

Generating 2D random unit steps

Comment on the algorithm n. 5 (p. 39 of the slides)

Indicating with x and y the individual displacements,

$$p(x) = \frac{1}{2\sqrt{2}} \text{ for } |x| < \sqrt{2} \text{ and } 0 \text{ otherwise; the same for } p(y);$$

the average step size is:

$$\sqrt{\langle x^2 + y^2 \rangle} = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2}}^{\sqrt{2}} (x^2 + y^2) p(x) p(y) dx dy = \dots = \frac{2}{\sqrt{3}}$$

Therefore, with x and y generated in this way, the behaviour of the simulated $\langle \Delta R_N^2 \rangle$ should be $\frac{4}{3}N$ (since $\langle \Delta R_N^2 \rangle = N\ell^2$).

In which extension you should generate x and y in order to have on average a unitary step size?