The Drude Theory of Metals

Outline

The Drude theory of metals

The model

The metallic state

- Most elements are metallic
  - more than $\frac{2}{3}$ of the periodic table
- Ductile
- Malleable
- Good conductors of heat and electricity
- Need a theory of the metallic state
  - to understand their properties
  - ... and the properties of insulators

The Drude theory of metals

The model

General remarks

- Apply the classical kinetic theory of gases to the conduction electrons
  - viewed as an electron gas
- Works well in some instances
  - because of cancellation errors
- Can be used for gross estimates of metallic properties

The Drude theory of metals

The model

Kinetic theory of gases: overview

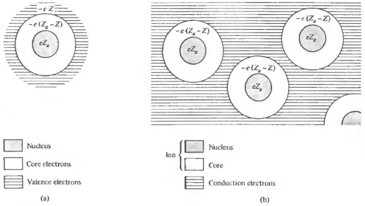
- Atoms/molecules modelled as spheres
- No forces acting between molecules
  - straight motion between collisions
- Collisions are istantaneous and elastic
  - among molecules or with the walls of the container
- Kinetic energy is a measure of T ($\frac{1}{2}mv^2 = \frac{3}{2}k_B T$)

The Drude theory of metals

The model

Valence and core electrons

- Atom of atomic number Z_a with Z valence electrons:
  - involved in the chemical bond
  - periodicity of chemical/physical properties of elements
  - conduction electrons in a metal
- Z_a-Z Core electrons:
  - tightly bound to the nucleous, essentially atomic
  - positive background confining the electron gas



(a) isolated atom; (b) model of the electron gas of conducting electrons

The Drude theory of metals

The model

Free electron densities

- Density of the electron gas:

$$n = \frac{N}{V} = 6.022 \times 10^{23} \frac{Z \rho_m}{A}$$

  - number of electron per cm^3
  - ρ_m: density of the element $(\frac{g}{cm^3})$
  - A: atomic mass of the element
- Alternative measure, r_s
  - $\frac{4\pi}{3} r_s^3 = \frac{V}{N}$
  - $r_s = (\frac{3}{4\pi n})^{\frac{1}{3}}$
  - $\frac{1}{n}$: volume per electron

The Drude theory of metals

The model

Free electron densities

FREE ELECTRON DENSITIES OF SELECTED METALLIC ELEMENTS[a]

ELEMENT	Z	n ($10^{22}/\text{cm}^3$)	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

The Drude theory of metals

The model

Assumptions and approximations

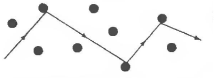
- free electron and independent-electron approximations
  - between collisions
  - free electron approximation: neglect electron-ionic cores interaction
  - independent electron approximation: neglect electron-electron interaction
- Uniform motion of electrons
  - in the absence of external fields
- Interaction with ionic cores is implicitly assumed (confinement of electrons)

The Drude theory of metals

The model

Assumptions and approximations

- Collisions are istantaneous
  - change abruptly the electron's velocity
  - assumed due to collision with ion cores
  - electron-electron scattering is not important
- The detailed scattering mechanism is actually not important



electron's trajectory scattering off the ionic cores

The Drude theory of metals

The model

Assumptions and approximations

- Probability of collision per unit time: $\frac{1}{\tau}$
- τ: relaxation time, or mean free time
    - average time between collisions
    - usually independent of electron momentum and position

The Drude theory of metals

The model

Assumptions and approximations

- Thermal equilibrium with the surroundings is reached through collisions
- Local temperature determines the electron velocity
  - after a collision
  - randomly directed
  - independent of the velocity just before the collision

DC electrical conductivity

Predictions from the Drude model

Ohm's law

$$V = RI$$

- V: potential drop
- R: resistance
  - depends on nature and shape of the material
  - proportionality constant btw V and I
- I: current
  - units of $\frac{C}{s}$

DC electrical conductivity

Predictions from the Drude model

Resistivity

$$\boldsymbol{E} = \rho \boldsymbol{j}$$

- $\boldsymbol{E}$: electric field vector
- $\boldsymbol{j}$: (induced) density current $\left(\frac{C}{cm^2 s}\right)$
- ρ: resistivity
  - depends only on the nature of the material
  - generally a tensor
- For uniform I flowing in a wire of length L and constant area A:

$$R = \rho \frac{L}{A}$$

DC electrical conductivity

Predictions from the Drude model

Current density

- If all e^- move with constant $\boldsymbol{v}$:

$$dq = -enA(vdt) \rightarrow \boldsymbol{j} = -ne\boldsymbol{v}$$

- when $\boldsymbol{E} = 0 \rightarrow \boldsymbol{v} = 0$
  - random orientations, no net flow of charge
- If $\boldsymbol{E} \neq 0$:

$$\boldsymbol{v}_{avg} = -e\frac{\boldsymbol{E}\bar{t}}{m} = -e\frac{\boldsymbol{E}\tau}{m}$$

  - $\boldsymbol{j} = (\frac{ne^2\tau}{m})\boldsymbol{E} = \sigma\boldsymbol{E}$
  - σ: conductivity

DC electrical conductivity

Predictions from the Drude model

Conductivity from Drude model

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

- All parameters are known except τ.
- Use measured σ's to estimate τ's
  - ρ linear in T at room temperature
  - τ are of the order of 10^{-14}–10^{-15}s

DC electrical conductivity

Predictions from the Drude model

Exp. resistivities (μOhm cm)

ELECTRICAL RESISTIVITIES OF SELECTED ELEMENTS[a]

ELEMENT	77 K	273 K	373 K	$\frac{(\rho/T)_{373\,K}}{(\rho/T)_{273\,K}}$
Li	1.04	8.55	12.4	1.06
Na	0.8	4.2	Melted	
K	1.38	6.1	Melted	
Rb	2.2	11.0	Melted	
Cs	4.5	18.8	Melted	
Cu	0.2	1.56	2.24	1.05
Ag	0.3	1.51	2.13	1.03
Au	0.5	2.04	2.84	1.02
Be		2.8	5.3	1.39
Mg	0.62	3.9	5.6	1.05
Ca		3.43	5.0	1.07
Sr	7	23		
Ba	17	60		
Nb	3.0	15.2	19.2	0.92
Fe	0.66	8.9	14.7	1.21
Zn	1.1	5.5	7.8	1.04
Cd	1.6	6.8		
Hg	5.8	Melted	Melted	
Al	0.3	2.45	3.55	1.06
Ga	2.75	13.6	Melted	
In	1.8	8.0	12.1	1.11
Tl	3.7	15	22.8	1.11
Sn	2.1	10.6	15.8	1.09
Pb	4.7	19.0	27.0	1.04
Bi	35	107	156	1.07
Sb	8	39	59	1.11

DC electrical conductivity

Predictions from the Drude model

Drude relaxation times $\tau = (\frac{0.22}{\rho_\mu})(\frac{r_s}{a_0})^3 \times 10^{-14}$s

DRUDE RELAXATION TIMES IN UNITS OF 10^{-14} SECOND[a]

ELEMENT	77 K	273 K	373 K
Li	7.3	0.88	0.61
Na	17	3.2	
K	18	4.1	
Rb	14	2.8	
Cs	8.6	2.1	
Cu	21	2.7	1.9
Ag	20	4.0	2.8
Au	12	3.0	2.1
Be		0.51	0.27
Mg	6.7	1.1	0.74
Ca		2.2	1.5
Sr	1.4	0.44	
Ba	0.66	0.19	
Nb	2.1	0.42	0.33
Fe	3.2	0.24	0.14
Zn	2.4	0.49	0.34
Cd	2.4	0.56	
Hg	0.71		
Al	6.5	0.80	0.55
Ga	0.84	0.17	
In	1.7	0.38	0.25
Tl	0.91	0.22	0.15
Sn	1.1	0.23	0.15
Pb	0.57	0.14	0.099
Bi	0.072	0.023	0.016
Sb	0.27	0.055	0.036

DC electrical conductivity

Predictions from the Drude model

Relaxation times and electron's velocities

- mean electron's velocity, $\boldsymbol{v}_0$:
  - $\frac{1}{2}mv_0^2 = \frac{3}{2}k_B T$
  - $v_0 \sim 10^7$ cm/s
  - 1 order of magnitude too small
- mean free path, l:
  - average distance btw scattering events
  - $l = v_0\tau \sim$1–10 Å (interatomic spacing)
  - τ is actually T dependent
  - $l \sim cm$ can be achieved (low T, low defects)

DC electrical conductivity

Predictions from the Drude model

Equation of motion in an external field, $\boldsymbol{f}$

- If $\boldsymbol{p}(t)$ is the total momentum per electorn
- $\boldsymbol{j} = -\frac{ne\boldsymbol{p}(t)}{m}$
- $\boldsymbol{p}(t + dt) = (1 - \frac{dt}{\tau})[\boldsymbol{p}(t) + \boldsymbol{f}(t)dt]$
- The equation of motion reads

$$\frac{d\boldsymbol{p}(t)}{dt} = -\frac{\boldsymbol{p}(t)}{\tau} + \boldsymbol{f}(t)$$

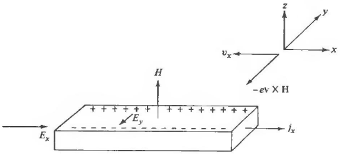
  - $\boldsymbol{f}$: average force per electron

Hall effect and magnetoresistance

Predictions from the Drude model

Experimental setup

- E_x and j_x: electric field and current density in the wire
- $\boldsymbol{H}$: constant magnetic field in the $\hat{\boldsymbol{z}}$ direction
- Lorentz force: $\boldsymbol{f} = -\frac{e}{c}\boldsymbol{v} \times \boldsymbol{H}$
  - electrons are deflected in the negative $\hat{\boldsymbol{y}}$ direction
  - E_y builds up to prevent further accumulation (balancing the Lorentz force)

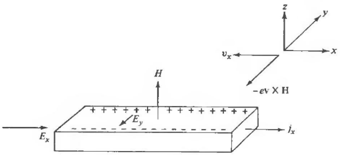


Hall effect and magnetoresistance

Predictions from the Drude model

Magnetoresistance and Hall coefficient

- Magnetoresistance: $\rho(H) = \frac{E_x}{j_x}$
  - transvere
- Hall coefficient: $R_H = \frac{E_y}{j_x H}$
  - proportional to E_y
  - must be negative for negative charge carriers ($E_y < 0$)
- In some metals, $R_H > 0$ (quantum effects)



Hall effect and magnetoresistance

Predictions from the Drude model

Magnetoresistance and Hall coefficient

- Consider the equations of motion:

$$\frac{d\boldsymbol{p}(t)}{dt} = -\frac{\boldsymbol{p}(t)}{\tau} + \boldsymbol{f}(t)$$

- with $\boldsymbol{f} = -e(\boldsymbol{E} + \frac{\boldsymbol{p}}{mc} \times \boldsymbol{H})$ (position independent)
- steady-state conditions: $\rightarrow \frac{d\boldsymbol{p}(t)}{dt} = 0$ $(\omega_c = \frac{eH}{mc})$

$$\begin{cases} 0 = & -eE_x - \omega_c p_y - \frac{p_x}{\tau} \\ 0 = & -eE_y + \omega_c p_x - \frac{p_y}{\tau} \end{cases}$$

Hall effect and magnetoresistance

Predictions from the Drude model

Magnetoresistance and Hall coefficient

- Multiply by $-\frac{ne\tau}{m}$ ($\sigma_0 = \frac{ne^2\tau}{m}$):

$$\begin{cases} \sigma_0 E_x & = \omega_c \tau j_y + j_x \\ \sigma_0 E_y & = -\omega_c \tau j_x + j_y \end{cases}$$

- If $j_y = 0 \rightarrow E_y = -(\frac{H}{nec})j_x$
- $R_H = -\frac{1}{nec}$
  - for a given metal depends only of n
  - independent of H (true for low T and high H)

Hall effect and magnetoresistance

Predictions from the Drude model

Comparison with the experiment

- $n_0 = -\frac{1}{R_H e c}$
- Data are in the form of $\frac{n_0}{n}$

Table 1.4
HALL COEFFICIENTS OF SELECTED ELEMENTS IN MODERATE TO HIGH FIELDS[a]

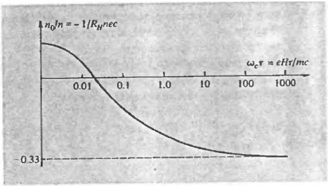
METAL	VALENCE	$-1/R_H nec$
Li	1	0.8
Na	1	1.2
K	1	1.1
Rb	1	1.0
Cs	1	0.9
Cu	1	1.5
Ag	1	1.3
Au	1	1.5
Be	2	−0.2
Mg	2	−0.4
In	3	−0.3
Al	3	−0.3

Hall effect and magnetoresistance

Predictions from the Drude model

Comparison with the experiment

- $n_0 = -\frac{1}{R_H ec}$
- Data are in the form of $\frac{n_0}{n}$
- Predicts one carrier per primitive cell with charge e



Hall effect and magnetoresistance

Predictions from the Drude model

Magnetoresistance and Hall coefficient

- Consider once again the equations:

$$\begin{cases} \sigma_0 E_x & = \omega_c \tau j_y + j_x \\ \sigma_0 E_y & = -\omega_c \tau j_x + j_y \end{cases}$$

- If $\omega_c \tau \to 0 \longrightarrow \boldsymbol{E} \parallel \boldsymbol{j}$
- In general they are at an angle ϕ
  - $\tan \phi = \omega_c \tau$ (Hall angle)
  - ω_c: cyclotron frequency

AC electrical conductivity

Predictions from the Drude model

Frequency dependent conductivity, $\sigma(\omega)$

- Equation of motion $\frac{d\boldsymbol{p}(t)}{dt} = -\frac{\boldsymbol{p}}{\tau} - e\boldsymbol{E}$
- $\boldsymbol{E}(t) = \mathrm{Re}(\boldsymbol{E}(\omega)e^{-i\omega t})$, $\boldsymbol{p}(t) = \mathrm{Re}(\boldsymbol{p}(\omega)e^{-i\omega t})$
  - steady-state solution
- $\boldsymbol{p}(\omega)$ satisfies

$$-i\omega\boldsymbol{p}(\omega) = -\frac{\boldsymbol{p}(\omega)}{\tau} - e\boldsymbol{E}(\omega)$$

- Therefore:

$$\boldsymbol{j}(\omega) = -\frac{ne\boldsymbol{p}(\omega)}{m} = \frac{(\frac{ne^2}{m})\boldsymbol{E}(\omega)}{(\frac{1}{\tau}) - i\omega}$$

AC electrical conductivity

Predictions from the Drude model

Frequency dependent conductivity, $\sigma(\omega)$

- $\boldsymbol{j}(\omega) = \sigma(\omega)\boldsymbol{E}(\omega)$
- The AC conductivity is

$$\sigma(\omega) = \frac{\left(\frac{ne^2\tau}{m}\right)}{1 - i\omega\tau} = \frac{\sigma_0}{1 - i\omega\tau}$$

  - σ_0: DC conductivity ($\omega = 0$)
  - useful for studying e.m. wave propagation

AC electrical conductivity

Predictions from the Drude model

Electromagnetic wave propagation in metals

- The effect of $\boldsymbol{H}$ can be neglected
  - $\frac{-e\boldsymbol{p}}{mc} \times \boldsymbol{H} \sim \frac{v}{c} \sim 10^{-10}$
  - $v \sim 0.1$ cm/sec (for $i = 1$ amp/mm^2)
- The field $\boldsymbol{E}$ is assumed constant over distances $\sim l$ (mean free path)
  - good approx. for visible and UV radiation ($\lambda \sim 10^3$–10^4 Å)

AC electrical conductivity

Electromagnetic wave propagation in metals

Maxwell's Equations ($\rho = 0$)

$$\begin{cases} \nabla \cdot \boldsymbol{E} & = 0 \\ \nabla \cdot \boldsymbol{H} & = 0 \\ \nabla \times \boldsymbol{E} & = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \\ \nabla \times \boldsymbol{H} & = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \end{cases}$$

AC electrical conductivity

Predictions from the Drude model

Electromagnetic wave propagation in metals

- Ansatz: $\boldsymbol{E} = \boldsymbol{E}(\boldsymbol{r}, \omega) e^{-i\omega t}$
- Use $\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$
- The wave-equation is obtained

$$-\nabla^2 \boldsymbol{E} = \frac{\omega^2}{c^2} \varepsilon(\omega) \boldsymbol{E}$$

  - $\varepsilon(\omega) = 1 + \frac{4\pi i \sigma}{\omega}$, complex dielectric constant
- If $\omega\tau >> 1 \longrightarrow \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$
  - $\omega_p^2 = \frac{4\pi n e^2}{m}$ (plasma frequency)

AC electrical conductivity

Predictions from the Drude model

Electromagnetic wave propagation in metals

- Wave propagation can only occur for $\omega > \omega_p$
- Provided $\omega\tau >> 1$ for $\omega \sim \omega_p$
- $\omega_p \tau = 1.6 \times 10^2 (\frac{r_s}{a_0})^{3/2} (\frac{1}{\rho_\mu})$
  - $\frac{r_s}{a_0} \in$2–6; $\rho_\mu \sim \mu\Omega$cm
- $\lambda_p = \frac{2\pi c}{\omega_p} = 0.26 \times (\frac{r_s}{a_0})^{3/2} \times 10^3$Å

Table 1.5
OBSERVED AND THEORETICAL WAVELENGTHS BELOW WHICH THE ALKALI METALS BECOME TRANSPARENT

ELEMENT	THEORETICAL[a] λ (10^3 Å)	OBSERVED λ (10^3 Å)
Li	1.5	2.0
Na	2.0	2.1
K	2.8	3.1
Rb	3.1	3.6
Cs	3.5	4.4

AC electrical conductivity

Predictions from the Drude model

Charge density oscillations (plasmons)

- Solutions of the type $\rho(\boldsymbol{r}, t) = \rho(\boldsymbol{r}, \omega)e^{-i\omega t}$, from
  - charge conservation$(\frac{dq}{dt} + \int_S \boldsymbol{j} \cdot \hat{\boldsymbol{n}} dS = 0)$
  - Gauss Theorem

$$\begin{cases} \nabla \cdot \boldsymbol{j} & = -\frac{\partial \rho}{\partial t} \\ \nabla \cdot \boldsymbol{E} & = 4\pi\rho \end{cases}$$

- Non trivial solutions for $1 + \frac{i4\pi\sigma(\omega)}{\omega} = 0 \Longrightarrow \omega = \omega_p$

AC electrical conductivity

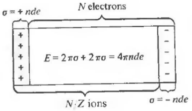
Predictions from the Drude model

Charge density oscillations (plasmons)

- Displace the electron charge by a distance d from the positive uniform background
- Charge separation, distributed on two opposite surfaces $(\sigma = \frac{-enAd}{A} = -nde)$
- Equation of motion:

$$Nm\ddot{d} = -4\pi ne^2 Nd$$

  - Harmonic motion with $\omega^2 = \frac{4\pi ne^2}{m} = \omega_p^2$



simple model of plasma oscillation

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

- For many metals $\frac{\kappa}{\sigma} \propto T$
  - σ: electrical conductivity
  - κ: thermal conductivity (positive quantity)
- $\frac{\kappa}{\sigma T}$ roughly the same for all metals (Lorentz number)

EXPERIMENTAL THERMAL CONDUCTIVITIES AND LORENZ NUMBERS OF SELECTED METALS

ELEMENT	273 K		373 K	
	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K^2)
Li	0.71	2.22×10^{-8}	0.73	2.43×10^{-8}
Na	1.38	2.12		
K	1.0	2.23		
Rb	0.6	2.42		
Cu	3.85	2.20	3.82	2.29
Ag	4.18	2.31	4.17	2.38
Au	3.1	2.32	3.1	2.36
Be	2.3	2.36	1.7	2.42
Mg	1.5	2.14	1.5	2.25
Nb	0.52	2.90	0.54	2.78
Fe	0.80	2.61	0.73	2.88
Zn	1.13	2.28	1.1	2.30
Cd	1.0	2.49	1.0	
Al	2.38	2.14	2.30	2.19
In	0.88	2.58	0.80	2.60
Tl	0.5	2.75	0.45	2.75
Sn	0.64	2.48	0.60	2.54
Pb	0.38	2.64	0.35	2.53
Bi	0.09	3.53	0.08	3.35
Sb	0.18	2.57	0.17	2.69

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

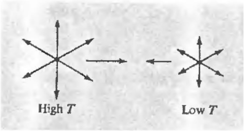
- The thermal current is carried by the conduction electrons
    - insulators do not conduct heat well
    - thermal conduction by the ions is less important
- Fourier's Law: $\boldsymbol{j}^q = -\kappa \nabla T$
    - steady-state
    - small temperature gradient (∇T)
    - $\boldsymbol{j}^q$: thermal current

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

- Consider a 1D model metal bar (uniform T drop along x)
- $j^q = -\kappa \frac{dT}{dx}$
  - $\frac{n}{2}$ e^- move $\rightarrow$
  - Temperature is a function of x ($T = T[x]$)
- Thermal energy $\epsilon = \epsilon(T)$ (per e^-)
  - assumption 4 of the model



net flow of heat to the low T side

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

- At point x, $j^q = \frac{1}{2}nv\{\epsilon(T[x - v\tau] - \epsilon(T[x + v\tau])\}$
- Therefore $j^q = nv^2\tau\frac{d\epsilon}{dT}(-\frac{dT}{dx})$
  - Taylor exp. around x, small variations of T
- In 3D: $\boldsymbol{j}^q = \frac{1}{3}v^2\tau c_v(-\nabla T)$
- $\kappa = \frac{1}{3}v^2\tau c_v = \frac{1}{3}lvc_v$
  - $\boldsymbol{v}$ distribution is isotropic $(< v_x^2 >=< v_y^2 >=< v_z^2 >= \frac{1}{3}v^2)$
  - v^2: mean square speed
  - $c_v = \frac{N}{V}\frac{d\epsilon}{dT} = \frac{1}{V}\frac{dE}{dT}$: specific heat capacity
  - Assume v^2 is T independent

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

- $\frac{\kappa}{\sigma} = \frac{\frac{1}{3} c_v m v^2}{n e^2}$
- Using ideal gas law: $\frac{\kappa}{\sigma} = \frac{3}{2} (\frac{k_B}{e})^2 T$
  - $c_v = \frac{3}{2} n k_B$
  - $\frac{1}{2} m v^2 = \frac{3}{2} k_B T$
- $\frac{\kappa}{\sigma T} = \frac{3}{2} (\frac{k_B}{e})^2$
  - $\frac{\kappa}{\sigma T} = 1.11 \times 10^{-8}$ watt·ohm/K^2
  - half the correct value

Thermal conductivity

Predictions from the Drude model

Explanation of the Wiedemar-Franz law

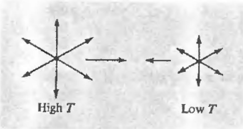
- The good agreement is fortuitous
- Two errors compensating each other
  - c_v is about 100 times smaller than the classical prediction
  - v^2 is about 100 times larger than the classical prediction
- Need quantum statistical mechanics (Fermi-Dirac distribution)

Thermoelectric effects

Predictions from the Drude model

The Seebeck effect

- A temperature gradient is accompained by $\boldsymbol{E}$ opposite to ∇T
- $\boldsymbol{E} = Q \nabla T$
  - Q: thermopower
- It cancels the effect of ∇T on v^2



net flow of heat to the low T side

Thermoelectric effects

Predictions from the Drude model

Estimation of Q

- $\boldsymbol{v}_Q$: mean $\boldsymbol{v}$ due to ∇T
- $\boldsymbol{v}_E$: mean $\boldsymbol{v}$ due to $\boldsymbol{E}$
  - $\boldsymbol{v}_E = -\frac{e\boldsymbol{E}\tau}{m}$
  - $\boldsymbol{v}_Q = -\frac{\tau}{6}\frac{dv^2}{dT}(\nabla T)$
- Put $\boldsymbol{v}_E + \boldsymbol{v}_Q = 0 \Longrightarrow Q = -\frac{c_v}{3ne}$

Thermoelectric effects

Predictions from the Drude model

Estimation of Q

- $Q = -\left(\frac{1}{3e}\right)\frac{d}{dT}\frac{mv^2}{2} = -\frac{c_v}{3ne}$
- If $c_v = \frac{3nk_B}{2} \rightarrow Q = -\frac{k_B}{2e} = -0.43 \times 10^{-4}$V/K
  - Exp. Q values are $\sim \mu$V/K
  - due to non-compensating error on c_v
  - In some cases the direction of the thermoelectric field is reversed
- classical statistical mechanics is inadequate
- There are also inadequacies in the free- and independent-electron model