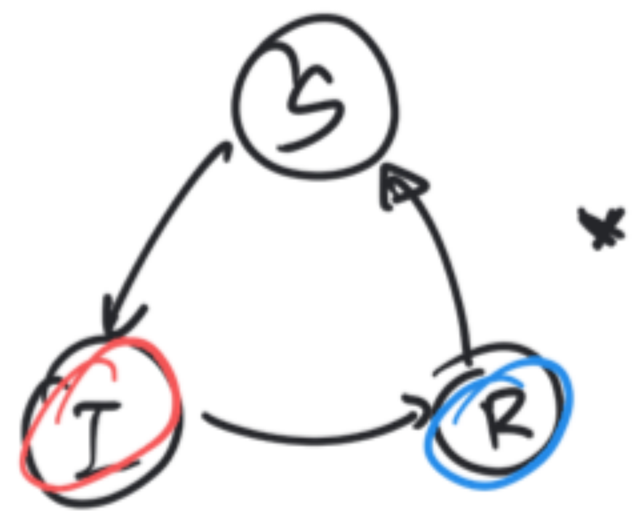
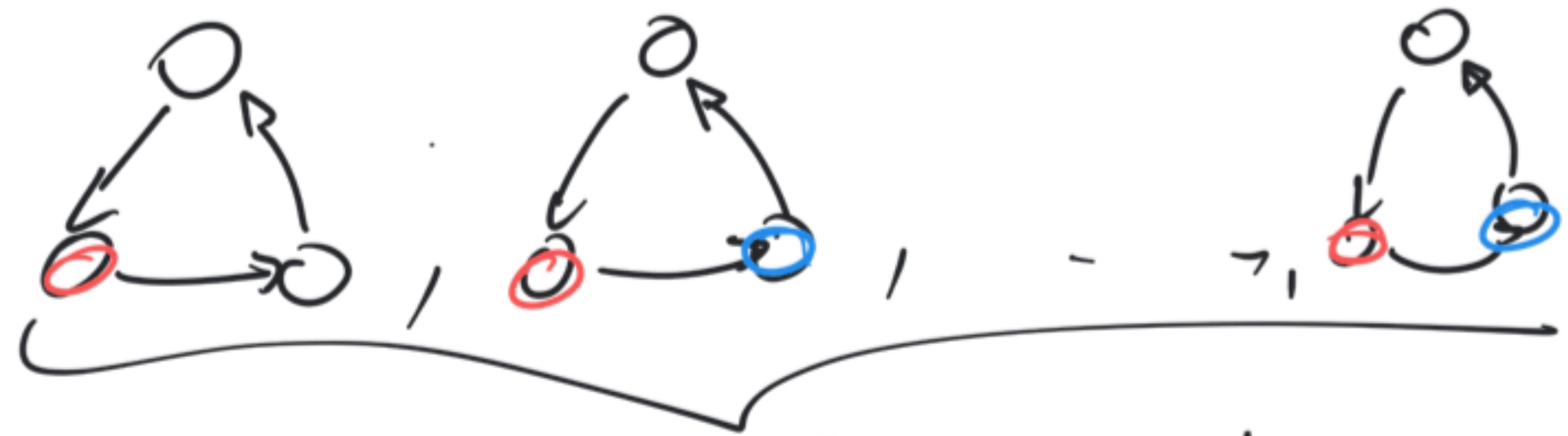
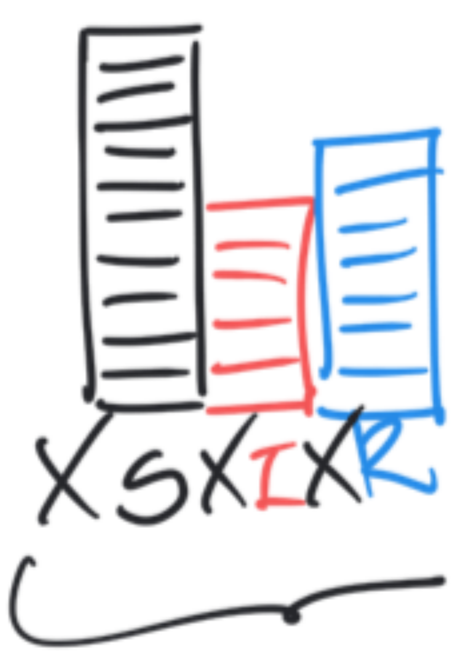
STOCHASTIC SIMULATION ALGORITHM (SSA)



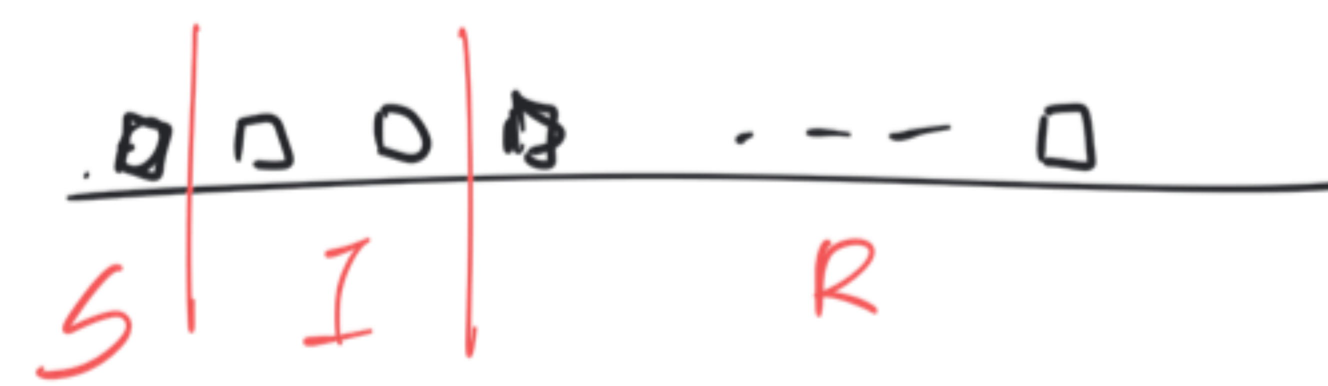
$* N$



N agenti 3^N stati



$X_j \in \{0, \dots, N\}$ #stati



$\binom{N+1}{2}$ agenti

$\Theta(N^2)$

DIRECT METHOD

FIX T_{max} TO the TIME HORIZON

$t \leftarrow 0$

$x \leftarrow$ initial state

WHILE $t < T_{max}$ or termination condition

$u_1, \dots, u_m \leftarrow$ SAMPLE_UNIFORM$(0,1)$

$\tau_j \leftarrow -\frac{1}{r_j(x)} \log(u_j)$, $j = 1, \dots, m$

$t \leftarrow t + \min_j \tau_j$

$x \leftarrow x + v_{\bar{j}}$, $\bar{j} = \arg\min_j \tau_j$

$\Rightarrow$ costo per iterazione: $\{$ m uniform r.v. $\}$

Population model

$X_1, \dots, X_n$ counting variables

$\eta_1, \dots, \eta_m$ transition

$\eta_j = (v_j, r_j(x))$

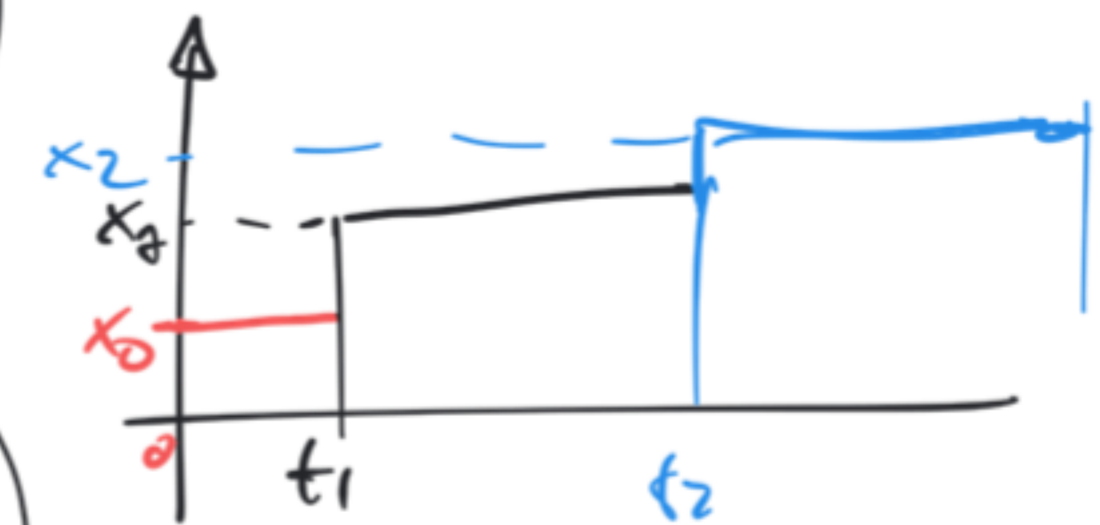
SSA - use jump chain + holding times

```
SSA (model, Tmax)
  t ← 0
  x ← init(model)
  WHILE t < Tmax
    calcola r_j(x), j = 1, ..., m
    r_0 ← SUM(r_j(x))
    t ← t - (1/r_0) log(u_1), u_1 ~ Unif(0,1)
    j ← SAMPLE FROM (r_1(x)/r_0, ..., r_m(x)/r_0)
    x ← x + v_j
```

$t_0, x_0, t_1, x_1, t_2, x_2, \ldots$: Traj

$X_{[0,T]}$

$T = T_{max}$



$X_{0:T}$

SAMPLE

$X^1_{0:T}, X^2_{0:T}, \ldots, X^N_{0:T}$

$f(X^1_{0:T})$, $f(X^2_{0:T})$, $f(X^N_{0:T})$

$= z_1$, $= z_2$, $= z_N$

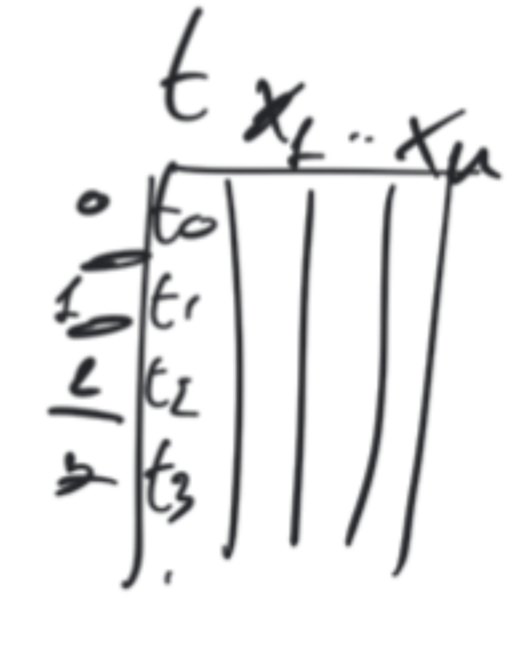
$\hat{p} = \frac{1}{N} \sum z_i$, plus confidence interval

Reachability in SIR

$$p = \mathbb{P}\left(\{\underbrace{X_I(t) \geq 0.4 \cdot N, \; t \in [0, T_{max}]}_{R}\}\right)$$

$$f(X_{0:T}) = \begin{cases} 1, & \text{if } X_{0:T} \text{ reaches } R \\ 0, & \text{otherwise} \end{cases}$$

$f \to \{0, 1\}$



CONJUGATE PRIORS are such that POSTERIOR IS "OF THE SAME CLASS"

Beta distribution is c.p of Bernoulli

$$p(q \mid a,b) = \frac{1}{W} q^{a-1}(1-q)^{b-1} \qquad W = \int_0^1 q^{a-1}(1-q)^{b-1}\,dq$$

$$f(\vec{z} \mid q) = \prod_i q^{z_i}(1-q)^{1-z_i}$$

$$f(\vec{z} \mid q)\, p(q \mid a,b) = \frac{1}{W} q^{(a-1)+\overbrace{\sum_i z_i}^{K}} (1-q)^{(b-1)+\overbrace{\sum_i (1-z_i)}^{N-K}}$$

$$\propto q^{a+K-1}(1-q)^{b+N-K-1}$$

$$p(q \mid \vec{z}, a,b) = \mathrm{Beta}(a+K,\, b+N-K)$$

$$E[q \mid \vec{z}, a,b] = \text{expectation} \overset{!}{=} \frac{a+K}{a+b+N}$$

$$\left(E[q] = \frac{a}{a+b} \qquad q \sim \mathrm{Beta}(a,b) \right)$$

$a=b=1$

$\Downarrow$

uniform