), so it may be a source of fiscal smoothing. But on average, its role in overall government revenue is small.

5 Reserves and central bank insolvency

So far, we have set outstanding reserves to zero at all dates. Being a liability of the central bank, they are a government liability. This section asks whether reserves (and banknotes) should be part of the public debt.

5.1 The intertemporal resource constraint of the central bank

With both seignorage and reserves present, then the equality between sources and uses of funds for the central bank gives:

$$s_t + v_t = (1 + r_t)v_{t-1} + d_t. \tag{18}$$

Iterating forward this gives:

$$(1 + r_t)v_{t-1} = \mathbb{E}_t\left[\sum_{j=0}^{\infty} m_{t,t+j}(s_{t+j} - d_{t+j})\right] + \lim_{T\to\infty} \mathbb{E}_t(m_{t,T}v_T). \tag{19}$$

Reserves are just another form of government liability. Banks must voluntarily choose to hold them and, like any other private agent, they will not be willing to hold an asset that is a Ponzi scheme. Of course the central bank can always just issue new reserves to pay for the interest on old reserves. But so can a standard consumer issue new debt to pay past debt in textbook microeconomics. The problem is not the ability to issue pieces of paper that stand for liabilities, but rather whether these can have a positive value in the sense that private agents are willing to give you positive real resources in return. Therefore, reserves cannot be a Ponzi scheme:

$$\lim_{T\to\infty} \mathbb{E}_t(m_{t,T}v_T) \leq 0. \tag{20}$$

Equivalently, the central bank faces an upper bound on its dividends:

$$\mathbb{E}_t\left(\sum_{j=0}^{\infty} m_{t,t+j}d_{t+j}\right) \leq \mathbb{E}_t\left(\sum_{j=0}^{\infty} m_{t,t+j}s_{t+j}\right) - (1 + r_t)v_{t-1}. \tag{21}$$

This is the intertemporal resource constraint facing the central bank. Together with the constraint facing the fiscal authority from section 2, in equation (5), they jointly define the overall government fiscal limits.

5.2 Reserves and the public debt

If there is no constraint on the dividends paid to the fiscal authorities, then the two resource constraints can be combined by replacing out for dividends. That is, if the central bank can always count on the fiscal authority to set a sequence of $\{d_{t+j}\}_{j=0}^{\infty}$ that satisfies the central bank's resource constraint, then of course this constraint is slack, and only the combined intertemporal resource constraint for the government as a whole is important. This may include negative dividends every year, implying that the central bank is being recapitalized annually, but this is what effectively happens in many government bodies, like the department of public works, which has many expenses and almost no revenues. Insofar as its liabilities are backed by the overall fiscal authorities, then a government agency cannot be insolvent separately from the solvency of the overall government (Sims, 2003).

In this case, replacing out for the present value of dividends in the government budget constraint in equation (5) and using the resource constraint of the central bank in equation (21) gives the joint constraint:

$$(1+r_t)v_{t-1} + \Phi_t \leq \mathbb{E}_t \left[\sum_{j=0}^{\infty} m_{t,t+j}(f_{t+j} + s_{t+j}) \right]. \tag{22}$$

In the previous section, where reserves were set to zero, the same equation held.

This equation makes clear that reserves are just another government liability. They count for the fiscal burden, and they could be included in the public debt. However, counting them as public debt would require making two more adjustments to public accounting. Once these are done, at least for the four countries in figure 1, including reserves would actually *lower* the calculated public debt.

First, adding reserves to the left-hand side comes with adding seignorage to the right-hand side in the source of funds. As discussed in section 4, the present value of U.S. seignorage is somewhere between 20% and 30% of GDP while, as displayed in figure 1, outstanding reserves peaked at 15% of GDP. Second, the issuance of reserves often happens to fund the purchase of public debt. When this happens, it leaves the right-hand side of the equation unchanged, since the higher v_{t-1} is immediately offset by a lower Φ_t. Section 8 will further explore this Modigliani-Miller neutrality whereby exchanging one form of public liability for another makes no difference to the overall fiscal burden. Currently, for the countries in figure 1, the value of their bond portfolio exceeds the value of outstanding reserves. Net of assets and seignorage, including reserves would then actually lower the public debt.

Moreover, starting from an initial situation where reserves were zero, then the future path of reserves from then onwards would make no difference to the fiscal burden. The ability to issue reserves does not alleviate per se the fiscal burden of the government; it is the present value of seignorage that matters. Intuitively, no net transfers result from issuing reserves because in equilibrium no arbitrage ensures that no matter what nominal interest rate is chosen, the price level will adjust and reserves will end up paying the market real return.

Therefore, issuing reserves ends up boiling down to another government liability, with no creation of new resources, so they do not alleviate the fiscal burden.

Finally, banknotes in circulation do not enter the resource constraint above. They are not a liability of the government or of the central bank. Rather, they are a durable good that the central bank produces and sells. Therefore, currency only affects the fiscal burden via the seignorage revenue discussed in the previous section.

5.3 Central bank independence and insolvency

The analysis of the previous sub-section implies that there is no independent resource constraint on the actions of the central bank. Whatever losses or gains, the fiscal authority would back the central bank, and as such the only binding constraint is the integrated budget constraint of the fiscal and monetary authorities. As Hall & Reis (2015) clarified, the institution of central bank independence breaks this situation. A minimal condition for independence of choices and actions of the central bank in terms of its setting of interest rates, is the ability of the central bank to finance its operations without needing approval from the Treasury (Amtenbrink, 2010; Masciandari & Romelli, 2017). This requires the central bank having its own budget, independently of fiscal authorities, and to not relay on the Treasury to approve recapitalizations of the central bank and impose conditions on monetary policy attached to these. Formally, central bank independence means a constraint on the sequence $\{d_{t+j}\}_{j=0}^{\infty}$. It is the joint combination of a constraint on the flow of dividends, $\{d_{t+j}\}_{j=0}^{\infty}$, and the no Ponzi scheme condition in equation (20), that leads to a meaningful intertemporal resource constraint for the central bank in equation (21).

There is no bankruptcy or insolvency procedure for a central bank. That is, there is no legal framework or institution that will judge if a central bank's liabilities exceed its assets and force it to reorganize or be liquidated. If any private agent wants to settle a claim on the central bank, or is willing to sell the central bank a good or asset, then as long as the claim or price are denominated in units of currency, the central bank can always pay by issuing more reserves and standing ready to exchange them for banknotes on request. But if the central bank cannot go legally insolvent, how can it be that it has a resource constraint, like we just derived?

The standard model of intertemporal consumer behavior taught in microeconomic textbooks also does not have insolvency legal proceedings, but it puts a strict constraint on the consumer. That comes from the no-Ponzi constraint, so that insolvency in a rational-expectations model ends up typically being an off-equilibrium event, that is not observed but is still relevant. Likewise for the central bank in the standard economic model described so far, the no Ponzi scheme condition is the constraint that provides meaning to central-bank insolvency.

If the central bank tries to have reserves shoot for infinity, as in a Ponzi scheme, then the real value of reserves for a private agent will be zero. Since, by definition the value of reserves is $1/p_t$, when no one wants to hold them, then $p_t = \infty$. This is what is special about central banks: their insolvency is synonymous with hyperinflation or currency reform. It is not impossible, or even rare, for central banks to go insolvent; it happens all the time around the world. Insolvency happens when people no longer accept to hold the central bank's liabilities and the country has to re-denominate its currency, an event which countries frequently go through.

Still, you may wonder, how is it that a central bank that can print currency is unable to pay all its nominal reserves debt? Since banknotes are legal tender to settle any debt, isn't reverting from interest-rate setting to exogenously choosing the supply of currency the answer to prevent insolvency? To formally evaluate this intuition, consider a pseudo final date, date t, when the central bank must pay a very large nominal dividend. After that, no more payments are made, but also no more reserves or banknotes are issued. The flow of funds, now written in nominal terms, is:

$$H_t - H_{t-1} = (1 + i_{t-1})V_{t-1} + D_t, \tag{23}$$

where V_{t-1} and D_t are reserves and dividends, respectively, in nominal terms. From date $t+1$ onwards, since there are no dividends or reserves, the central bank can close its doors, and the currency supply stays fixed at H_t forever. This equation formalizes the intuition behind the questions in this paragraph because, not matter how large V_{t-1} or D_t are, the central bank could always choose H_t large enough to make this hold. Moreover, since reserves would be zero from date t onwards, clearly the no Ponzi scheme condition would be satisfied. It might seem therefore that a central bank that switches from the current policy of setting interest rates to exogenously printing banknotes would not be insolvent.

This thinking is incorrect because it keeps inflation and V_{t-1} fixed, when in fact the act of printing banknotes would change them. As already noted, given a stable demand function $L(.)$, raising the supply of banknotes must come with higher inflation.

If this inflation was unexpected, then the ex post real payment on the reserves:

$$1 + r_t = \frac{(1 + i_{t-1})p_{t-1}}{p_t} \tag{24}$$

will be unexpectedly low. Therefore, at this final date, holders of reserves will receive a much lower return than expected. The extra inflation is equivalent to the decline in the recovery rate of a government bond during sovereign insolvency. One may not call it insolvency, but in fact it is just like it: the payment on reserves is well below what was promised and expected.

Alternatively, if the inflation was expected, then agents would have asked for a much larger i_{t-1} the period before. But this leads to a larger needed payment on reserves this period, which in turn requires even more printing of banknotes in the current period, and the process continues. As we saw in the previous section, ultimately there is an upper bound on the real resources that can be generated by seignorage, such that the central bank simply cannot pay the real amount required by markets on its reserves at date $t - 1$. But in that case, banks will not want to accept reserves at date $t - 1$, so their real value would be zero and p_{t-1} would be infinity.

The argument that the ability to print banknotes without bound excludes insolvency is therefore incorrect. Ex post or ex ante, the ability to print banknotes does not stop the two manifestations of insolvency: the ex ante unwillingness of lenders to roll over the debt, or the ex post large haircuts on defaulting debt. An alternative way to state this is that being solvent can be interpreted as being able to keep one's promises. A central bank that promised a nominal interest rate on reserves and that promised to hit a certain inflation target has promised a real return on its reserves. The central bank is insolvent when that actual return r_t is well below what was expected. This happens when inflation is well above what was expected.

To sum up, it is the combination of central bank independence and central bank insolvency that puts a fiscal resource constraint on the actions of the central bank. Central bank design (Reis, 2013) must spell out precisely what are the constraints imposed on $\{d_{t+j}\}_{j=0}^{\infty}$ in the relation between the central bank and the fiscal authorities. The next four sections consider four separate constraints on dividends that match different central banks, and discuss their implications for central bank insolvency and the resulting fiscal burden of monetary policy actions.

6 Exchange-rate income risk and period solvency

Most central banks across the world hold large reserves of foreign assets. Foreign assets come with what is perhaps the major source of income risk to the central bank, that associated with fluctuations in exchange rates. This section studies the implications this has for the dividend flow to the fiscal authorities and how it interacts with the constraints that determine central bank solvency.

6.1 Period solvency

The charters of the European Central Bank have no explicit allowance for automatic recapitalizations. For the ECB to receive a transfer from the fiscal authorities (a negative d_t) there must be an explicit agreement by all of the nation members of the Eurosystem. In most circumstances, this is politically very difficult. Therefore, the ECB can be interpreted as having the weakest of fiscal support from the European fiscal authorities, with negative dividends being ruled out, or $\{d_{t+j}\}_{j=0}^{\infty} \geq 0$.

At the same time, the charters of the ECB state that it must rebate its net income to the national central banks of the Eurosystem every year, and many of these are then required by national law to send them as dividends to their respective fiscal authorities. Therefore, to a first approximation, the dividend rule of the ECB can be written as:

$$d_t = \max\{s_t - r_t v_{t-1}, 0\}. \tag{25}$$

For our simple central bank that so far only issues reserves and prints banknotes, net income is equal to the seignorage from printing banknotes minus the interest paid on reserves. When paid interest exceeds seignorage, net income is negative, and the central bank does not receive any recapitalization form the fiscal authorities. In this extreme reading of the charters of the ECB, every time the central bank records a positive net income it must rebate it to governments, but when net income is negative it can expect no transfer from the fiscal authority.

Over the past century, across most of the developed world, annual seignorage has usually been positive. Nominal income growth has been positive in most years, and the velocity of currency seems to be stationary, so s_t is positive by the end of the year. However, consider what would happen to our hypothetical central bank if in one year, seignorage turned out negative. Starting from zero reserves and zero asset, that year the central bank would have to borrow to offset this fall in income. As banks come to deposit some of their currency at the central bank, they receive reserves in return. Interest must now be paid on these reserves forever since positive net income is sent to the fiscal authorities and cannot be used to retire reserves. Every time that net income is negative in the future, which happens more often when reserves are higher, again reserves grow. At some point, the total interest on reserves exceeds seignorage at all dates, and the central bank must issue reserves every year just to pay for the interest. The central bank is running a Ponzi scheme; it is insolvent to start with.

More formally, combining the dividend rule with the resource constraint in equation (18) leads to the law of motion for reserves

$$v_t = v_{t-1} + \max\{0, r_t v_{t-1} - s_t\}. \tag{26}$$

Therefore, $v_t \geq v_{t-1}$ almost surely. Moreover, once v_t is large, so s_t is negligible with respect to it, v_t grows exponentially at the real interest rate. At this point, the expected discounted value of reserves is strictly positive, violating the no-Ponzi scheme condition in equation (20).

Reis (2015a) called this *period insolvency*. Under the asymmetric dividend rule that does not allow for any negative dividends, then as long as there is a positive probability of negative seignorage in one period, the central bank becomes insolvent. Hall & Reis (2015) characterize the circumstances where $d_t < 0$ for the United States and the ECB, by measuring the necessary recapitalizations of the central bank to cover the possible losses. Understanding the period solvency of the central bank reduces then to inspecting its net income, and gauging the circumstances under which this can be negative, since these define the cases where the central bank becomes a fiscal burden.

6.2 Risky assets and central bank net worth

Let the central bank now buy risky assets in the amount z_t. For simplicity, imagine that this asset pays no dividend but has a non-zero real price e_t, as in the case of foreign currency, although one could extend the analysis to have a risky dividend with no significant changes to the conclusions. In this case, e_t would stand for the real exchange rate, such that when this rises, the foreign currency appreciates relative to the domestic currency. No arbitrage with respect to this asset imposes that:

$$\mathbb{E}_t\left[\frac{m_{t,t+1}e_{t+1}}{e_t}\right] = 1. \tag{27}$$

The resource constraint of this central bank now is:

$$v_t = (1 + r_t)v_{t-1} - s_t + d_t + e_t(z_t - z_{t-1}). \tag{28}$$

The new term reflects the fact that funding new purchases of the risky asset must be done by issuing reserves. The risk comes because while z_{t-1} is chosen last period, its real payment today depends on e_t, so reserves today may have to be used to offset losses. The no-Ponzi scheme condition on reserves now is:

$$\lim_{T\to\infty} \mathbb{E}_t[m_{t,T}(v_T - e_T z_T)] \leq 0, \tag{29}$$

reflecting the central bank's assets that can be used to pay the reserves.

Having introduced assets on the central bank balance sheet, there is also now a definition of central bank net worth as the difference between the value of assets and the value of liabilities. In real terms:

$$n_t = e_t z_t - v_t. \tag{30}$$

By itself, this definition does not mean much. Unlike private corporations, central banks cannot be closed down by their creditors demanding a sale of the assets in order to pay for the liabilities, so whether net worth is positive or negative has in itself no economic implications. However, when combined with the no Ponzi scheme condition, note that if net worth is constant, as long as $m_{t,T}$ declines with T, which is true in almost all models, then the central bank will always be solvent. Requiring constancy of net worth is sufficient, but not necessary, to ensure solvency.

The pervasiveness of the concept of net worth has led to it creeping into the rules for the operations of central banks and for distribution of dividends to the fiscal authority. For instance, the way in which net income is calculated is often following the accounting convention of it being the first difference in the balance sheet. In other words, net income is such that if it was paid as a dividend, net worth would be constant. In this case, net income is equal to:

$$x_t = s_t - r_t v_{t-1} + (e_t - e_{t-1})z_{t-1}. \tag{31}$$

The first two terms are familiar. They are the income from seignorage and the expenses from paying interest on reserves. Both are expressed in real terms using market prices. The last term measures the marked-to-market capital gain from owning foreign reserves. Because net worth is constant, then the central bank is solvent as long as negative net income comes with a recapitalization by the fiscal authorities.

6.3 Foreign exchange and the fiscal burden

Replacing out reserves in the expression for net income using the law of motion for reserves gives a reduced-form expression for dividends:

$$d_t = s_t + r_t n_{t-1} + \left[\frac{e_t}{e_{t-1}} - (1 + r_t)\right] e_{t-1} z_{t-1}. \tag{32}$$

When this is negative, either the central bank is period insolvent, or it must be recapitalized, increasing the fiscal burden.

The first term in the expression is the same as in the previous subsection. By selling banknotes, the central bank earns seignorage. The second term is new, and equal the interest earned on the net worth. Insofar as the central bank started with assets exceeding reserves, it will earn a return on this endowment. This can be used as a buffer against fluctuations in the other components of income to prevent period insolvency. Even though this term is small for most central banks, it can be large enough to offset some of the rare instances where seignorage is negative. Central bank net worth can therefore eliminate the insolvency that we discussed in the previous subsection and, together with periodic recapitalization, account for why the Fed, the ECB or the Bank of England have not experienced insolvency in their history.

The last term is zero in risk-adjusted expectation; just multiply by the stochastic discount factor and take expectations using the no-arbitrage pricing conditions to see this. On average, the central bank does not earn excess returns on its holdings of risky assets. Still, given fluctuations in the price of those assets, if the portfolio of assets is large, the central bank could earn a large positive or negative income in any given year. In the case of foreign currency, losses happen if the domestic real exchange rate appreciates more than expected. With the very large amount of reserves that were accumulated by the central banks in many emerging countries so far this century, even small fluctuations in currency can make the central bank have losses, and even end up with negative net worth.

The situation becomes more extreme when there are large fluctuations in exchange rates during crisis. Countries often accumulate large foreign reserves in order to defend their currency pegs. When the markets expect the currency to depreciate and the central bank successfully defends the peg, the exchange rate depreciates by less than expected, so the central bank accumulates a loss in its large foreign holdings. When the peg is let go, the surprise depreciation should give the central bank a large gain. But this is typically done by the time when z_t is already very low, so the gain is small and not enough to offset the accumulated losses from the months defending the peg. By the end of the year, the central bank of a country that went through a currency crisis often records very large losses. Ironically, if the central bank is successful at defending the peg, the losses are even larger.

In the other direction, the Swiss National Bank in 2013-15 tried to keep the Swiss franc from appreciating relative to the euro by buying a larger amount of euro reserves. Towards the end of 2014, it became clear that the exchange rate peg would have to be eventually let go, especially as the real exchange rate of the Swiss franc was appreciating because the euro was depreciating relative to the dollar. Moreover, the holdings of foreign reserves were so large relative to Switzerland's GDP that there was no hope of a government recapitalization for when this happened. The announcement of a further round of quantitative easing by the ECB, which was expected to lead to a further rise in z_t to defend the peg, led the SNB to abandon the peg and realize the losses rather than risk its own solvency.

It is therefore not surprising that in many countries the central bank has stronger fiscal support when it comes to its foreign reserves holdings. Often, charters commit the fiscal authority to recapitalize them in case of losses in foreign reserves. In other countries, the central bank is an agent holding the foreign currency in the name of the fiscal authority which ultimately own it and bears any losses associated with them. The central banks of countries with large foreign reserve holdings and exchange rate pegs can sometimes not count the income risk from these holdings as part of their net income, transferring losses automatically to the fiscal authorities. Because foreign reserves expose the central bank to such large income risk, they are brought back into the accounts of the fiscal authorities. In this case, the central bank's actions in managing their portfolio of foreign exchange can add to or diminish the overall fiscal burden.

7 Private default risk and rule solvency

In normal times, the central banks of advanced economies stay away from holding assets that are credits on the private sector. This is seen as potentially interfering with the allocation of credit across sectors in a way that goes beyond the mandate of the central bank. Yet, many central banks in emerging countries routinely do so, and from 2013 onwards the Bank of Japan started buying large amounts of corporate bonds as part of its QQE (qualitative and quantitative easing) policies. This section considers the risk that buying bonds that may default brings to the solvency of the central bank, and thus to its ability to generate fiscal resources for the government.

7.1 Bonds and default risk

I assume that private bonds are denominated in real terms, or alternatively that they are indexed to inflation. This is clearly unrealistic but plays only a pedagogical role. I already partly considered the effect of inflation on the value of bonds in section 3, and section 8 will have more to say about government bonds that are assumed to be nominal. Inflation would not affect private bonds in a different interesting way than the way it affects government bonds.

A bond promises to give one real unit tomorrow in exchange for a real price q_t today. This promise may be unfulfilled if the bonds defaults, and instead pays a fraction of what was promised. With a slight abuse of notation, let δ_{t+1} be the repayment rate, as we did for government bonds. This equals 1 in most states of the world, but it is between 0 and 1 in some of them. No arbitrage as usual dictates that the risk-adjusted expected gross return on this bond is one:

$$\mathbb{E}_t[m_{t,t+1}\delta_{t+1}] = q_t. \tag{33}$$

As before, the central bank issues reserves v_t, promises to pay a nominal interest rate on them, which after inflation leads to a real return r_t, collects seignorage from selling banknotes s_t and pays a dividend to the government d_t. I drop the risky foreign assets, but they would just lead to an extra additive term in net income, which was already dissected in the previous section. The novelty is that the central bank buys b_t bonds in real terms and collects the repayment on the bonds it bought the previous period. Therefore, the law of motion for reserves is:

$$v_t = (1 + r_t)v_{t-1} - s_t + d_t + q_t b_t - \delta_t b_{t-1}. \tag{34}$$

Finally, the new no Ponzi scheme condition on reserves again just states that reserves minus the value of assets cannot be rolled over forever:

$$\lim_{T\to\infty} \mathbb{E}_t \left[m_{t,T}(v_T - q_T b_T)\right] \leq 0. \tag{35}$$

7.2 Rule solvency and accounting net worth

Hall & Reis (2015) first combined the no-Ponzi scheme on reserves with the link to central bank independence to come up with a workable definition of central bank solvency. They interpreted central bank independence in terms of a rule for the dividends, leading to a definition of *rule solvency*: a central bank is solvent while it can stay committed to the rule for dividends that is in its charters without reserves exploding. This definition is explicitly grounded on the key role of fiscal support by emphasizing how the rule for dividends fiscally connects the central bank and the Treasury.

Period solvency, which was studied in the previous section, is a special case of rule solvency when the rule is that the central bank dividend can never be negative. But this rule is more general because it can accommodate other cases. The first rule considered by Hall & Reis (2015) came from an economic reading of most central bank charters: that dividends should be paid as net income. In that case, as the previous section showed, net worth would be constant. Reserves would equal:

$$v_t = q_t b_t - n_0 \tag{36}$$

at all dates. The central bank is solvent, and reserves track the purchases of bonds.

However, while for most central banks the rule is to pay out net income, this is interpreted differently via the accounting rules used to calculate net income. Most central banks differ in their accounting from a proper economic measure of net worth. First, they count their output of banknotes as a liability. Second they measure net worth in nominal, rather than real terms. Therefore, accounting nominal net worth is measured as:

$$W_t = p_t(q_t b_t - v_t) - H_t. \tag{37}$$

As a result, the rule that tells the central bank to keep its net worth constant over time ($W_t = W_{t-1}$) leads to a dividend payment equal to net income that is calculated as:

$$d_t = \left(\frac{i_{t-1}p_{t-1}}{p_t}\right) n_{t-1} + \left[\frac{\delta_t}{q_{t-1}} - (1 + r_t)\right] q_{t-1}b_{t-1}. \tag{38}$$

The first difference relative to before is that seignorage disappears. Because banknotes are recorded as a liability, printing and selling them is not counted as a revenue. Of course, this accounting definition does not change the reality that there is a seignorage revenue, but it simply allows the central bank to keep this revenue instead of distributing it to the fiscal authorities.

The second difference is that it is now the nominal return to net worth that raises dividends. Because the nominal interest rate is always positive, this return is always positive. The reason is that by calculating net income in terms of nominal accounting net worth, the calculation suffers from a version of money illusion. For a fixed real interest rate, higher inflation raises the real dividend that gets paid. With these accounting rules, if the central bank holds no risky assets, then its dividend is sure to be always positive, so it is never period insolvent.

With a bond, or other risky assets, income might be negative. The last term in the expression is zero under risk-adjusted expectations on average, which means that in the states when the bond pays in full, the central bank will earn a small positive revenue on its assets. But, when it defaults, there will be a loss, which is larger the smaller the repayment rate and the larger is the price of the bond (for instance because default was less likely). If the rule for fiscal transfers is still the max rule, then declines in central bank net worth will measure the extent of the recapitalization that the central bank needs. Private default therefore ends up affecting central bank solvency and the impact of central bank actions on fiscal burden in a similar way as exchange-rate risk: both can lead to fluctuations in net income and in the regular remittances to the fiscal authorities which, if large enough, can trigger central bank insolvency.

7.3 Provisioning and the fiscal burden

Many central banks have the ability to retain earnings as provisions against future losses on risky assets. This consists of holding on to positive net income, instead of distributing it as a dividend, so it can be offset against future negative net income. Provisioning raises the net worth of the central bank, so that when losses happen, then net worth can fall back without leading to insolvency of the central bank. For instance, the ECB statute allows it to build a "risk provision" and a "general reserve" by retaining profits up to 100% of the statutory capital.

By focusing on net worth, provisioning may at first seem peculiar since net worth should not be a relevant concept for a public agency. Yet, for an independent public body that issues its own liabilities like the central bank, provisioning is a sensible way to accommodate future income losses from investments in risky private assets, without needing future recapitalizations from the fiscal authority. That is, provisioning is a form to relax the institutional independence constraint on $\{d_{t+j}\}$ and in doing so prevent central bank insolvency.

Returning to the fiscal burden, including both foreign reserves and bond holdings, the present value of dividends from the central bank to the fiscal authority is still just equal to the net liabilities of the central bank. Therefore, combining the resource constraints gives:

$$\Phi_t \leq \mathbb{E}_t \left[\sum_{j=0}^{\infty} m_{t,t+j+1}(f_{t+j} + s_{t+j}) \right] + e_t z_{t-1} + \delta_t b_{t-1} - (1 + r_t) v_{t-1}. \tag{39}$$

The current value of the assets with which the central bank enters this period reduces the overall debt burden, while the promised payments on reserves increase it. At period t, the central bank does not choose any of the variables on the right-hand-side, so none of its present actions this period affect the debt burden today.

As for past actions, we can re-write:

$$e_t z_{t-1} + \delta_t b_{t-1} - (1 + r_t) v_{t-1} = n_{t-1} + (e_t - e_{t-1}) z_{t-1} + (\delta_t - q_{t-1}) b_{t-1} - r_t v_{t-1}. \tag{40}$$

By issuing liabilities (reserves) to hold assets (foreign and private) the central bank has a net worth of $n_t = q_t b_t + e_t z_t - v_t$. These are net assets that are ultimately owned by the government and which it can use to pay down some of its debt. Therefore, a higher n_{t-1} reduces the fiscal burden.

Yet, the discussion in this section and the previous one emphasized two properties of net worth that severely limit its ability to relax the fiscal burden. First, in theory, for a central bank that follows a net income rule to pay its dividends, net worth is constant over time. Thus, $n_t = n_0$ and since the initial net worth was given to the central bank by the fiscal authority, it will have added to the fiscal burden then. The intertemporal effect is therefore neutral. Second, in practice, the net worth of most central banks is quite small at most dates, and two orders of magnitude smaller than the public debt, so this effect is for all purposes quantitatively irrelevant.

The remaining terms on the right-hand side of the equation above are all zero in risk-adjusted expectation. This is a more general lesson that applies to risky investment by the central bank: the central bank does not earn excess returns. Therefore, holdings of private assets by the central bank does not lead to a reduction of the fiscal burden of the fiscal authorities. Some periods, the central bank will return a positive income beyond seignorage, some periods it will require a recapitalization. On average these will be zero, and if the fiscal authority only receives the positive income and refuses to proceed with the recapitalizations, then the central bank will become insolvent. The balance sheet of the central bank is only important through the income risk it generates, and how this interacts with central bank independence.

8 Sovereign risk: public bonds

Continuing to consider default, I now turn attention to public nominal bonds. These are interesting and distinct from the previous discussion for two reasons. First, because the central bank is a branch of the government, so issuing new public liabilities (reserves) to buy old public liabilities (government bonds) is a purely intra-governmental transaction. Second, because sovereign default leads to a transfer from one branch of the government to the other. This section introduces central bank holdings of one-period government bonds, which I will denote by B_t^{1c} from the total B_t^1 outstanding.

8.1 Wallace neutrality with no default

The resource constraint for a central bank that holds these public bonds as its only asset is:

$$v_t = (1 + r_t)v_{t-1} - s_t + d_t + \left(\frac{Q_t^1 p_{t+1}}{p_t}\right)\left(\frac{B_t^{1c}}{p_{t+1}}\right) - \delta_t \left(\frac{B_{t-1}^{1c}}{p_t}\right). \tag{41}$$

From the sole perspective of the central bank, this is almost the same as its resource constraint when it held private bonds. The only difference is that the bonds are nominal and so their return has inflation risk.

This nominal versus real difference makes a slight difference for the capital gains calculation in net income. Under a proper constant real net worth rule for calculating net income, dividends would equal:

$$d_t = s_t + r_t n_{t-1} + \left[\frac{\delta_t}{Q_{t-1}^1} - (1 + i_{t-1})\right] \frac{Q_{t-1}^1 B_{t-1}^{1c}}{p_t}. \tag{42}$$

The difference in the last term relative to when bonds were real (compare with equation (38)) is that capital gains now accrue in nominal terms.

If there is never any default—so $\delta_t = 1$ in all states of the world—then because no arbitrage implies that $1/Q_{t-1}^1 = 1 + i_{t-1}$, the last term in the expression above is always equal to zero. The intuition is that in this case reserves and government bonds have identical payoffs in the model: they are both denominated in nominal terms, and they both pay one nominal unit next period. In this case, the size of the balance sheet of the central bank is irrelevant to its own net income. Liabilities and assets are perfectly matched in payoffs and maturity, so that any returns earned from the government bonds are immediately paid to the reserve holders. This case roughly matches the experience of the U.S. Federal Reserve over the fifty years before the 2007-8 financial crisis.

This neutrality extends beyond the central bank balance sheet and applies to all variables in most monetary equilibrium models (Wallace, 1981; Benigno & Nistico, 2015). When the central bank issues reserves and buys government bonds, it is substituting one type of government liability for another. Because the two have exactly the same payoff, they make no difference to any choice or constraint of anyone in the economy. Conventional open market operations by central banks, where they issue reserves to buy short-term government bonds, are neutral because they simply change the composition of the overall government liabilities between two identical elements.

In reality, there are two important distinctions between reserves and short-term government bonds that lead to non-neutralities. First, reserves tend to be overnight, whereas the more liquid government bonds traded in most countries are typically of maturities of at least a few months. Therefore, there is some, even if short, maturity mismatch in the central bank's balance sheet which on the one hand makes the central bank affect real outcomes by engaging in maturity transformation but on the other hand expose it to interest-rate risk (Greenwood et al., 2016). This will be the topic of the next section. Second, reserves can only be held by banks, whereas government bonds are widely traded. This segmentation of the two markets may break the no-arbitrage condition linking their returns, especially during times of financial crisis when, by holding reserves, banks maybe able to relax liquidity and collateral constraints (Reis, 2016b).

8.2 Transfers to the government with sovereign default

Replacing the central bank resource constraint into the government's by assuming that the institutional restrictions on dividends do not bind gives the consolidated constraint:

$$\frac{(1+i_{t-1})V_{t-1}+\delta_t(B^1_{t-1}-B^{1c}_{t-1})}{p_t} \leq \mathbb{E}_t\left[\sum_{j=0}^{\infty} m_{t,t+j}(f_{t+j}+s_{t+j})\right]. \tag{43}$$

Buying government bonds changes only the left-hand side. Wallace neutrality can also be seen here: with no default, then any expansion in reserves comes by buying government bonds, which therefore reduces the bonds being held by the private sector. The left-hand side is completely unchanged by this change, and thus the fiscal burden is unchanged as well.

With sovereign default though, there is a clear difference. The reason is that reserves are default free. Because they are the unit of account, they are always nominally paid. When the central bank buys more sovereign bonds, there is a shift in the composition of the overall public debt away from risky-default bonds and towards default-free reserves. Insofar as the government intended to use default as a way to lower its fiscal burden, then by holding government bonds, the central bank makes this harder. To achieve the same real reduction in the debt burden, the government's default must be greater in the sense that the recovery rate on the bonds δ_t is smaller. This all applies to fundamental default; if instead the sovereign default is driven by multiplicity of equilibrium, then central bank purchases can instead make default less likely (Corsetti & Dedola, 2016).

In terms of central bank net income, as equation (42) makes clear, after the sovereign default the central bank makes a loss. Its loss is the gain of the fiscal authority, and it is exactly matched by a lower stream of dividends from the former to the latter, so that this leads to no net reduction in the fiscal burden. Another redistribution that occurs as a result of default and central bank intervention is from the holders of the public debt towards banks (Reis, 2016b). Because only banks can hold reserves, only they benefit from being immune to sovereign default. Of course, this redistribution towards banks is perhaps more than offset by the extensive financial repression that governments often adopt when they find themselves in a fiscal crisis.

9 Interest-rate risk and QE

The Federal Reserve holds almost entirely domestic assets, issued or backed by the U.S. Federal government, and this has been the case for many decades (Toporowski, 2017). It is exposed to almost no exchange-rate or private default risk. If the Federal Reserve were to hold only short-term Treasuries (as it did until 2008), then the interest paid on reserves would be almost always precisely offset by the interest earned on the Treasuries bought with these reserves. This would result in no change in the fiscal burden of the government and no risk to the Fed's solvency.

However, through its quantitative easing policies, the Fed increased the average maturity of the Treasuries it held by buying longer-term securities. This section studies the risk that this brings by including longer-term government bonds.

9.1 Longer-term bond and interest rate risk

I assume that aside from default-free government bonds of maturity 1 period, there are also 2-period bonds. Extending the analysis to a full array of bonds of arbitrary maturities would just involve keeping track of more involved notation, with little gain in insights.

A 2-period bond sells for price Q_t^2 today and promises to pay one nominal unit in two periods. Tomorrow, this bond is identical to a one-period bond issued tomorrow, so its price is Q_t^1 then. Assuming no default, these two prices are linked by the no-arbitrage conditions:

$$\mathbb{E}_t\left[\frac{m_{t,t+1}p_t}{Q_t^1 p_{t+1}}\right] = \mathbb{E}_t\left[\frac{m_{t,t+1}Q_{t+1}^1 p_t}{Q_t^2 p_{t+1}}\right] = 1, \tag{44}$$

The central bank's resource constraint now is:

$$v_t = (1+r_t)v_{t-1} - s_t + d_t + \frac{1}{p_t}(Q_t^1 B_t^{1c} + Q_t^2 B_t^{2c} - \delta_t B_{t-1}^{1c} - Q_t^1 B_{t-1}^{2c}). \tag{45}$$

The Ponzi scheme condition still applies to reserves, but can now be offset by the market value of both types of bonds:

$$\lim_{T\to\infty} \mathbb{E}_t\left[m_{t,T}\left(v_T - \frac{Q_T^1 B_T^{1c} + Q_T^2 B_T^{2c}}{p_T}\right)\right] \leq 0. \tag{46}$$

The central bank issues 1-period liabilities and uses some of them to buy 2-period assets. In doing so, it is engaging in maturity transformation. In fact, since in reality post-QE the central banks of the major advanced economies borrow very short (reserves are overnight) and invest in government bonds of maturities of several years, the balance sheet of the central bank resembles that of a short-long investment fund. With this activity comes risk and fiscal consequences.

9.2 Yield curve risk

Using again the constant net worth rule that rebates every period net income to the fiscal authority, and assuming no risk of default, the dividends are:

$$d_t = s_t + r_t n_{t-1} + \left[\frac{Q_t^1}{Q_{t-1}^2} - (1+i_{t-1})\right] Q_{t-1}^2 B_{t-1}^{2c}. \tag{47}$$

The novelty relative to before is the last term on the right-hand side. As before, there is income risk from seignorage that is partly buffered by the return on net worth. Because I assumed there is no default risk, the return on one-period bonds is exactly the same as the interest paid on reserves, so they generate no income. The new feature are the capital gains on the long-term bonds as a result of changes in their price.

The arbitrage conditions in the equations (44) dictate that the expected risk adjusted real value of this term is zero. There are no expected returns from maturity transformation in this simple model because there is no term premium to reward this activity. However, ex post, this term may not be zero depending on how interest rates and inflation envolve. To see this clearly, note that the arbitrage conditions dictate that this term is equal to:

$$\left[\frac{1}{1+i_t} - (1+i_{t-1})\,\mathbb{E}_{t-1}\left[\frac{m_{t-1,t}p_{t-1}}{(1+i_t)p_t}\right]\right] B^{2c}_{t-1} \tag{48}$$

It is surprises about inflation and the evolution of interest rates in the future that lead this term to be non-zero. A sudden unexpected steepening of the yield curve forces a loss over the central bank (Bassetto & Messer, 2013; Hall & Reis, 2015).

9.3 Intertemporal solvency and the solvency upper bound

The Federal Reserve has a particular way to deal with these losses. The Federal Reserve Act (section 7, 12 USC 290) allows it to, when net income is negative, record a “negative liability“ in the sense of a balance in a deferred account. Then, when in a future period, net income is positive, the balance of the deferred account is first brought to zero before any disbursements are made to the Treasury. This rule has never been tested by a large balance in the deferred account, and it is unclear to what extend the Federal Reserve could keep to it if there was strong political opposition. It has the virtue though of providing fiscal backup by the fiscal authorities without requiring actions by them. They no longer need to approve a recapitalization of the central bank, or to transfer funds to it. Instead, it is the fiscal authority’s future claim on the central bank’s net income that is used to provide this fiscal backup.

To see this mathematically, let k_t be the balance on the deferred account and recall that x_t denotes net income. The adjusted rule for dividends to the fiscal authority then is:

$$d_t = \max\{x_t - (1+r_t)k_{t-1}, 0\}. \tag{49}$$

Net income is first used to pay off the balance in the deferred account. Only after that, is the remainder distributed as dividends.

The law of motion for the account balance then is:

$$k_t = \max\left\{0, \min\left\{\bar{k}, (1+r_t)k_{t-1} - \max(x_t - d_t, 0) + \max(-x_t, 0)\right)\right\}\right\}. \qquad (50)$$

If there is a limit to the size of the deferred account, $k_t \leq \bar{k}$, either explicit in the rules of the central bank, or implicit in terms of how far the fiscal authority will allow it, then $\bar{k} - k_t$ provides a measure of the solvency of the central bank. Once it is close to zero, then any future loss will imply insolvency in precisely the same way as before. Because the size of $\bar{k} - k_t$ may correlate with the accounting net worth of the central bank, it being negative is not a problem per se but may be indicative of solvency risks in the horizon.

Because the central bank will keep on generating seignorage from printing banknotes, a natural economic limit for $\bar{k}$ is the present value of this seignorage. In that case, summing up the central bank resource constraint and using the no Ponzi scheme condition gives the intertemporal constraint:

$$\mathbb{E}_t\left(\sum_{j=0}^{\infty} m_{t,t+j} d_{t+j}\right) \leq \frac{\delta_t B^{1c}_{t-1} + Q^1_t B^{2c}_{t-1}}{p_t} - (1+r_t)v_t + \mathbb{E}_t\left(\sum_{j=0}^{\infty} m_{t,t+j} s_{t+j}\right). \qquad (51)$$

The right-hand side is the sum of the accounting value of the central bank and the present value of seignorage. This is the fiscal capacity of the central bank, the upper limit on the dividends that it can achieve. Reis (2015a) and Benigno & Nistico (2015) took this constraint to define the *intertemporal solvency* of the central bank. As long as the right-hand side is positive, the central bank is solvent. This is a weaker requirement than period or rule solvency. Even if the central bank has periods of negative net income, and even if it deviates from some rule, as long as it can use a deferred account then it can smooth out these income shocks by driving its reserves up and down. As long as the right-hand side of the expression above is positive, it can ensure that it will always pay zero or positive dividends to the fiscal authority, although this may require at times very large balances in the deferred account.

10 Fiscal redistribution

The fifth and final channel through which central banks can affect fiscal burdens is by redistributing them across different fiscal authorities. In public finance, the ability of fiscal authorities to generate resources from taxation and then spend them in different public projects is as studied as the government's ability to redistribute a fixed amount of resources to different groups. Likewise, a study of the fiscal role of the central bank is not complete without also discussing the fiscal redistributions that the central bank can engender.

At the same time, such a discussion is limited by two opposing forces. On the one hand, unlike fiscal authorities, central banks have no redistribution mandate, and in fact are often explicitly disallowed from actively pursuing redistributive policies. In fact, one of the main reasons why central banks are dissuaded from buying risky assets, as in sections 6 and 7, is the redistribution that they may lead to across the private issuers of these securities. On the other hand, *any* central bank action that has some effect on inflation or real activity will likely have some redistributive effects. Households and firms are differentially exposed to inflation risk, through the multiple contracts they sign or the portfolios they choose, and likewise they are hurt differently during recessions.

As a result of these two forces, a more institution-centered discussion of fiscal redistribution by central banks might rule it out from the start, while a more consequentially-driven one would require a whole article in itself. The compromise solution in this section is to focus only one one source of risk—public default—and to consider only redistribution across regions within a currency union.

10.1 Redistribution via dividends and income

In a currency union, the central bank faces many fiscal authorities. On top of whether it alleviates the fiscal burden of the union as a whole, the central bank might or might not be able to transfer fiscal burdens from one region to another. This section extends the model of the central bank to a currency union to investigate this possibility. For concreteness, I consider only the case of a short-term bond with default risk, although the other sources of risk discussed so far could be included.

Consider a world with 2 regions, labelled a and b. With only one-period real bonds with default risk, and assuming for concreteness that the central bank follows a net-income dividend rule, total dividends paid are:

$$d_t^a + d_t^b = s_t + r_t n_t + \left[\frac{\delta_t^a}{q_{t-1}^a} - (1+r_t)\right] q_{t-1}^a b_{t-1}^a + \left[\frac{\delta_t^b}{q_{t-1}^b} - (1+r_t)\right] q_{t-1}^b b_{t-1}^b. \qquad (52)$$

With no restrictions on how to choose the dividends to each section, the central bank could promote unlimited redistribution across areas. It would just have to set d_t^a to a very large number, and d_t^b to a correspondingly negative amount. Under the no-recapitalization constraint discussed in section 6, the constraint that $d_t^b \geq 0$ would limit this redistribution.

But this would still leave significant room for a sizable transfer.

Most central banks in advanced countries do not have this discretion. They must return their dividends to a single authority at the national level rather than to multiple regions. An exception is the Eurosystem. There is no single union-wide fiscal authority to send dividends to in the euro zone, and they are instead distributed to the member central banks, and so effectively to regions. Each country's central bank then has its rules of distribution to the fiscal authorities. Yet, the Eurosystem itself has close to *zero* discretion in choosing how to distribute these dividends to each member central bank and region. There is a strict formula in the founding Treaty stipulating a fixed rule such that each country's share of its dividends must equal the average of the country's population share and GDP share in the Eurozone, averaged over the last three years. Therefore, there is no scope for this discretionary redistribution.

There is still discretion though in how the overall d_t is calculated, that is on how net income is calculated, which can be subtly manipulated to lead to redistribution. Imagine that the expected repayment on country a's bonds is quite low perhaps because the probability of default is quite high. As a result, the price of its bonds q_t^a is low as well. Yet, ex post, default may not materialize. In this case, the return on the bond δ_t^a/q_{t-1}^a will ex post turn out to be well above $1 + r_t$. Even if on average the central bank earns no excess returns on these bonds, for some states of the world the return may be high.

In this situation, it may turn out to be politically difficult to justify the high returns by the central bank at the expense of a country going through a fiscal crisis and paying a high interest rate on its bonds. The ECB in 2014 agreed to raise the dividends to Greece above their stipulated key, by including in the Greek dividend all the gains in holding Greek bonds. This is a form of redistribution. If default materializes instead, the redistribution goes in the other direction. Country a reaps the benefits from not paying its bonds, but the fall in net income lowers the dividends to both country a and b. Ex post, this is a transfer from country b to country a.

The only way to fully prevent redistributions through net income would be to replace equation (52) with two separate dividend rules for each region. One way to achieve this in the Eurozone is to have national central banks hold the bonds of their country, so that any losses and gains affect the net income and dividends of that country alone. The ECB moved partly in this direction from 2015 onwards, when it instituted as a rule for its government bonds purchase program that 92% of net profits would stay at the national central banks. Whether in case of a loss, there would be provisioning or deferred accounts at the national or at the Eurozone level is less clear.

10.2 Redistribution via assets and reserves

Even if the net income rules completely sever the two regions apart, they still share a resource constraint via the central bank. Its no-Ponzi scheme version is:

$$\lim_{T\to\infty} \mathbb{E}_t \left[m_{t,T} \left(v_T^a + v_T^b - q_T^a b_T^a - q_T^b b_T^b \right) \right] \leq 0. \tag{53}$$

This allows for one final form of redistribution. If the central bank holds a larger share of country a's bonds than its share of reserves, and does so forever, then this amounts to letting country a run a Ponzi scheme at the expense of country b. Country a is on net borrowing from the central bank, and country b is on net lending to it, with this debt rolled over forever.

The ECB's charters forbid this redistribution on the side of assets. The ECB until 2012 did not choose which bonds to hold. Rather, it conducted its monetary policy operations by accepting in repurchase agreements any collateral from a wide list. Therefore, it did not effectively control the compositions of its portfolio across regions. Once the ECB started its securities market program (SMP) it bought the debt of countries in trouble. The German and European constitutional courts assessed whether these bond holdings would lead to a redistribution contrary to the charter of the ECB. Equation (53) shows clearly that the answer to that question depends on a single criteria: whether the SMP was a temporary or permanent program. In the former case, no redistribution results from it. In the latter, redistribution happens.

On the side of reserves, the ECB was also set up to prevent this redistribution. In the Eurosystem, when one financial institution moves its deposits from the central bank of one country to another, the liabilities to the private sector of the first central bank fall, while the liabilities of the second central bank rise. Via the TARGET2 system, this transaction is recorded in the Eurosystem as a liability of the first country to the second. The total liabilities and reserves of each country are unchanged. The crisis again led to a deviation from this principle. The ECB allowed the national sovereign banks to give emergency lending assistance (ELA) around this system. Banks can get reserves from national central bank, and print euros with the ECB's authorization. Again, if this program is temporary, it implies no redistribution. The no-Ponzi scheme has real economic bite.

The bottom line is that there are many channels through which a common central bank in a fragmented fiscal union can redistribute resources. Because the central bank is a fiscal actor, with a resource constraint shared by all the regions, it can be used to voluntarily redistribute resources across regions. Even if it is not used this way, it will still lead to ex post involuntary transfers. This is the inevitable consequence of sharing a fiscal actor with a common resource constraint. Many of the ECB's rules were designed to prevent this redistribution on average, yet its unconventional policies put these rules to the test. Using the principles laid out in this article, one can measure and assess this redistribution in a disciplined way.

11 Conclusion

The central bank produces and supplies a public good (banknotes) for which it collects a revenue (seignorage), it affects the path of a variable that works as a tax (inflation), it issues a public liability (reserves), it holds other public liabilities (government bonds), it provides a stream of dividends to the fiscal authorities that can fluctuate significantly with income (remmittances), and its policies interact with those of fiscal authorities to determine equilibrium outcomes that affect tax revenue and public spending. From more than one perspective, the central bank is a fiscal agent. In a broad sense, any study of central banks and monetary policy must therefore consider their interaction with fiscal policies.

There are several surveys on how the interaction between monetary and fiscal policy determines inflation (Sims, 2013; Leeper & Leith, 2016). This article reviewed a different set of results, from a more recent literature, which has explored the monetary-fiscal interactions from the perspective of the resource flows from the central bank to the fiscal authorities. As important as highlighting that the central bank is a fiscal agent, is to understand that it is also subject to a fiscal limit. This clarifies a few dangerous fantasies that are commonly stated because they are partly correct.

The first one is that "central banks can never go insolvent." This is partly correct in that there is no legal insolvency procedure for a central bank. The danger associated with it is that it leads to claims that central banks can always pay for any public spending, and even if that raises inflation a little, this may be desirable when inflation is below target anyway. As this article clarified, central banks can and do become insolvent, not in a legal sense, but in an economic sense. This happens whenever the central bank does not have support from the fiscal authorities to cover its losses, and the reserves outstanding are no longer fully covered by its assets and the stream of income that it can retain post remittances to the fiscal authority. When this happens, banks no longer wish to roll over reserves at the central bank, their value falls to zero, so hyperinflation and currency reform result. The central bank is insolvent in the economic sense that no one accepts its liabilities, and their value must be written down. This paper discussed how the extent of fiscal support conceptually leads to different forms of insolvency (period, rule and intertemporal) and how in practice different asset holdings some with different forms of income risk that may put solvency in question.

The second fantasy is that “a country that prints its own money can never have a sovereign debt crisis.” This is partly correct because the central bank can issue default-free reserves or banknotes to buy government bonds. But this statement is dangerous and not quite right for many reasons linked to different conclusions in this article. First, as we saw in section 5, printing currency does not prevent central bank insolvency. Second, the actual real revenues from printing currency are quite small, as discussed in section 4. Third, as emphasized in section 2, printing money raises inflation, which increases the interest rate that nominal government bonds must pay. A country with a central bank may be able to commit to never default, but if this comes with expected high inflation, the interest rates on its government bonds will be just as large. Fourth, we saw that it is not enough to generate inflation to devalue the debt: in a fiscal crisis where the maturity of the debt usually becomes very low, this must be done so suddenly as to make it almost infeasible without hyperinflation, as discussed in section 3. Finally, as section 8 discussed, if to prevent sovereign default the central bank buys most of the government bonds this will just lead the default to happen through central bank insolvency, or currency reform.

The third claim is that the central bank can freely redistribute across regions. The ECB’s misguided choices are then blamed for not doing more to help Greece during its debt crisis post-2010. Section 10 shows that indeed the central bank can redistribute across regions in a currency union. But, the institutional design of the ECB (and the Federal Reserve or the Bank of England) make these extremely difficult without breaking the legal mandate of the central bank.

The overall point of this paper is not to say that central banks can not play a role in a fiscal crisis. Neither is it to argue that monetary policy cannot constrain fiscal policy in many ways. Rather, it was to analyze the resource flows between the central bank and the fiscal authorities. From that analysis comes the conclusion that these resource flows are typically small and do not come for free.

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