

Geothermics

Course Outline:

1. Thermal conditions of the early Earth and present-day Earth's structure
2. Thermal parameters of the rocks
3. Thermal structure of the lithospheric continental areas (steady state)
4. Thermal structure of the lithospheric oceanic areas
5. Thermal structure of the lithosphere for transient conditions in various tectonic settings
- 6. Heat balance of the Earth**
7. Thermal structure of the sedimentary basins
8. Thermal maturity of sediments
9. Mantle convection and hot spots
10. Magmatic processes and volcanoes
11. Heat transfer in hydrogeological settings

Global Energy Budget

$$\frac{d(U + E_c + E_g)}{dt} = - \int_S \mathbf{q} \cdot \mathbf{n} dS + \int_V H dV - p_a \frac{dV}{dt} + \int_V \Phi dV$$

Different energy components

All processes contributing to energy changes

U=internal energy, E_c =kinetic energy, E_g =gravitational potential energy

n=unit normal vector, S=outer surface of the Earth, V=total volume of the Earth

q=surface heat flux, H=internal heat generation, p_a =the work of atmospheric pressure (due to planet's contraction), ϕ =energy transfers to or from external systems (e.g., tidal dissipation).

Energy components

| | Value | Units |
|--|-------------------------------|---------------------|
| Rotational energy | 2.1×10^{29} | J |
| Internal energy (for 2500 K average temperature) | 1.7×10^{31} | J |
| Gravitational energy (uniform sphere) | 2.2×10^{32} | J |
| Rotation angular velocity | 7.292×10^{-5} | rad s ⁻¹ |
| Polar moment of inertia | 8.036×10^{37} | kg m ² |
| Total mass | 5.974×10^{24} | kg |
| Total volume | 1.08×10^{21} | m ³ |
| Mass mantle | $\approx 4.0 \times 10^{24}$ | kg |
| Mass crust | $\approx 2.8 \times 10^{22}$ | kg |
| Mass core | $\approx 1.95 \times 10^{24}$ | kg |

Energy components

E_g = energy required to bring matter to infinity

For a sphere with uniform density and same mass of the Earth:

$$E_g = - \int_0^R \rho(r)g(r)r4\pi r^2 dr \qquad E_g = -\frac{3}{5} \frac{GM^2}{R}$$

- For the Earth E_g is negative because the accretion process releases energy
- E_g changes, due to thermal contraction, chemical differentiations, and vertical variations of the Earth's surface are compensated by strain energy (E_s , the energy required to compress matter to its present local pressure) changes.

Kinetic energy

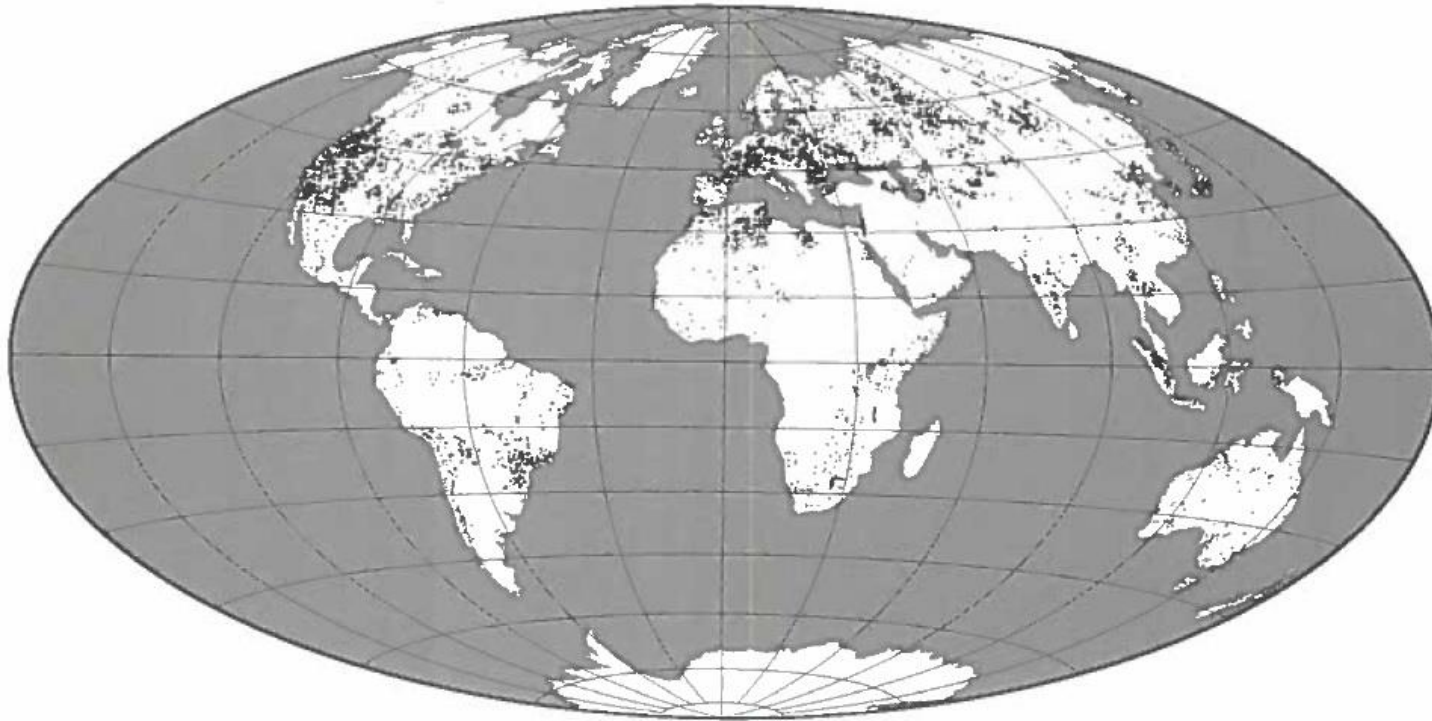
$$E_k = E_{\text{rot}} + E_{\text{contr}} + E_{\text{conv}}$$

(E_{contr} and E_{conv} are negligible)

E_{rot} = energy due to Earth's rotation, E_{contr} = energy due to radial contraction, E_{conv} = energy due to internal convective motions

Heat loss through continents

- Continental heat flux data are uneven distributed (most of data are in the north hemisphere) and the mean is 80mWm^{-2} .
- Weighing the heat flux data by area sampling, bias are removed and the mean becomes $\sim 65\text{ mWm}^{-2}$
- The contribution of all the continental areas ($210 \times 10^6\text{ km}^2$) to the energy loss of the Earth is $\sim 14\text{TW}$.



| | $\mu(Q)$ mW m^{-2} | $\sigma(Q)$ mW m^{-2} | $N(Q)$ |
|-----------------------------------|--------------------------------|-----------------------------------|--------|
| World | | | |
| all values | 79.7 | 162 | 14123 |
| averages $1^\circ \times 1^\circ$ | 65.3 | 82.4 | 3024 |
| averages $2^\circ \times 2^\circ$ | 64.0 | 57.5 | 1562 |
| averages $3^\circ \times 3^\circ$ | 63.3 | 35.2 | 979 |
| USA | | | |
| all values | 112.4 | 288 | 4243 |
| averages $1^\circ \times 1^\circ$ | 84 | 183 | 532 |
| averages $2^\circ \times 2^\circ$ | 78.3 | 131.0 | 221 |
| averages $3^\circ \times 3^\circ$ | 73.5 | 51.7 | 128 |
| without USA | | | |
| all values | 65.7 | 40.4 | 9880 |
| averages $1^\circ \times 1^\circ$ | 61.1 | 30.6 | 2516 |
| averages $2^\circ \times 2^\circ$ | 61.6 | 31.6 | 1359 |
| averages $3^\circ \times 3^\circ$ | 61.3 | 31.3 | 889 |

μ =average heat flux

σ =standard deviation

N =number of measurements

Heat loss through continents

- Present surface heat flux data from submerged and recently active continental areas ($92 \times 10^6 \text{ km}^2$ ~45% of the continental surface) reflect the heat from the mantle in the past 200 Myr.
- In zone of extension lithospheric thinning enhance heat flux, reflecting the contribution of the transient component and mantle usually shows short wavelength variations, due to cooling of shallow magmatic bodies and/or groundwater movements.
- In compressional orogens, lithospheric thickening result in reduced temperature gradients and heat flux, but total heat production is larger.
- The average heat flux of the margins (78 mW m^{-2}), higher than in stable regions despite the thinner crust, is explained by the cooling of the stretched lithosphere and is reflected in thermal subsidence.

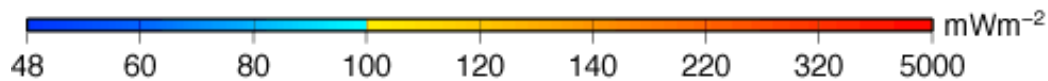
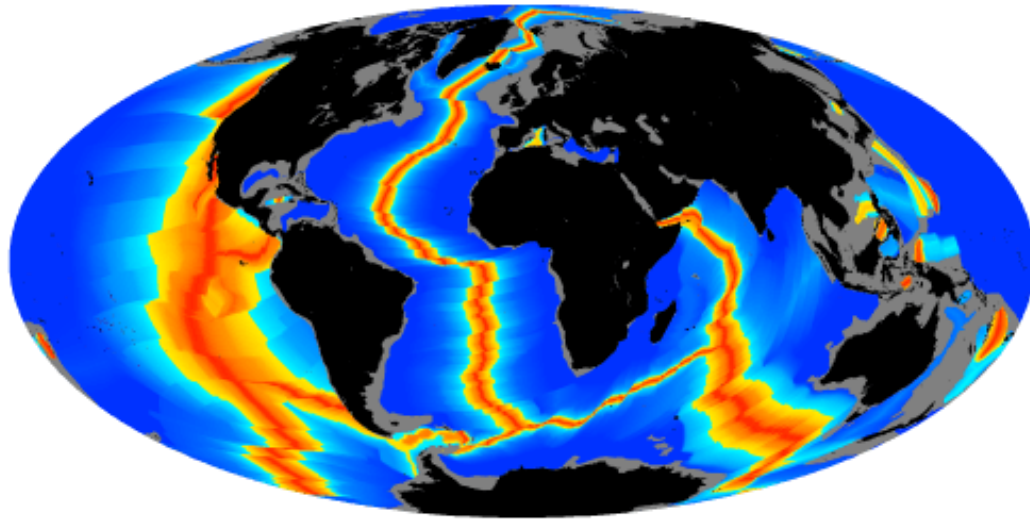
| Age group | Heat production $\mu\text{W m}^{-3}$ | Total (40 km crust) mW m^{-2} | % Area † |
|-------------------------|---|---|------------|
| Archean | 0.56–0.73 | 23–30 | 9 |
| Proterozoic | 0.73–0.90 | 30–37 | 56 |
| Phanerozoic | 0.95–1.21 | 37–47 | 35 |
| Total Continents | 0.79–0.99 | 32–40 | 100 |

† Fraction of total continental surface, from Model 2 in Rudnick and Fountain (1995).

| | Area | Total heat flux |
|--------------------------------|--|-----------------|
| Oceans | | |
| Oceanic | $273 \times 10^6 \text{ km}^2$ | |
| Marginal basins | $27 \times 10^6 \text{ km}^2$ | |
| Total oceans | $300 \times 10^6 \text{ km}^2$ | 32 TW |
| Continents | | |
| Precambrian | $95 \times 10^6 \text{ km}^2$ | |
| Paleozoic | $23 \times 10^6 \text{ km}^2$ | |
| Stable continental | $118 \times 10^6 \text{ km}^2$ | |
| Active continental | $30 \times 10^6 \text{ km}^2$ | |
| Submerged (margins and basins) | $62 \times 10^6 \text{ km}^2$ | |
| Total continental | $210 \times 10^6 \text{ km}^2$ | 14 TW |

Oceanic heat flux

To reduce/cancel errors measurements heat flux values are binned in age intervals of 2Myr



Surface heat flux (Q) decreases with age (τ):

$$Q(\tau) = C_Q \tau^{-1/2}$$

$$Q(\tau) = \lambda T_M / \sqrt{\pi \kappa \tau}$$

For constant properties

For age > 80Myr heat flux is almost constant: $q_{80} \sim 48 \text{ mWm}^{-2}$

C_Q = constant valid for arbitrary temperature-dependent physical properties

T_M = Potential T of oceanic upper mantle

Potential temperature of the oceanic upper mantle

| $T, ^\circ\text{C}$ | Reference | Method |
|---------------------|-------------------------------|--------------------------------------|
| 1315 | McKenzie <i>et al.</i> (2005) | Depth + heat flux with cooling model |
| 1280 | McKenzie and Bickle (1988) | Average basalt composition |
| 1315–1475 | Kinzler and Grove (1992) | Basalt composition |

Heat flow measurements on the sea floor are done (1) by dropping a probe (consisting of 15 m long shaft fitted with thermistors), penetrating the soft sediments of sea floor and measuring T ; (2) deep-sea drill-holes (only in a small number of sites).

Oceanic heat loss

$$Q_0 = \int_0^{\tau_{\max}} q(\tau) \frac{dA}{d\tau} d\tau, \quad \frac{dA_1}{d\tau} = C_A (1 - \tau/\tau_m)$$

$A(\tau)$ = distribution of sea floor with age (obtained from maps of magnetic anomalies)

$\tau_m = 180$ Myr

C_A (plate accretion rate) can be different if marginal basins are included or not in the calculations. Marginal basins represent 3% of the oceanic surface and cause an uncertainty on the heat loss estimate of $\sim 1\%$.

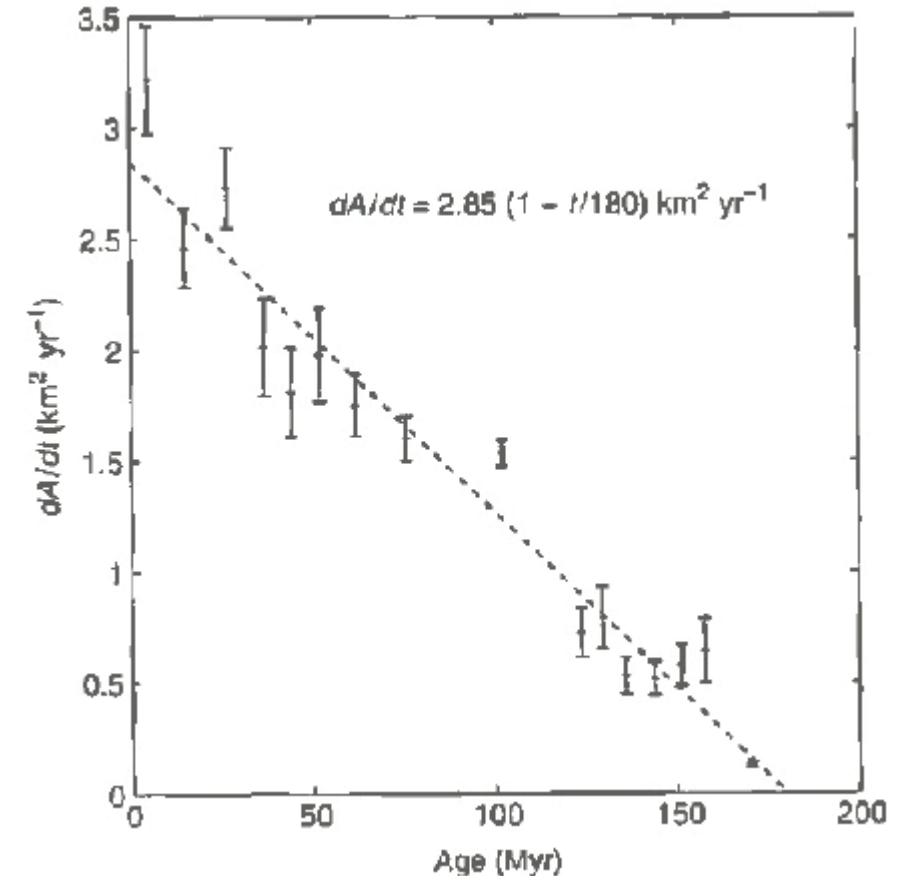
Estimates of the continental and oceanic heat flux and global heat loss

| | Continental mW m^{-2} | Oceanic mW m^{-2} | Total TW |
|--------------------------------|-----------------------------------|-------------------------------|-------------|
| Williams and von Herzen (1974) | 61 | 93 | 43 |
| Davies (1980) | 55 | 95 | 41 |
| Sclater <i>et al.</i> (1980a) | 57 | 99 | 42 |
| *Pollack <i>et al.</i> (1993) | 65 | 101 | 44 |
| Jaupart <i>et al.</i> (2007)† | 65 | 94 | 46 |

† The average oceanic heat flux does not include the contribution of hot spots. The total heat loss estimate does include 3 TW from oceanic hot spots.

*These estimates were based on $T_M = 1725$ K and $C_Q = 510 \text{ mW m}^{-2} \text{ My}^{1/2}$

Distribution of sea floor area with age



Oceanic area (e.g., marginal basins): $257 \times 10^6 \text{ km}^2$

Oceanic heat loss

$$Q_0 = \int_0^{\tau_{\max}} q(\tau) \frac{dA}{d\tau} d\tau, \quad \frac{dA}{d\tau} = C_A (1 - \tau/\tau_m)$$

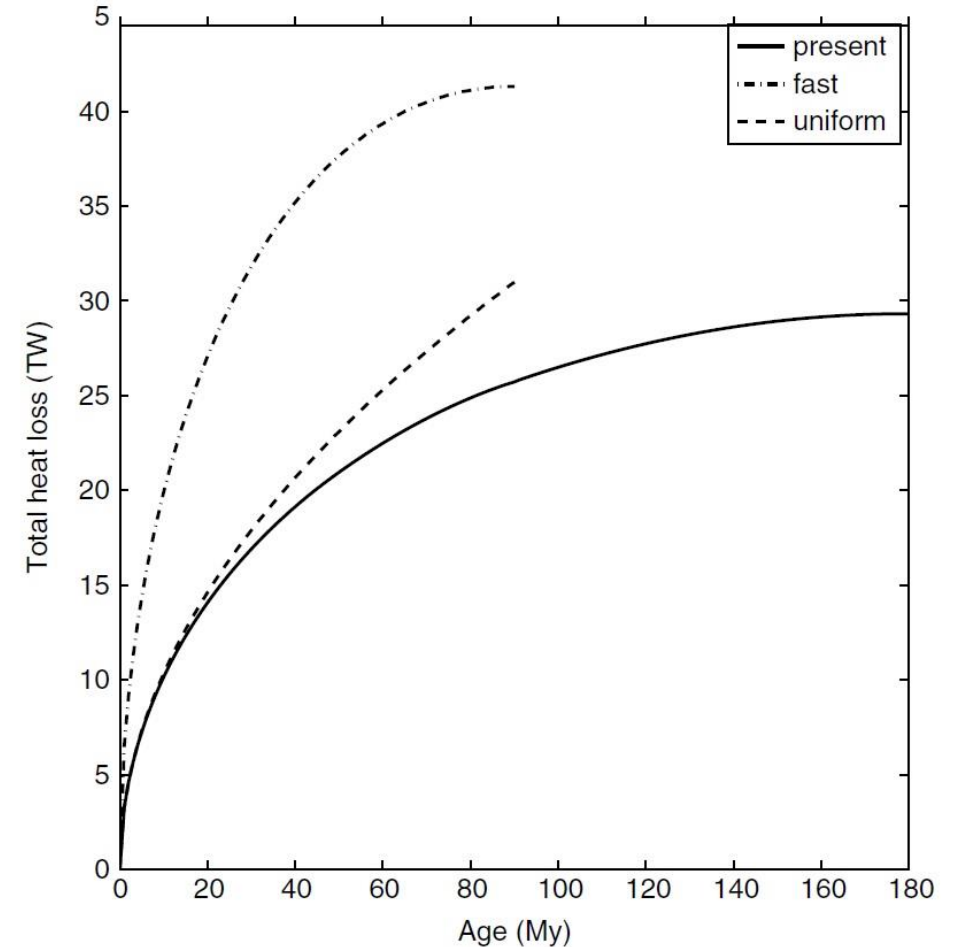
$$Q(\tau) = C_Q \tau^{-1/2}$$

Integrating sea floor younger and older than 80 Myr:

$$Q_{80-} = \int_0^{80} C_Q \tau^{-1/2} C_A (1 - \tau/180) d\tau = 24.3 \text{ TW}$$

$$Q_{80+} = q_{80} \int_{80}^{180} C_A (1 - \tau/180) dt' = 4.4 \text{ TW}$$

$$Q_{\text{oceans}} = 29 \pm 1 \text{ TW}$$

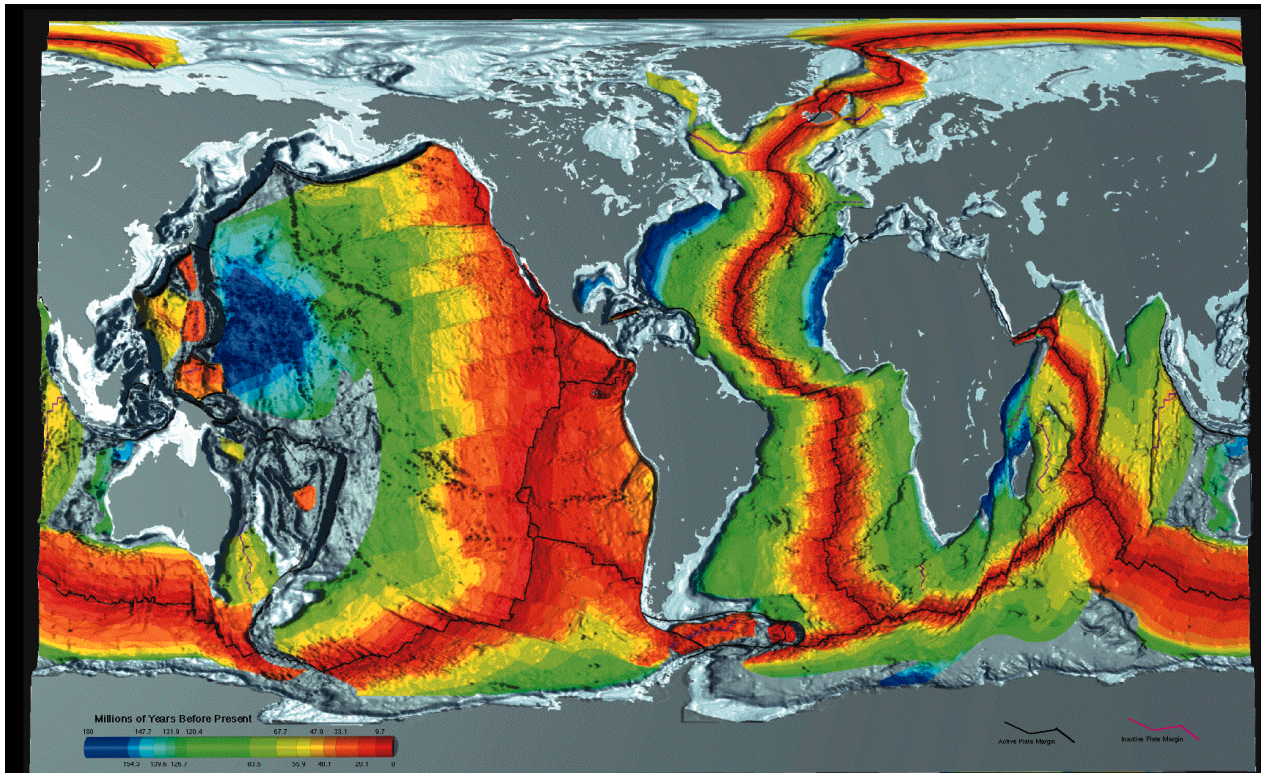


Fast=Twice the present spreading rates and oldest sea floor 90 Myr.
Uniform=uniform distribution of ages from 0 to 90 Myr with the same spreading rate as present.

- Oceanic heat loss depends on the distribution of sea-floor ages (rate of sea-floor creation/destruction)
- Additional contribution of hot spots (areas of enhanced heat flux) to the heat loss are not included (2-4 TW)

Oceanic heat loss

- Changes of oceanic heat loss can occur when a new ridge forms (it enhances heat loss, due to an increase of area of young sea floor), when a new subduction zone appears (it reduces heat loss) and, more in general, when the spreading rates change.
- The Pacific ocean alone accounts for almost 50% of the oceanic total and 34% of the global heat loss of the planet, due to its high spreading rate. The Atlantic ocean started to open 180 Myr and its heat loss is ~ 6 TW, 17% of the oceanic total.
- Using the half-space cooling model, the distribution of the sea-floor age f is a function of the ratio τ/τ_m .



$$\frac{dA}{d\tau} = C_A f\left(\frac{\tau}{\tau_m}\right)$$

$$Q_{oc} = A_o \frac{\lambda T_M}{\sqrt{\pi \kappa \tau_m}} \gamma(f)$$

A_o =total ocean surface, C_A =plate accretion rate, τ =time, τ_m =maximum age, $\gamma(f)$ =coefficient depending on the dimensionless age distribution.

$$Q_{oc}^T = \frac{8A_o}{3} \frac{\lambda T_M}{\sqrt{\pi \kappa \tau_m}}$$

$$Q_{oc}^R = 2A_o \frac{\lambda T_M}{\sqrt{\pi \kappa \tau_m}}$$

Triangular age distribution

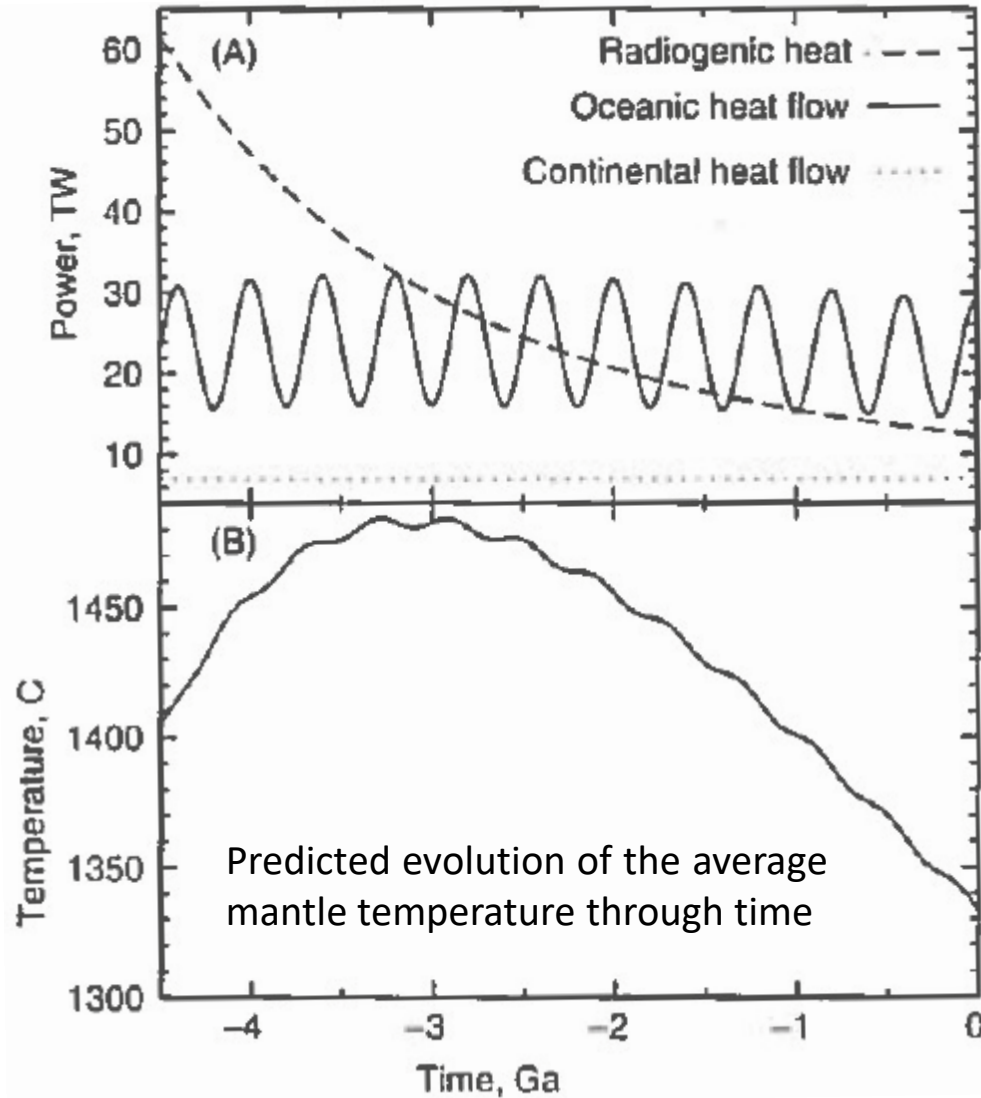
Rectangular age distribution

Heat loss increases by 30%

- Changing that sea-floor age distribution can change the oceanic heat loss. The sea-floor spreading seems to proceed with a rectangular age distribution in ocean basins that do not exceed a certain size (younger than 110 Myr): In the Atlantic Ocean the age distribution is rectangular up to 80 Myr, and then the area per unit age decreases for older age, while the Pacific ocean shows no simple age distribution.

Oceanic heat loss

The evolution of the oceanic plates leads to significant short-term variations of heat loss and mantle temperatures



$$Q_{oc} = A_o \frac{\lambda T_M}{\sqrt{\pi \kappa \tau_m}} \gamma(f)$$

τ_c = time scale for the cooling of the Earth through the oceans

$$\tau_c = \frac{MC_p T_M}{Q_{oc}} = \frac{MC_p \sqrt{\pi \kappa \tau_m}}{\lambda A_o \gamma(f)}$$

τ_c = time required for both temperature and heat flux to drop by a factor e if all sources are instantly suppressed and is ~ 10 Gyr.

Fluctuation of heat loss due to plate reorganizations of time scale $\tau_w \sim 400$ Myr does not affect significantly the secular cooling trend, neither the mantle temperatures.

Mantle Thermal evolution

- Over the Earth history, heat sources have decreased by a factor of ~ 4 . The decay time of bulk radiogenic heat production (the weighted average of the individual decay times of the main isotopes) is 3Gyr.
- The efficiency of the Earth's convective engine in losing the heat is given by the Urey (Ur) ratio (ratio of the rate of heat production over the rate of heat loss).

$$Ur = \frac{\int_V H dV}{\int_A \mathbf{q} \cdot \mathbf{n} dA} = \frac{H_T}{Q}$$

Q =total rate of heat loss

H_T =Total heat production rate

For $Ur=1$, mantle T remains constant

From the present-day energy budget, the cooling rate is $\sim 120 \text{ Kgyr}^{-1}$ ($< 50 \text{ Kgyr}^{-1}$): cooling rate is increased with time

$$M \langle C_p \rangle \frac{d\langle T \rangle}{dt} = -Q + H_T$$

$$\frac{Q_{av} - H_{Tav}}{Q - H_T} = \frac{(dT/dt)_{av}}{d\langle T \rangle/dt}$$

$Ur \sim 0.4$ (rate of Q varies less rapidly than that of H)

By integrating over the age of the Earth

Q_{av} = time-averaged of heat loss

H_{Tav} = time-averaged of heat production

$(dT/dt)_{av}$ = average cooling rate

$\langle C_p \rangle$ = "effective" heat capacity which accounts for the variation of temperature with depth

How the continental growth influences the cooling of the Earth

Continental growth reduces the Earth's cooling rate, since (1) it acts to reduce the amount of internal heat generation driving mantle convection (by the extraction of radioelements from the mantle), (2) it increases the size of continental domain at the expenses of the oceanic one, (3) convection cells stretch over larger horizontal distance implying a lower heat flux than with the shorter cells.

Internal temperatures that are required to evacuate the amount of heat produced in the planet by convection in a fluid with viscosity μ over thickness h , and conduction in a rigid crust enriched in radioactive elements over thickness d_c :

If the total amount of heat generation is the same in both cases, $H_{conv}h = H_{cond}d_c$, and no exchange of heat exists between the two layer, the surface heat flux is the same, $Q = H_{conv}h = H_{cond}d_c$

$$\Delta T_{conv} = C_Q^{-3/4} \left(\frac{H_{conv}h}{\lambda} \right)^{3/4} \left(\frac{\kappa\mu}{\rho_0 g \alpha} \right)^{1/4}$$

$$\Delta T_{cond} = \frac{H_{cond}d_c^2}{2\lambda}$$

$$\frac{\Delta T_{cond}}{\Delta T_{conv}} = \frac{C_Q^{3/4} d_c}{2 h} \left(\frac{\rho_0 g \alpha H_{conv} h^5}{\lambda \kappa \mu} \right)^{1/4} = \frac{C_Q^{3/4} d_c}{2 h} Ra_H^{1/4}$$

μ =viscosity

h =thickness of the convective layer

d_c =thickness of the crustal conductive layer

λ =thermal conductivity

κ =thermal diffusivity

α =thermal expansion coefficient

C_Q =constant

Ra =Reyleigh number

$$C_Q^{3/4} \sim 0.2 \quad d_c = 30 \text{ km} \quad h = 3000 \text{ km}$$

Since $Ra = 10^9$, $\Delta T_{cond}/\Delta T_{conv} \sim 0.4$, then conduction in a thin enriched crust evacuates radioactive heat with a lower internal T than by a convective layer

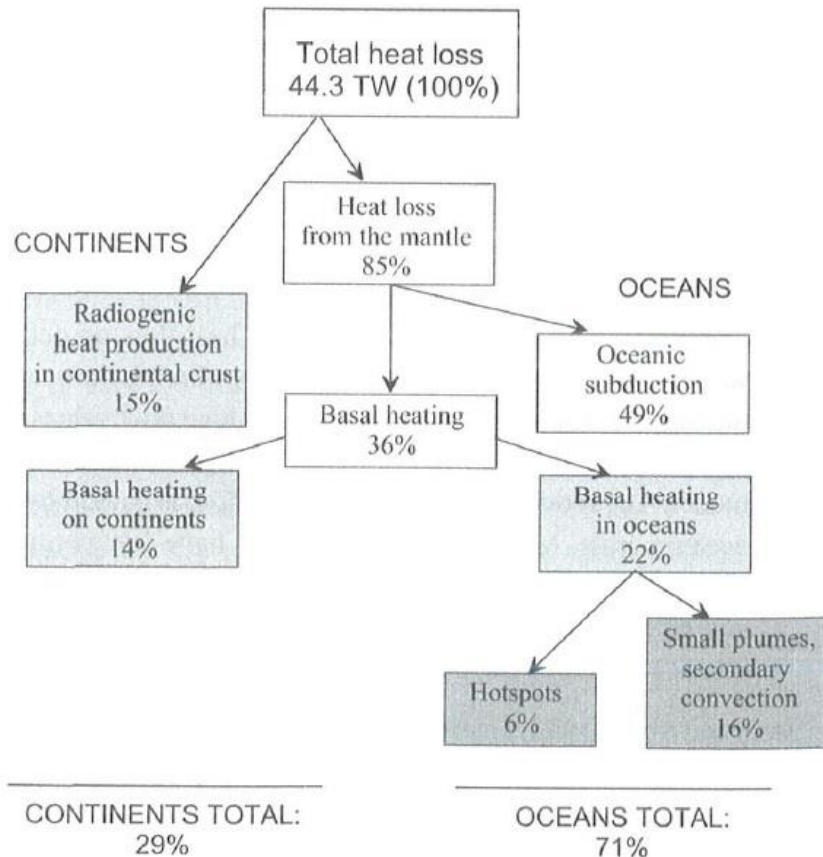
Heat loss of the Earth

Total heat loss from ocean = 32TW

Total heat loss from continents = 14 TW

Total heat loss from continental crust = 6-7 TW (resulting from a crustal range of heat production $0.79\text{-}0.99\mu\text{Wm}^{-3}$ and volume $0.73\times 10^{10}\text{ km}^3$)

Total heat loss from cont. lithospheric mantle = 0.5 TW (resulting from a mantle range of heat production 0.02 mWm^{-3} and volume $3.1\times 10^{10}\text{ km}^3$)



| Heat flow balance of the Earth | | | |
|--|---|---------------------|-------------------------------|
| Source of heat | Mean surface heat flow* (mW/m ²) | Heat loss (TW)** | Total global heat loss (%) |
| Continents (area $2.0 \times 10^8\text{ km}^2$) | 65.0 ± 1.6 | 13.0 ± 0.3 | 29.3 |
| <i>Radiogenic heat production in continental crust</i> | 34 ± 8 | 6.8 ± 1.6 | 15.3 |
| <i>Basal heating of continental lithosphere</i> | 31 ± 8 | 6.2 ± 1.6 | 14.0 |
| Oceans and marginal basins (area $3.1 \times 10^8\text{ km}^2$) | 101.0 ± 2.2 | 31.3 ± 0.7 | 70.7 |
| <i>Subduction of oceanic lithosphere</i> | 70 ± 8 | 21.7 ± 2.5 | 49.0 |
| <i>Basal heating of oceanic lithosphere</i> | 31 ± 8 | 9.6 ± 2.5 | 21.7 |
| Total global | 86.9 ± 2.0 | 44.3 ± 1.0 | 100 |

(after Malamud and Turcotte, 1999)
 * Sources: Sclater *et al.*, 1980; Pollack *et al.*, 1993
 ** 1TW= 10^{12} W

Continental heat loss has more uncertainty, due to the poor knowledge of radiogenic heat production

Heat flow out of the core

- The core loses heat to an unstable boundary layer which grows at the base of the mantle (D'' layer).
- The T difference across this 200 km thick layer is about $\delta T = 1000\text{K}$.

The energy content of the D'' layer, transferred to the mantle when the boundary layer goes unstable, is:

$$U = \rho C_p 4\pi b^2 h \frac{\delta T}{2} \simeq 7.5 \cdot 10^{28} \text{J}$$

b =radius of the core 3480 km, ρ = $\sim 5 \times 10^3 \text{kg m}^{-3}$ h =200 km C_p = $1000 \text{J kg}^{-1} \text{K}^{-1}$

Time scale of the energy release is that of conductive thickening of the layer: $\tau = h^2 / \pi \kappa \sim 400 \text{ Myr}$, for $\kappa = 10^{-6} \text{m}^2 \text{s}^{-1}$

- Dividing the total energy in the thermal boundary layer by the time scale of 400 Myr indicates that the variations of heat input at the base of the mantle is $\sim 5 \text{TW}$.
- **Total heat flux across the CMB = 7-14 TW** (sum of secular cooling, latent heat from inner core crystallization, compositional energy due to chemical separation of the inner core, radiogenic heat generation).