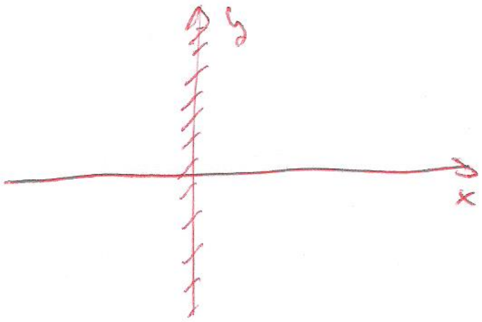
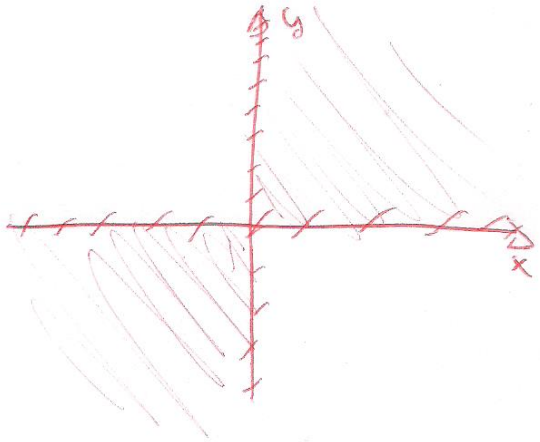
1) $f(x,y) = \sqrt{\ln \frac{y}{x}}$, $D.E = \{(x,y) \in \mathbb{R}^2 : x \neq 0,\ \frac{y}{x} > 0,\ \ln \frac{y}{x} \geq 0\}$

a)

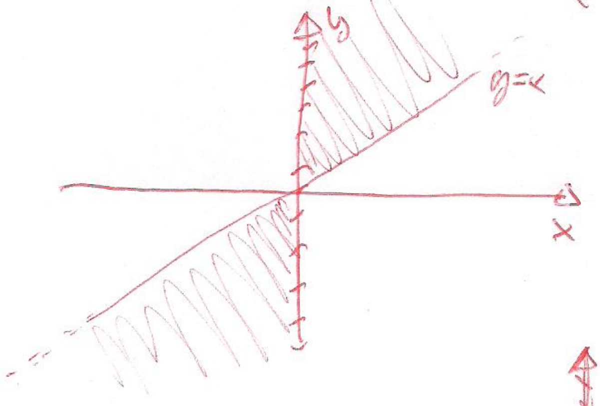
1- $x \neq 0$:



2- $\frac{y}{x} > 0 \iff (x>0 \wedge y>0) \vee (x<0 \wedge y<0)$

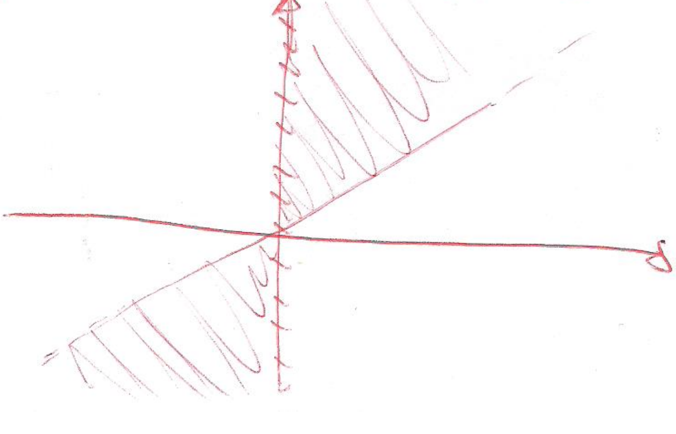


3- $\ln \frac{y}{x} \geq 0 \iff \frac{y}{x} \geq 1 \iff \begin{cases} y \geq x & \text{se } x>0 \\ y \leq x & \text{se } x<0 \end{cases}$



A = D.E

Si ottiene :



b) A né aperto né chiuso, è illimitato

c) Punti d'acc.: $(0,1)$ e $(-1,-2)$

$$\lim_{(x,y)\to(0,1)} \sqrt{\ln\Big(\underbrace{\frac{y}{x}}_{\to +\infty}\Big)} = +\infty \qquad y \to 1,\quad x \to 0^+$$

$$\lim_{(x,y)\to(-1,-2)} f(x,y) \underset{\uparrow}{=} f(-1,-2) = \sqrt{\ln 2}$$

f è continua in $(-1,-2)$

d) $$\partial_x f(x,y) = \frac{1}{2\sqrt{\ln \frac{y}{x}}} \cdot \frac{\cancel{x}}{\cancel{y}} \cdot \frac{-\cancel{y}}{x^{\cancel{2}}}$$

$$\Rightarrow \partial_x f(1,e) = -\frac{1}{2}$$

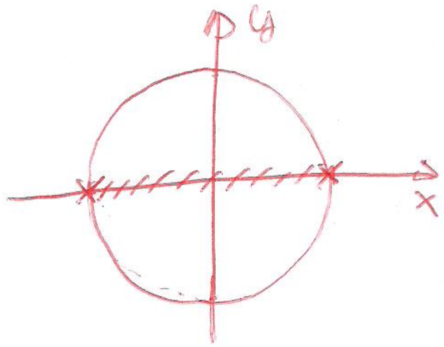
$$\partial_y f(x,y) = \frac{1}{2\sqrt{\ln \frac{y}{x}}} \cdot \frac{\cancel{x}}{y} \cdot \frac{1}{\cancel{x}}$$

$$\Rightarrow \partial_y f(1,e) = \frac{1}{2e}$$

$$f(1,e) = 1$$

$$\Rightarrow P_{(1,e)}(x,y) = f(1,e) + \partial_x f(1,e)(x-1) + \partial_y f(1,e)(y-e)$$
$$= 1 - \frac{1}{2}(x-1) + \frac{1}{2e}(y-e)$$

② $f(x,y) = \dfrac{\sqrt{1-x^2-y^2}}{y^2}$



$D.E. = \{(x,y) \in \mathbb{R}^2 : 1-x^2-y^2 \geq 0\}$

- P.ti interni:

$$\begin{cases} \partial_x f(x,y) = \dfrac{-2x}{2\sqrt{1-x^2-y^2}\, y^2} = 0 \quad (\iff x=0) \\[2ex] \partial_y f(x,y) = \dfrac{\dfrac{-2y}{2\sqrt{1-x^2-y^2}}\, y^2 - 2y\sqrt{1-x^2-y^2}}{y^4} \\[2ex] \qquad = \dfrac{-y^3 - 2y(1-x^2-y^2)}{y^4\sqrt{1-x^2-y^2}} = 0 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} x = 0 \\ -y^2 - 2\cancel{y}(1-y^2) = 0 \iff -2+y^2=0 \\ \qquad\qquad\qquad\qquad\quad \iff y = \pm\sqrt{2} \end{cases}$$

Ma i p.ti $(0,-\sqrt{2})$ e $(0,\sqrt{2}) \notin D.E.$

$\Rightarrow \nexists$ p.ti di estremo interni.

- P.ti sulla frontiera:

$C = \{(x,y) \in \mathbb{R}^2 : y \neq 0,\ 1-x^2-y^2 = 0\}$

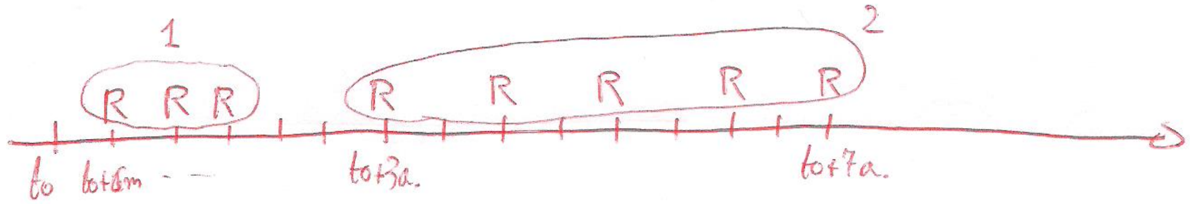
$$\Updownarrow$$

$x^2 = 1-y^2$

$$f|_C(x,y) = f(1-y^2, y) = 0 \quad \forall y$$

Siccome $f(x,y) \geq 0 \quad \forall (x,y) \in$ D.E. , allora tutti i punti di C sono p.ti di minimo assoluto in senso debole.

(3)



a) $1'000'000 = A_1(t_0) + A_2(t_0)$

$A_1(t_0)$: v.a. rendita periodica di 3 rate cost. con periodo 6m.

$i_2 = (1+i_4)^2 = 2,01\%$

$A_2(t_0)$: v.a. rendita periodica di 5 rate cost. con periodo 1 a.

$i_1 = (1+i_4)^4 = 4,06\%$

$$A_1(t_0) = a_{\overline{3}|0,0201} \cdot R = \frac{1 - 1,0201^{-3}}{0,0201} R = 2,8833 \cdot R$$

$$A_2(t_0) = a_{\overline{5}|0,0406} \cdot (1,0406)^{-2} \cdot R =$$

$$= \frac{1 - 1,0406^{-5}}{0,0406} (1,0406)^{-2} \cdot R = 4,10429 \cdot R$$

$$\Rightarrow R = \frac{1'000'000}{2,8833 + 4,10429} \simeq 143'110,32$$

b) $V(t_0 + 30m) = V(t_0 + 2,5\text{ a.}) = 1'000'000\,(1 + 0,0406)^{2,5}$

$= 1'104'611,48$

Capit. int. semplici: $t = \left(\frac{M}{C} - 1\right)/i$

$$\Rightarrow t = \frac{\frac{1'040'341}{1'104'611,48} - 1}{0,04} = -1,45 \text{ anni}$$

(tempo negativo!)

a)

4 Si considerino le restrizioni:

$C_1 = \{(x,y) \in A : y = x\} = \{(x,y) \in \mathbb{R}^2 : y = x, x \neq 0\}$

$C_2 = \{(x,y) \in A : y = 2x\} = \{(x,y) \in \mathbb{R}^2 : y = 2x, x \neq 0\}$

$f|_{C_1}(x,y) = f(x,x) = 0 \xrightarrow[x \to 0]{} 0$

$\neq$

$f|_{C_2}(x,y) = f(x,2x) = \sqrt{\ln 2} \xrightarrow[x \to 0]{} \sqrt{\ln 2}$

. Per l'unicità del limite e il th. del limite della restrizione, $\lim_{(x,y) \to (0,0)} f(x,y) \nexists$.

b) i_1 è un TIR, perchè calcolando $A(t_0)$ con i_1 risulta $A(t_0) = 1'000'000$ e quindi $1 - A(t_0) = 0$

• È l'unico TIR positivo in quanto gli importi dei flussi di cassa cambiano segno una sola volta.