

# Test of Mathematics

June 22nd, 2015

Name:.....Surname:.....

Matriculation number:.....

1. Determine the domain of the function

$$f(x) = \log(1 - e^{-x})$$

2. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} \log(x+1) & \text{if } -1 < x \leq 0 \\ e^{-x} & \text{if } x > 0 \end{cases} .$$

Determine the inverse function  $x = f^{-1}(y)$ .

3. Determine the following limit:

$$\lim_{x \rightarrow +\infty} \frac{e^{\frac{1}{x}} - 1}{\sin \frac{2}{x}} .$$

4. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{1}{x}} + \frac{1}{x} .$$

5. Determine the following indefinite integral:

$$\int \cos x \sqrt{\sin x} \, dx .$$

6. Determine the derivatives  $f'_x(x, y)$  and  $f'_y(x, y)$  of the following real-valued function of two real variables:

$$z = f(x, y) = \frac{x}{x - y} .$$