EXPONENTIAL DISTRIBUTION

$T \sim EXP(\lambda)$ on $\mathbb{R}_{\geq 0}$

$$p(t) = \lambda e^{-\lambda t}$$

, cdf $P(T<t) = F(t) = \int_0^t p(t)\,dt = 1 - e^{-\lambda t}$

$$\mathbb{E}[T] = \frac{1}{\lambda} \; ; \; STD[T] = \frac{1}{\lambda}$$

$$\underbrace{P(T>t)}_{\text{SURVIVAL FUNCTION}} = 1 - P(T \leq t) = e^{-\lambda t}$$

λ = expected frequence

$\frac{1}{\lambda}$ = expected time

T = models the time at which an event "fires".

MEMORYLESS PROPERTY

The Exponential is the only distribution in $\mathbb{R}_{\geq 0}$ to be memoryless

$$\underbrace{P(T>s+t \mid T>s)}_{\frac{P(T>s+t,\, \cancel{T>s})}{P(T>s)}} = P(T>t)$$

$T>s+t \Rightarrow T>s$

$$\frac{P(T>s+t,\, \cancel{T>s})}{P(T>s)} = \frac{P(T>s+t)}{P(T>s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = P(T>t)$$

$t \quad t+dt$

Assumption: firing probability in $[t, t+dt)$ is $\underline{\lambda \cdot dt}$

$$P(t) = P(T > t)$$

$$P(t+dt) = P(t)\,(1 - \lambda dt)$$

$$\Downarrow$$

$$\frac{P(t+dt) - P(t)}{dt} = -\lambda P(t) \overset{dt \to 0}{\Longrightarrow} \underbrace{\frac{d}{dt} P(t) = -\lambda P(t)}$$

$$\underline{P(t) = e^{-\lambda t}} \quad (P(0) = 1)$$

RACE CONDITION

$T_k \sim \text{EXP}(\lambda_k)$, $k \in I$ countable, $\lambda = \sum_{k \in I} \lambda_k < \infty$

$T = \inf_k T_k$, $K = \arg\min_k T_k$

[• $K \in I$ with prob. 1 (inf is a min with prob. 1)]

• T and K are INDEPENDENT

• $T \sim \text{EXP}(\lambda)$

• $P(K = i) = \dfrac{\lambda_i}{\lambda}$

dim: $P(K=k, T \geq t) =$

$= P(T_k \geq t \text{ AND } T_j \geq T_k \ \forall j \neq k) = \dots = \dfrac{\lambda_k}{\lambda} e^{-\lambda t}$

$P(T \geq t)$

$P(K=k)$

$=$