Calculate the burn-out time for $L \times L$ forests with different values of $L$. Show that the peak at $p_c$ becomes larger as the forest is made bigger and try to extrapolate your results to estimate the burn-out time as a function of $p$ for an infinite lattice. Hint: In making an extrapolation to the case $L \to \infty$ it is useful to plot things as a function of $L^{-1}$ so that $L = \infty$ is on the graph.



The fractal nature of the percolating cluster at $p_c$ has many interesting consequences. For example, if we are modeling the mechanical properties of a porous material and the unoccupied sites are voids in the system, the strength will be strongly dependent on the connectivity of the spanning cluster. The flow of a fluid like water or oil through such a system would also be a sensitive function of the connectivity. Another interpretation is to suppose that the occupied sites are trees in a forest. If a fire is somehow set at one edge of the forest, we can use a percolating network to study how the fire will propagate into the forest. This can be modeled by assuming that all of the trees (that is, occupied sites) on one edge of the lattice are set to burn at time $t = 0$. At the next step all of the trees adjacent to a burning tree will themselves start to burn, while the trees burning at $t = 0$ will burn out. This process is then repeated; at step $t_n$ the previously unburned trees that are adjacent to a burning tree will start to burn, and the trees burning at $t_{n-1}$ will burn out.

It is instructive to calculate the time it takes for a fire to burn out completely . For small $p$ the fires burn out quickly since the clusters of connected trees are on average very small. The only trees to burn will be those near the edge where the fire is lit, and most of the forest will be spared. For large $p$ the fire burns across the system rapidly. Since essentially all of the trees are connected in this case, the number of time steps for a fire to burn-out is $\sim L$ where $L$ is the size of the lattice, and nearly all of the trees will be engulfed. However, for $p$ near $p_c$ the fractal structure of the spanning cluster forces the fire to follow a tortuous path through the system, and the fire takes many time steps to burn out completely. Some results for the fire burn-out time are shown in Figure 7.35, which shows precisely this behavior. There is a large peak in the fire lifetime at $p_c$, which directly reflects the fractal connectivity of the critical cluster at $p_c$. If we were to study this peak as a function of lattice size (a task we will leave for the exercises), we would find that the burn-out time *diverges* for an infinitely large lattice. The nature of this divergence contains important information about the fractal nature of the critical cluster, a topic we will leave for the references. Our results also imply that a forest whose concentration of trees places it below $p_c$ has a better chance of surviving a fire than a more concentrated forest.

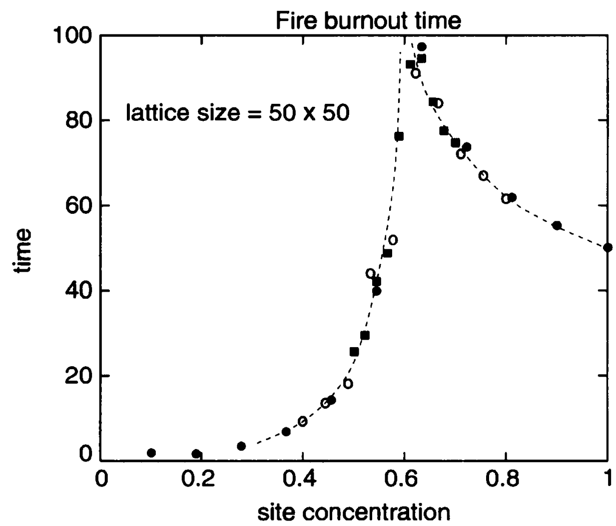


**Figure 7.35:** Forest fire burn-out times for a $50 \times 50$ square lattice. The different symbols were obtained from independent simulations. All three simulations computed the average burn-out time for several forests of each size. The dashed curves are guides to the eye that suggest a singularity near $p_c \sim 0.593$.