Basics of radiative transport in atmospheres and Earth's bulk energy balance

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With the term “specific radiation intensity” *I(P,k)* (even called *spectral radiance* in the International System or simply *intensity* in Astrophysics or even *brightness* in old literature) coming out from an infinitesimal surface *ds* with position in space *P(x,y,z)*, we refer to the flux of the energy *dE* in a given direction *k*, across the infinitesimal surface perpendicular to the direction *r*, in an interval of time *dt,* in an infinitesimal solid angle *dω* and in an infinitesimal interval of wavelengths *dλ* (or frequency *dv*). In formulas we would have

$$dE = I(P,r)ds\, d\omega\, d\lambda dt \qquad [1]$$

Because of its definition, mathematically speaking specific radiation intensity belongs to the family of “distributions”, which encompasses in a sub-set even functions, but it is not strictly a function because it has a meaning only in conjunction with the differentials that are used for its construction. The main consequence of the fact that specific radiation intensity is a distribution is that if we want to switch over its dependence from the wavelength to its dependence from frequency, we have to take into account even the switch of the frequency differential over the wavelength. In formulas

$$I_\lambda(P,r)\,d\lambda = I_v(P,k)\frac{v^2}{c}\,dv \quad .$$

One of the interesting aspects of specific radiation intensity is that it does not depend from the distance of radiation source, but only from the position *P* and from the direction *r*. This differently form the radiation flux over the solid angle Ω (or better *density of radiation flux* over the solid angle Ω, called in the SI *emittance*), which is related to specific radiation intensity by the relationship

$$F_{\lambda,r}(P) = \int_\Omega I_\lambda(P,r)\, r\cdot j\, d\omega_j \qquad [2]$$

where the scalar product $r\cdot j$ represents the cosine of the angle between the two unitary directions *r* and *j*.

If specific radiation intensity does not depend from the position *P*, then it is called **homogeneous**, if it does not depend from the direction *r*, it is called **isotropic**.

Knowing the specific intensity of radiation of the Sun, we might calculate the flux of solar radiation over a unit surface, e.g., of the Earth, simply integrating it in this way

$$F_{\lambda,r}(P) = \int_{sun} I_\lambda(P,r)\, r\cdot j\, d\omega_j$$

If we substitute the solar specific radiation intensity with its average value over the solar disk, i.e., $I_\lambda(P,r) \sim I_{\lambda,sun}(r)$ into the above integral and, because of the distance between Earth and Sun , we consider constant the cosine $r\cdot j \sim \cos(\alpha)$ remembering that this cosine is equal to the sin of the solar declination, i.e., $\cos(\alpha) = \sin(\delta)$, we would have

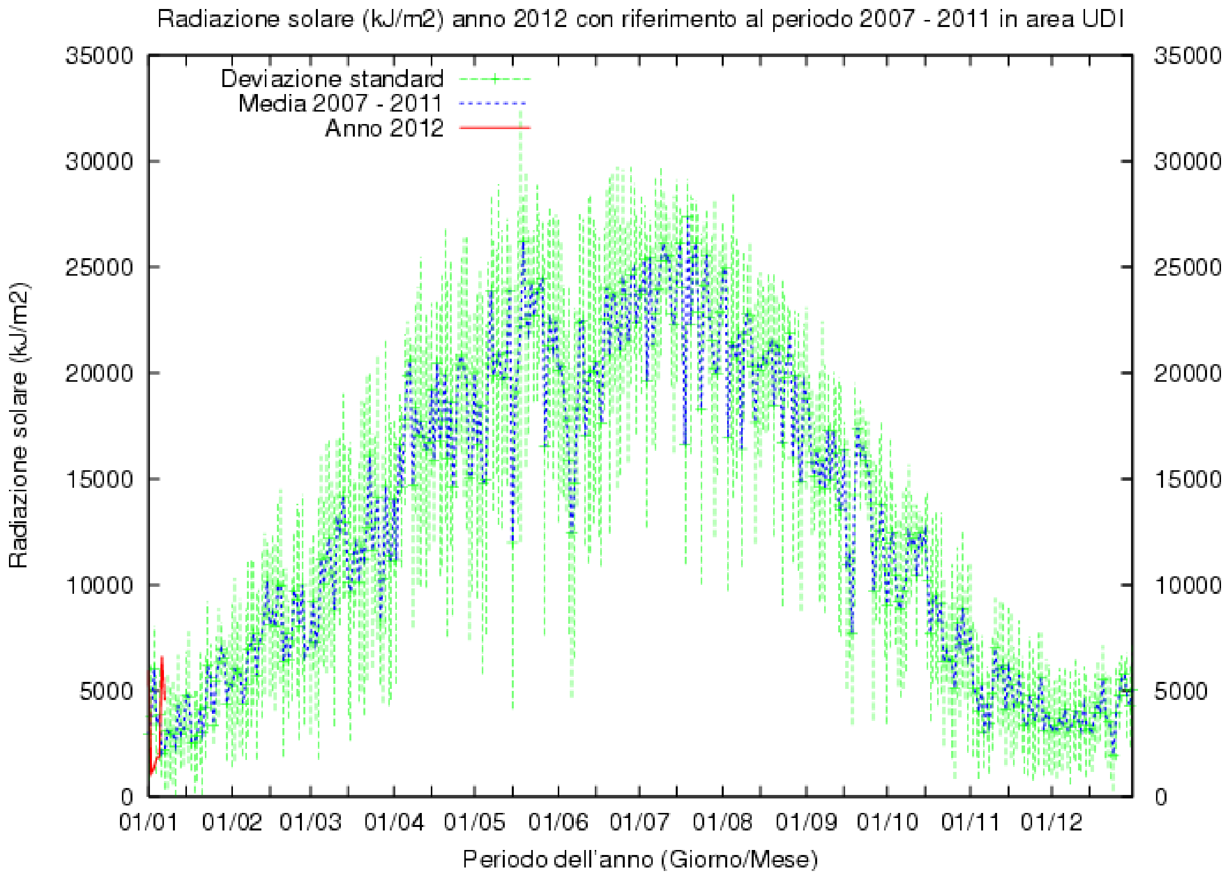
$$F_{\lambda,r}(P) \sim \int_{sun} I_\lambda(P,r)\, r\cdot j\, d\omega_j = I_{\lambda,sun} \sin(\delta) \int_{sun} d\omega_j$$

then

$$F_{\lambda,r}(P) \sim I_{\lambda,sun} \sin(\delta) \int_{sun} d\omega_j I = \sin(\delta) I_{\lambda,sun} \Delta\Omega_{sun}$$

where $\Delta\Omega_{sun}$ is the apparent angular extension, that is the solid angle covered by the sun disk (the apparent diameter of the sun is roughly half a degree).

The quantity $I_{\lambda,sun} \Delta\Omega_{sun}$, when integrated over all the wavelengths, then taking into account the contribution of the whole solar spectrum, is the "solar constant", which is not constant, indeed, because it changes according to the variations of $I_{\lambda,sun}$ due to the solar variability and according to the variations of $\Delta\Omega_{sun}$, which are mainly related to astronomical factors like the eccentricity variation of Earth's orbit discovered by Milankovich. The current average value of the "solar constant" is of the order of 1373 Wm^{-2}.



Blue dashed line represents the five years (2007-2011) average time series of the daily solar radiation over Udine (roughly LAT. 46 N), green shadow represents the above five years average daily standard deviation.

Another important parameter for the definition of the radiative budget is represented by energy density of the radiation field. To evaluate the energy density we have to compute the amount of energy contained into an infinitesimal cylinder parallel to the direction k with length $dr = c\ dt$, where dz is the distance run by a photon in the time interval dt. Since the amount of energy that crosses the surface ds is given by equation [1], we will have that the amount of energy contained into the cylinder will be

$$dE = I(P,r)\, ds\, d\omega\, d\lambda dt \frac{c}{c}$$

then, dividing by the infinitesimal volume $ds\, c\, dt$, we will obtain the energy density

$$du = \frac{I(P,r)}{c} d\omega d\lambda$$

If we remember that, according to the definition [2], the flux of radiation is represented by

$$F_{\lambda,r}(P) = \int_{\Omega} I_{\lambda}(P,r) r\cdot j\, d\omega_j$$

the convergence (negative of divergence) of this flux will represent the heating h_λ of the volume (then the cooling of the matter), which in formulas become

$$-\nabla F_{\lambda} = \frac{\partial F_{\lambda,x}}{\partial x} + \frac{\partial F_{\lambda,y}}{\partial y} + \frac{\partial F_{\lambda,z}}{\partial z} = h_{\lambda}$$

then, thanks to equation [2], we will have

$$-\nabla F_{\lambda} = \frac{\partial \int_{\Omega} I_{\lambda}(P,r) r\cdot x\, d\omega_x}{\partial x} + \frac{\partial \int_{\Omega} I_{\lambda}(P,r) r\cdot y\, d\omega_y}{\partial y} + \frac{\partial \int_{\Omega} I_{\lambda}(P,r) r\cdot z\, d\omega_z}{\partial z} = h_{\lambda}$$

then

$$-\nabla F_{\lambda} = \int_{\Omega} \left(r\cdot x \frac{\partial I_{\lambda}(P,r)}{\partial x} + r\cdot y \frac{\partial I_{\lambda}(P,r)}{\partial y} + r\cdot z \frac{\partial I_{\lambda}(P,r)}{\partial z}\right) d\omega_r = h_{\lambda}$$

then, remembering the definition of gradient in Cartesian coordinates, we can write

$$-\nabla F_{\lambda} = \int_{\Omega} (r\cdot\nabla I_{\lambda}(P,r))\, d\omega_r = h_{\lambda} \qquad [3]$$

if heating, i.e., the convergence of flux is null, then the situation is called of (monochromatic) radiative equilibrium.

Under the mathematical point of view, the term $r\cdot\nabla I_{\lambda}$ represents the projection of the intensity gradient on the *r* direction (remember that *r* is a versor), then it is the directional derivative, which sometimes is represented in this way

$$r\cdot\nabla I_{\lambda} = \frac{d I_{\lambda}}{d r} \qquad [4]$$

Before to treat the aspects of extension and emission, it is important to spend a few more words on the suffix λ in the above formulas, which ~~is~~ refers them to a specific wavelength. If we want to consider the whole radiative budget, we have to integrate on the whole spectrum (then on *dλ* or *dv*) the above expressions. This is relevant because almost always atmospheres interact differently with different parts of radiation spectrum

Extinction and emission

When radiation crosses a portion of matter (say, the atmosphere) changes its properties. All their changes can be divided in two classes: those which correspond to a reduction of intensity in the direction of propagation (called "***extinctions***") and those which correspond to an increase in the intensity of radiation in the direction of propagation (called "***emissions***").

Dealing with extinctions and emissions, the fundamental approximation, which holds for almost all the relevant atmospheric processes, is that called with the name of the two pioneers that dealt with this problem, i.e., Lamberts and Bouguet. This approximation says that the amount of extinction and emission are linear, i.e., proportional to the radiation intensity, with $e_{V,\lambda}$ the specific (for the specific wavelength) ***emission coefficient*** per unit volume *V* and $-e_{V,\lambda}$ the specific extinction coefficient per unit volume *V* provided that the state of the medium (i.e., pressure, temperature and composition) is kept constant. Then, according to the Lamberts-Bouguet approximation, we will write that the amount of extinct intensity ΔI_λ along to the path Δr is given by

$$\Delta I_\lambda = -e_{V,\lambda} I_\lambda \Delta r$$

while the increase in the intensity ΔI_λ along the same path Δr is given by

$$\Delta I_\lambda = e_{V,\lambda} J_\lambda \Delta r$$

Merging the two terms, we will have

$$\Delta I_\lambda = -e_{V,\lambda} I_\lambda \Delta r + e_{V,\lambda} J_\lambda \Delta r$$

then, moving to the limit when Δr tends to zero, we will have

$$\frac{d I_\lambda}{d r} = -e_{V,\lambda}(I_\lambda - J_\lambda) \qquad [5]$$

which, remembering equation [4] can even be written as

$$r \cdot \nabla I_\lambda = -e_{V,\lambda}(I_\lambda - J_\lambda) \quad .$$

Equation [5] is called the ***equation of transfer*** of ***Schwazschild equation*** and it is the fundamental mathematical tool to deal with radiation problems. The physical content of the equation of transfer is mainly related to the extinction/emission factor $e_{V,\lambda}$ and to the source function $J(\lambda)$ which, in the approximation of thermal emission, is often approximated by the Plank's distribution.

If we remember equation [3], we can substitute in it the equation of transfer, obtaining

$$h_\lambda = \int_\Omega (r \cdot \nabla I_\lambda(P,r))\, d\omega_r = -e_{V,\lambda} \int_\Omega (I_\lambda - J_\lambda)\, d\omega_r$$

or, in a much more compact form, defining $4\pi(\bar{I}_\lambda - \bar{J}_\lambda) = \int_\Omega (I_\lambda - J_\lambda)\, d\omega_r$, we will have

$$h_\lambda = -e_{V,\lambda} 4\pi (\bar{I}_\lambda - \bar{J}_\lambda)$$

This expression is the base to explain the so called greenhouse mechanism[(a)]. In fact, heating springs

(a) Some people considers that the expression "greenhouse mechanism" is not epistemologically correct, because the mechanism why greenhouses remain warmer than the surroundings is because they damp the convective transport of heating and they only barely interact under the radiative point of view.

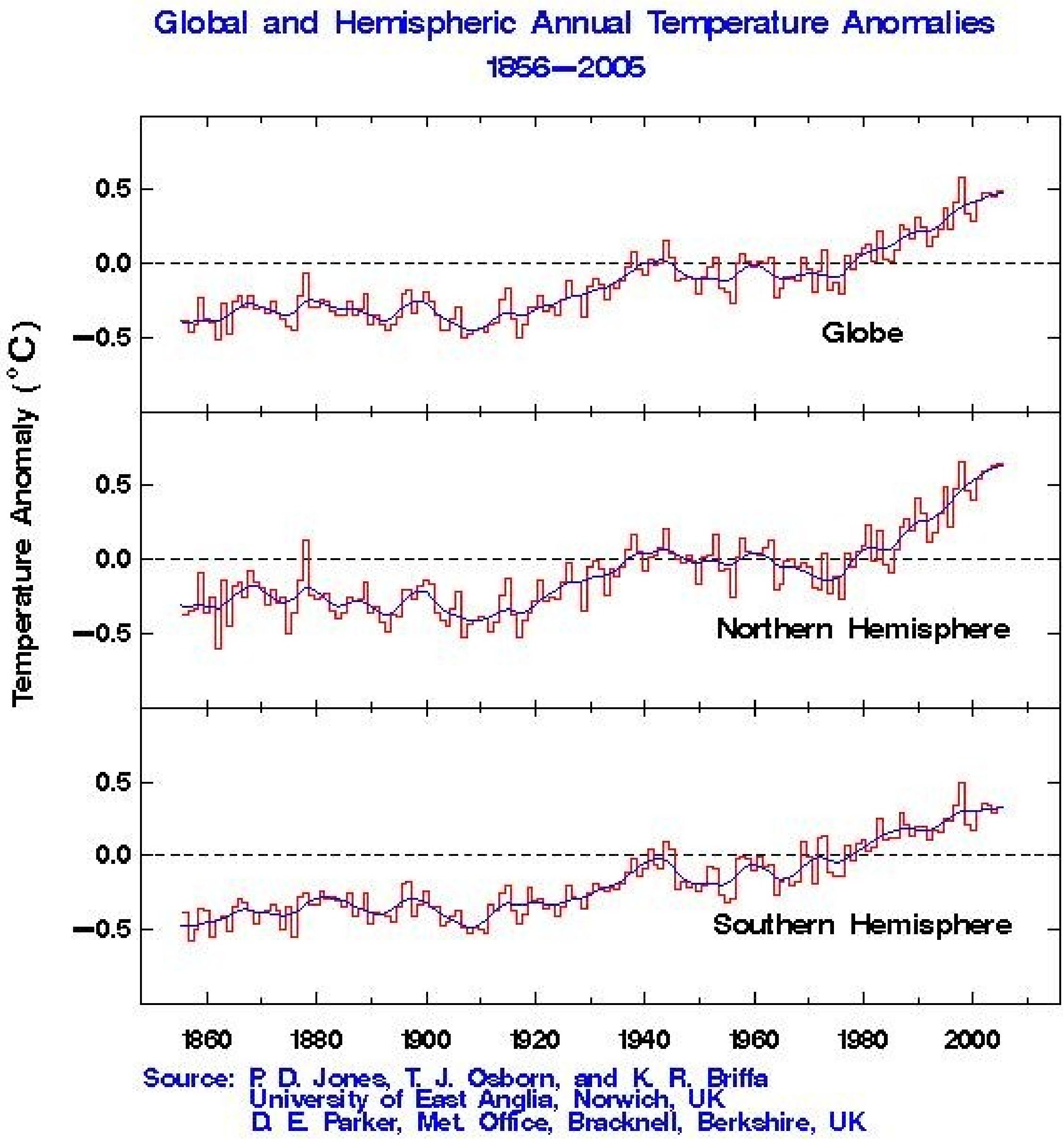
out from a difference between the monochromatic extinction and emission. Under this point of view, in fact, Earth's atmosphere has an almost null extinction on the visible band (say among 300 and 700 nm), while it is strongly emitting in the infrared band (say above 700 nm) mainly because of the presence of water vapour and even for the presence of carbon dioxide and of the other greenhouse gases (methane, CH4 and dinitrogen oxide, N2O).

Scattering mechanism

An important mechanism present in atmosphere is that related to the interactions of radiation with matter that do not change the amount of energy in the radiation field. If there is no net change of energy, the interaction is defined as "***scattering***". If the amount of energy absorbed by matter is sudden re-emitted at the same wavelength (frequency), then the scattering is called ***coherent***, if there is a cascade of energy at different wavelengths, the scattering is called ***incoherent***. Even if in the scattering mechanism the amount of energy absorbed and released is the same, the direction in which this radiation is re-emitted is different from the previous one, for this reason a scattering mechanism corresponds often to an extinction of the radiation field as soon as it penetrates into the atmosphere. This is, for example, what happens in the cumulonimbus clouds, which are particularly white if seen from above, but extremely dark is seen from below.

Thermal absorption

In the scattering mechanism, energy is absorbed by matter, but even sudden re-emitted. Because there is no interaction between the matter constituents (molecules) during the interval of time in which radiation interacts with a single molecule, this process is essentially related to the single molecule. But what happens if the amount of time during which the interaction between radiation and matter is enough large (or the interactions between matter constituents are enough frequent) to admit contemporary interactions between molecules?. In this case the amount of radiant energy absorbed by a single molecule can pass to another molecule and so on and so forth before than it might be re-emitted and then radiation is distributed among molecules. This is what happens during thermal absorption.



Planetary and emispheric average temperatures. It is interesting to notice the stationary or slightly decrease of temperatures of the northern emisphere in the 60es and 70es.

References

Goody R. M, and Yung Y. L., 1989. *Atmospheric Radiation, theoretical basis (Second Edition)*. Oxford University Press, 519 pp.