

EXPONENTIAL DISTRIBUTION

$$T \sim \text{EXP}(\lambda) \text{ on } \mathbb{R}_{\geq 0}$$

$$p(t) = \lambda e^{-\lambda t}$$

$$\text{cdf } P(T < t) = F(t) = \int_0^t p(t) dt = 1 - e^{-\lambda t}$$

$$E[T] = \frac{1}{\lambda}; \text{STD}[T] = \frac{1}{\lambda}$$

$$P(T > t) = 1 - P(T \leq t) = e^{-\lambda t}$$

SURVIVAL FUNCTION

λ \equiv expected frequency

$\frac{1}{\lambda}$ \equiv expected time

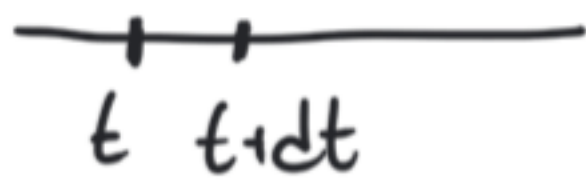
T \equiv models the time at which an event "fires".

MEMORYLESS PROPERTY

The Exponential is the only distribution in $\mathbb{R}_{\geq 0}$ to be memoryless

$$P(T > s+t | T > s) = P(T > t)$$

$$\frac{P(T > s+t, T > s)}{P(T > s)} = \frac{P(T > s+t)}{P(T > s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = P(T > t)$$



Assumption: firing probability in $[t, t+dt)$ is $\lambda \cdot dt$

$$P(t) = P(\text{no firing by } t)$$

$$P(t+dt) = P(t) (1 - \lambda dt)$$

\Downarrow

$$\frac{P(t+dt) - P(t)}{dt} = -\lambda P(t) \xrightarrow{dt \rightarrow 0} \frac{d}{dt} P(t) = -\lambda P(t)$$

$$P(t) = e^{-\lambda t} \quad (P(0) = 1)$$

RACE CONDITION

$$T_k \sim \text{EXP}(\lambda_k), \quad k \in I \text{ countable}, \quad \lambda = \sum_{k \in I} \lambda_k < \infty$$

$$T = \inf_k T_k, \quad K = \arg \min_k T_k$$

[• $K \in I$ with prob. 1 (inf is a min with prob. 1)]

• T and K are INDEPENDENT

• $T \sim \text{EXP}(\lambda)$

• $P(K=i) = \frac{\lambda_i}{\lambda}$

dim: $P(K=k, T \geq t) =$

$= P(T_k \geq t \text{ AND } T_j \geq T_k \quad \forall j \neq k) =$

$\frac{P(T \geq t)}{P(K=k)} = \frac{\lambda e^{-\lambda t}}{\lambda} = e^{-\lambda t}$