Funzioni reali di più variabili

- CASO NOTO: funzioni di 1 variabile

$f: A(\subseteq \mathbb{R}) \longrightarrow B(\subseteq \mathbb{R})$

esempi: $f(x)=x$; $f(x)=x^2+x$

$f(x)=e^x$; $f(x)=\ln(x)$

- funzioni di più variabili

$f: A(\subseteq \mathbb{R}^n) \longrightarrow B(\subseteq \mathbb{R})$

A: DOMINIO, B: CODOMINIO

esempi:

- funzione di produzione:

x: ammontare di capitale impiegato

y: " " lavoro "

$f(x,y)$: quantità prodotta

- voto finale in concorso:

x_i: voto i-esimo commissario $(i=1,\dots,n)$

$$f(x_1,\dots,x_n)=\frac{x_1+\dots+x_n}{n}$$

$\downarrow$

MEDIA ARITMETICA

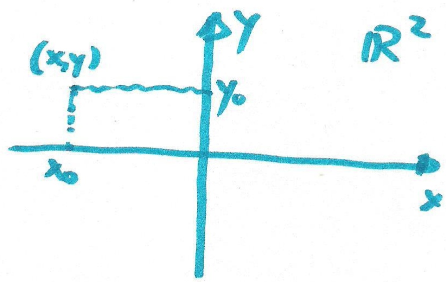
- **notazione**:

$f(x_1, \dots, x_n)$ se $f: A(\subseteq \mathbb{R}^n) \longrightarrow B(\subseteq \mathbb{R})$

$f(x, y)$ se $f: A(\subseteq \mathbb{R}^2) \longrightarrow B(\subseteq \mathbb{R})$

- piano cartesiano:

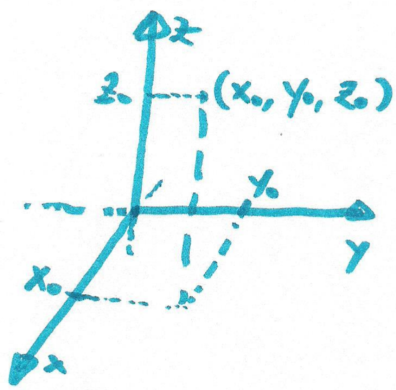
$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$



$\mathbb{R}^2$

- "spazio" cartesiano:

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$



$\mathbb{R}^3$

- prodotto cartesiano:

$$\mathbb{R}^n = \underbrace{\mathbb{R} \times \dots \times \mathbb{R}}_{n \text{ fattori}}$$

$$= \{(x_1, \dots, x_n) \underset{\text{tale che}}{:} x_i \in \mathbb{R} \ \underset{\text{per ogni}}{\forall} i = 1, \dots, n\}$$

Dominio di esistenza:

- Dominio di esistenza (DE): "il massimo, inteso come più grande, sottoinsieme A per il quale f e' ben definita.
- problema: data l'espressione algebrica di f, determinare il suo campo di esistenza
- ESEMPI:

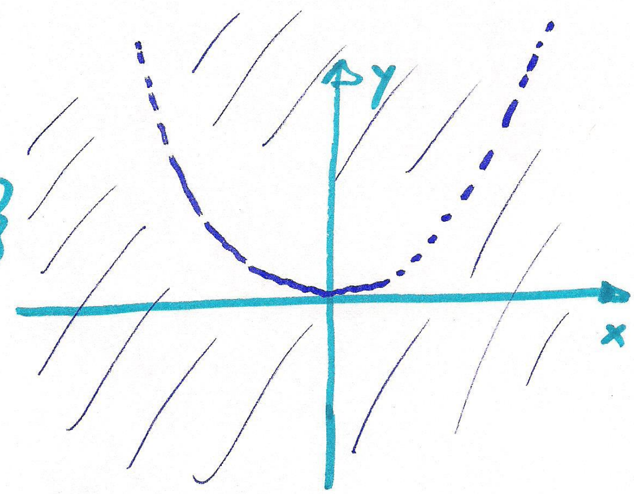
– $f(x,y) = \dfrac{1}{x^2-y}$

$D.E = \{(x,y) \in \mathbb{R}^2 : x^2 - y \neq 0\}$

$x^2 - y = 0 \iff y = x^2$

$\Downarrow$

PARABOLA



– $f(x,y) = x^2 + y^2 - 4$

$D.E. = \mathbb{R}^2$

La funzione e' definita $\forall\, x \in \mathbb{R}^2$

– $f(x,y) = \ln(x^2+y^2-4)$

$D.E. = \{(x,y) : \underline{x^2+y^2-4>0}\}$

$x^2+y^2-4=0$

circonf. di raggio 2 e centro $(0,0)$

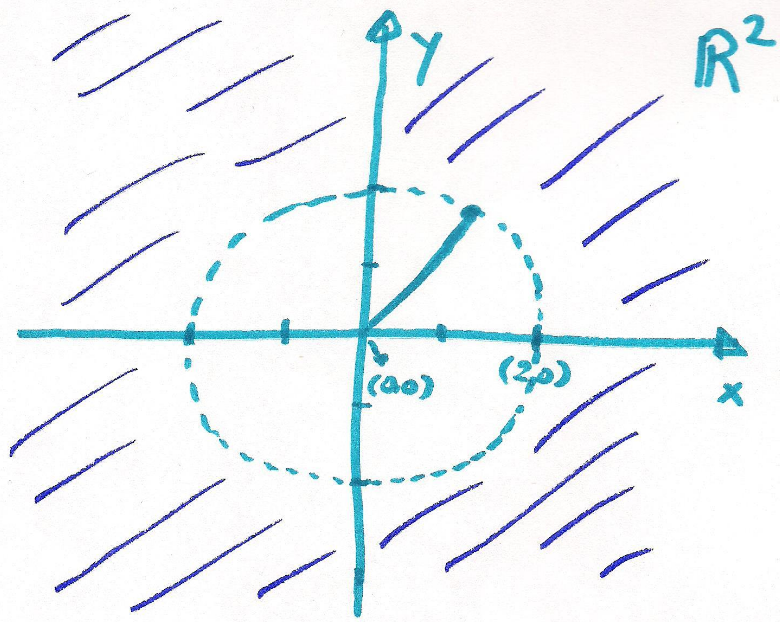
$\downarrow$

$x^2+y^2=4$

$\Updownarrow$

$\sqrt{x^2+y^2} = 2$

$\left(\begin{array}{c} \| \\ \text{dist.}((0,0),(x,y)) \end{array} \right)$



• Scelgo un punto fuori dalla circ. (es. (0,0)) e vedo se soddisfa ~~l'equazione~~ la disequaz.
$(0^2+0^2-4>0 \rightarrow$ FALSA$)$

– $\ln(\sqrt{x^2-y})$

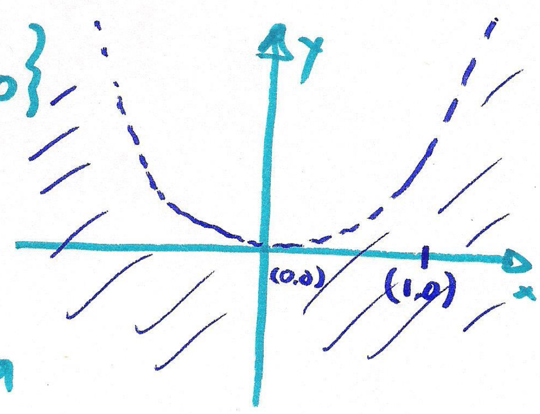
$D.E. = \{(x,y) \in \mathbb{R}^2 : x^2-y>0\}$

$x^2-y=0 \rightarrow$ PARABOLA

$\Updownarrow$

$y = x^2 = 0$

• in $(1,0)$: $1^2-0>0 \rightarrow$ vera



– $f(x,y) = \sqrt{4-x^2-y^2} + \ln(x^2+y^2-1)$

Abbiamo: $f(x,y) = f_1(x,y) + f_2(x,y)$,

dove: $f_1(x,y) = \sqrt{4-x^2-y^2}$

$f_2(x,y) = \ln(x^2+y^2-1)$

• Il D.E. e' l'insieme dei punti di $\mathbb{R}^2$ tali che $f_1(x,y)$ e $f_2(x,y)$ sono entrambe ben definite, ovvero:

$$\begin{cases} 4-x^2-y^2 \geq 0 & (1) \\ x^2+y^2-1 > 0 & (2) \end{cases}$$

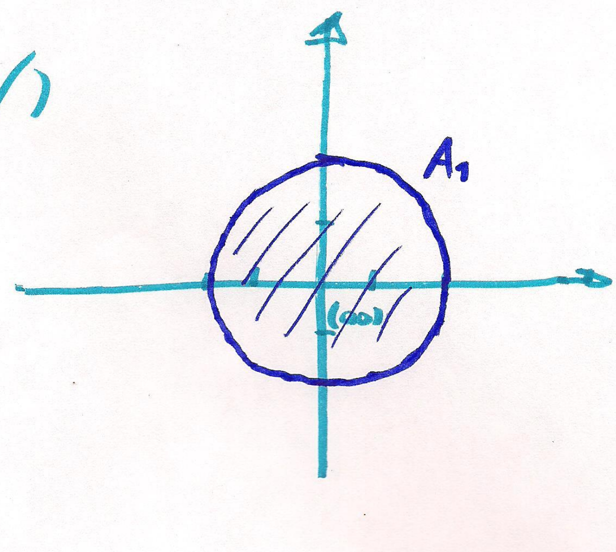
• Soluzioni di (1):

circonf. $4-x^2-y^2=0$

$y^2+x^2=4$

in $(0,0)$: $4-0^2-0^2 \geq 0$ (✓)

Quindi A_1 e' la parte interna della circonf.



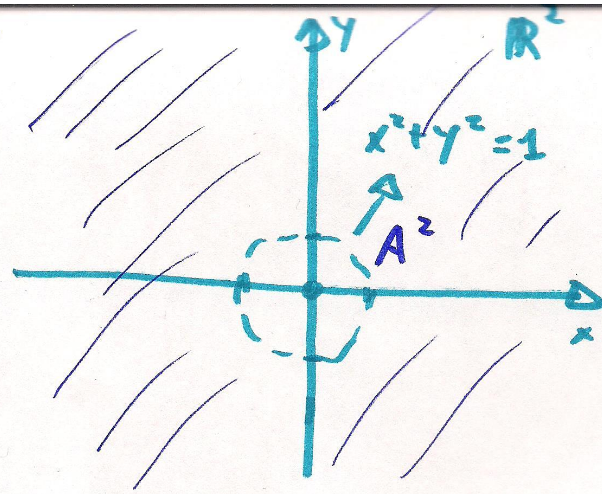
- $x^2+y^2-1=0$

circonf. di raggio 1

centrata in $(0,0)$

- in $(0,0)$: $0^2+0^2-1 > 0$

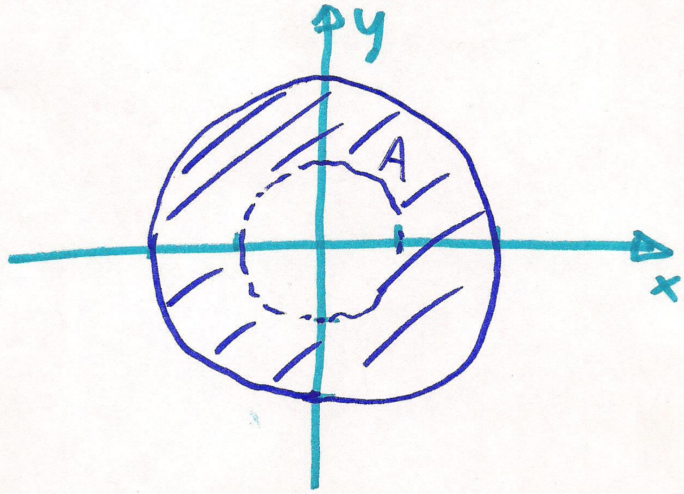
(F.)



- Soluzione del sistema:

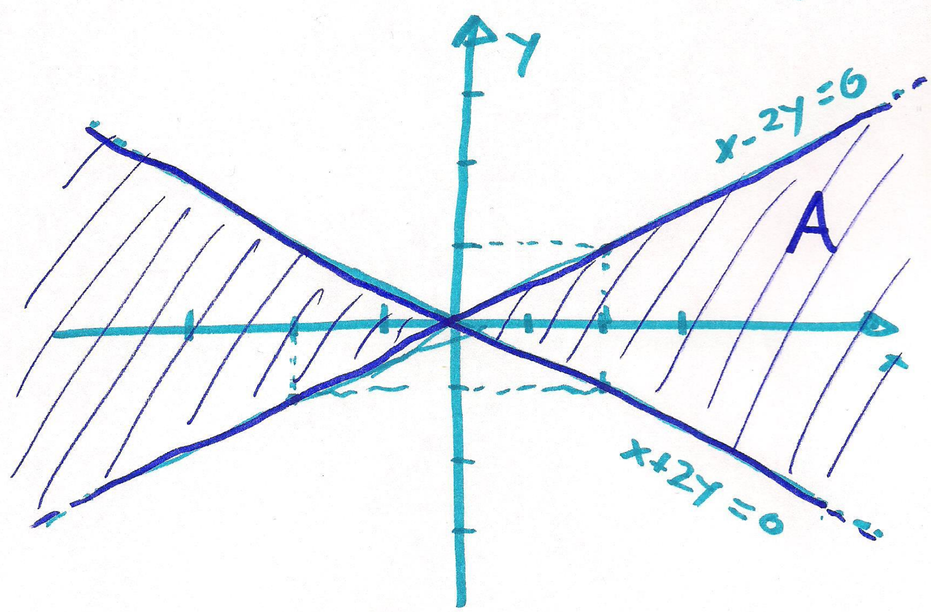
$A = A_1 \cap A_2$

D.E.



• $f(x,y) = \sqrt{x^2 - 4y^2}$

- D.E. $= \{(x,y) \in \mathbb{R}^2 : (x+2y)(x-2y) \geq 0\}$



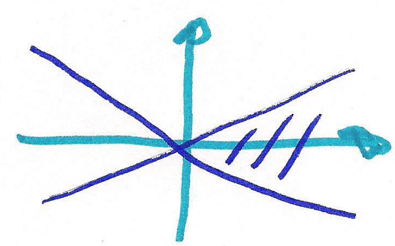
- D.E. $= A = A_1 \cup A_2$, dove:

$A_1 = \{(x,y) \in \mathbb{R}^2 : (x+2y) \geq 0, (x-2y) \geq 0\}$

$A_2 = \{(x,y) \in \mathbb{R}^2 : (x+2y) \leq 0, (x-2y) \leq 0\}$

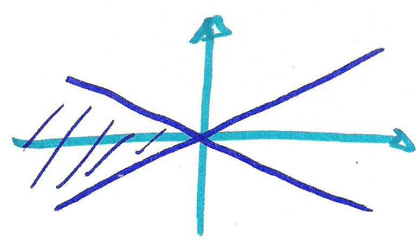
- A_1 sol. di:

$$\begin{cases} x+2y \geq 0 \\ x-2y \geq 0 \end{cases}$$



- A_2 sol. di:

$$\begin{cases} x+2y \leq 0 \\ x-2y \leq 0 \end{cases}$$



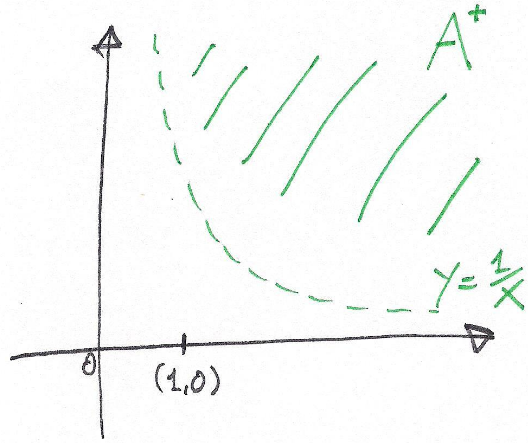
• $f(x,y) = \ln(xy-1)$

- D.E. $xy-1>0 \Leftrightarrow xy>1$
- Distinguiamo 2 casi:

1- Se $x>0$, $xy>1 \Leftrightarrow y>\frac{1}{x}$

• in $(1,0)$: $1\cdot 0-1>0$

(F.)



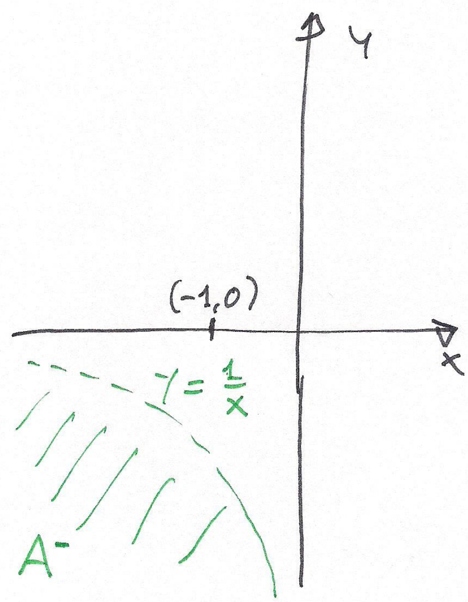
2- Se $x<0$,

$xy>1 \Leftrightarrow y<\frac{1}{x}$

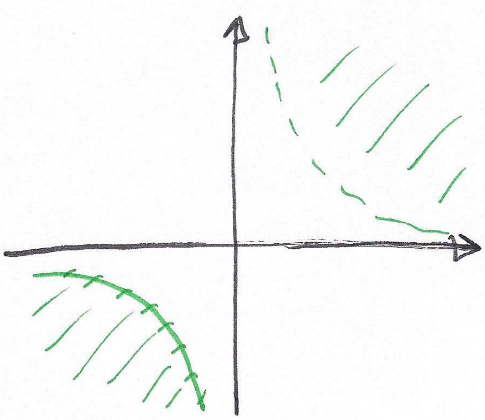
• in $(-1,0)$:

$(-1)\cdot 0-1>0$

(F.)



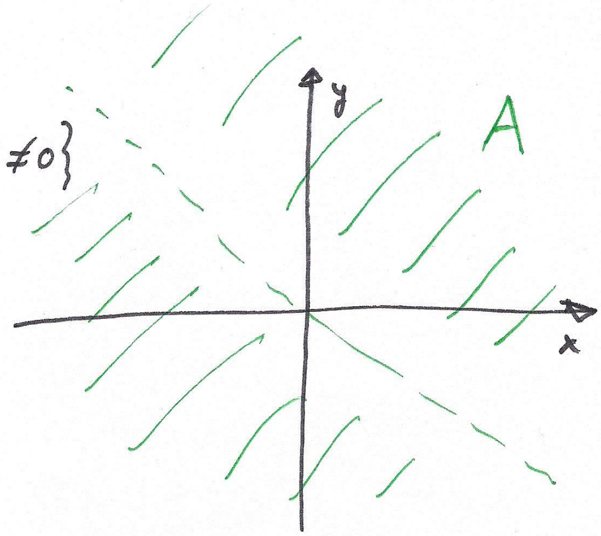
- Quindi $A=A^+\cup A^-$



- $f(x,y) = \dfrac{e^{x+y}}{x+y}$

- $D.E. = \{(x,y) \in \mathbb{R}^2 : x+y \neq 0\}$

$x+y=0 \Leftrightarrow y=-x$



- $f(x,y) = \ln\left(1 + \frac{y}{x}\right)$

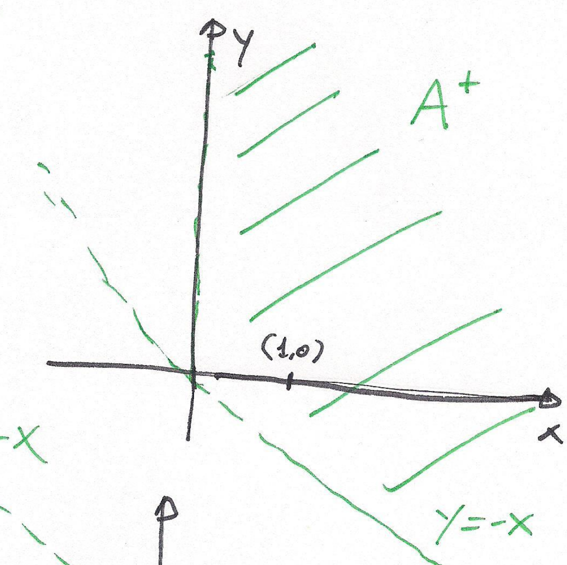
$D.E. = \{(x,y) \in \mathbb{R}^2 : x \neq 0,\ 1 + \frac{y}{x} > 0\}$

- La retta $x=0$ non appartiene al D.E.
- Per $x \neq 0$ distinguiamo 2 casi:

  - $x>0$: $1 + \frac{y}{x} > 0 \Leftrightarrow$

    $y > -x$

    Consid. la retta $y=-x$
    in $(1,0)$: $1 + \frac{0}{1} > 0$
    (V.)

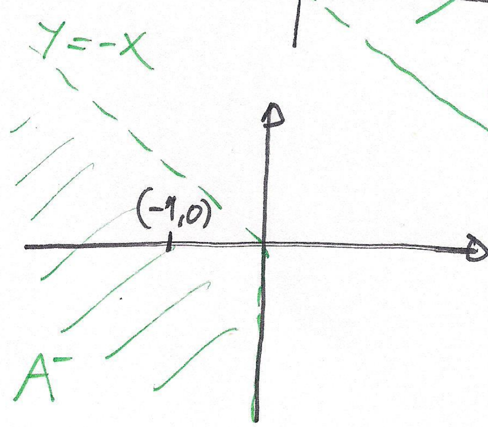


  - $x<0$: $1 + y/x > 0$

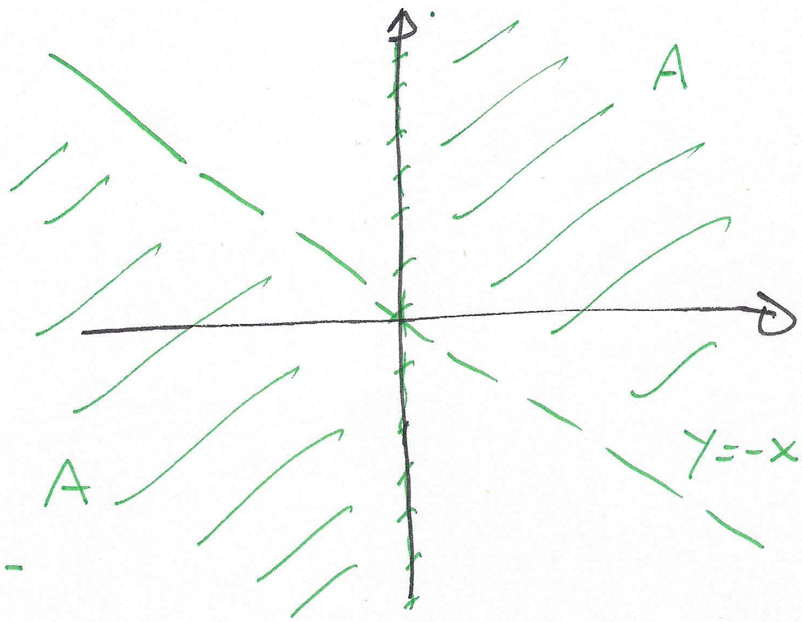
    $\Updownarrow$

    $y < -x$

    Considero $y=-x$
    in $(-1,0)$: vera



- Riunendo i 2 casi:



$A = A^+ \cup A^-$