Hénon–Heiles potential: The potential and Hamiltonian

$$V(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + x^2 y - \frac{1}{3}y^3, \quad H = \frac{1}{2}p_x^2 + \frac{1}{2}p_y^2 + V(x, y), \tag{12.52}$$

are used to describe three interacting astronomical objects. The potential binds the objects near the origin but releases them if they move far out. The equations of motion follow from the Hamiltonian equations:

$$\frac{dp_x}{dt} = -x - 2xy, \quad \frac{dp_y}{dt} = -y - x^2 + y^2, \quad \frac{dx}{dt} = p_x, \quad \frac{dy}{dt} = p_y.$$

a. Numerically solve for the position $[x(t), y(t)]$ for a particle in the Hénon–Heiles potential.
b. Plot $[x(t), y(t)]$ for a number of initial conditions. Check that the initial condition $E < \frac{1}{6}$ leads to a bounded orbit.
c. Produce a Poincaré section in the (y, p_y) plane by plotting (y, p_y) each time an orbit passes through $x = 0$. ▌