

## BATCH MEANS

PROBLEM: samples  $x_1, \dots, x_N$  of the SMC for MCMC method are CORRELATED!

How do I estimate  $E[f(x)]$  using correlated samples?

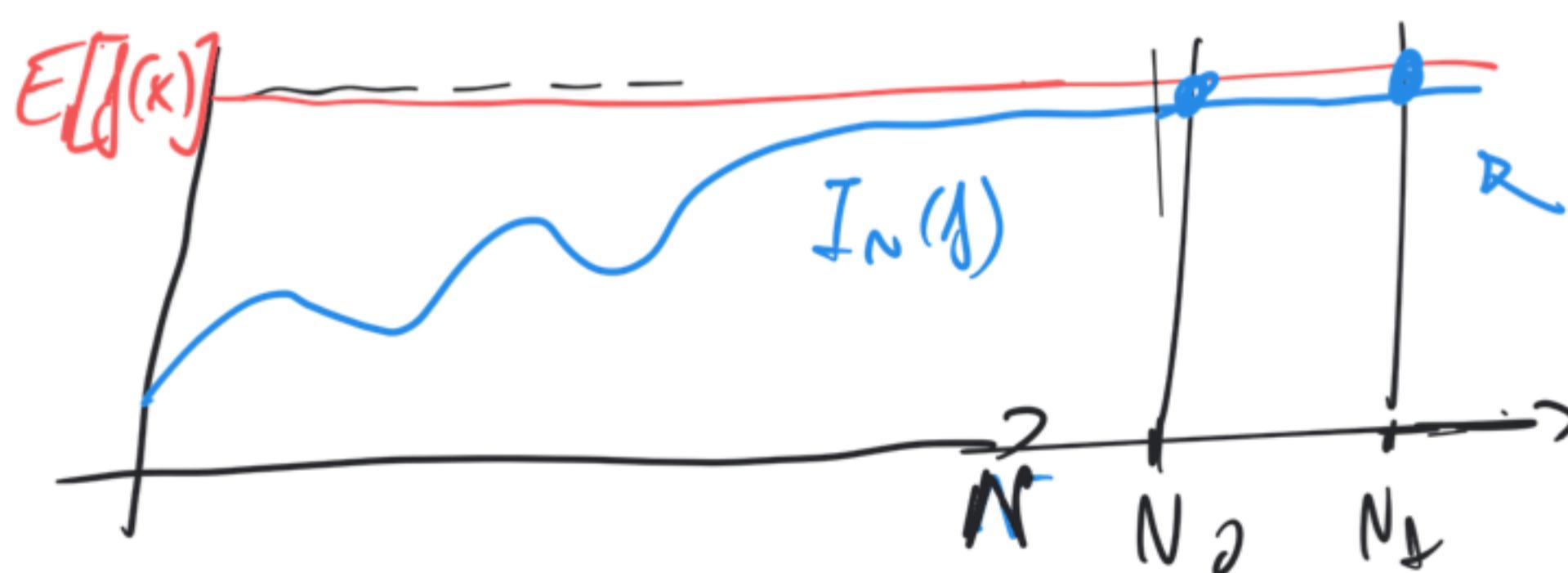
ISSUE: I need my chain to be at steady state

SOLUTION: DISCARD the first few samples. (few thousand)

ERGODIC THEOREM:  $E_p [f(x)] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N f(x_i) = I_N(f)$

PLOT  $I_N(f)$  AS A FUNCTION OF  $N$

$$\frac{1}{N} \sum_{i=1}^N f(x_i) \quad \text{TIME AVERAGE}$$



I can check stability of  $I_N(f)$ !

lag- $k$  AUTOCOVARIANCE

$\nwarrow$  we assume  $x_i$  at steady state  
OR

$x_n$  is STATIONARY.

I can estimate it as

$$\hat{f}_k = \frac{1}{N} \sum_{i=1}^{N-k} [f(x_i) - \bar{f}_n(f)][f(x_{i+k}) - \bar{f}_n(f)]$$

One can prove that the variance of  $\bar{f}_n(f)$  is  $\sigma^2/N$

$$\sigma^2 = \text{VAR}[f(x_i)] + \sum_{k>1} f_k$$